Package ‘Umoments’

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Type Package

Title Unbiased Central Moment Estimates

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Description Calculates one-sample unbiased central moment estimates and two-sample pooled estimates up to 6th order, including estimates of powers and products of central moments. Provides the machinery for obtaining unbiased central moment estimators beyond 6th order by generating expressions for expectations of raw sample moments and their powers and products.
Gerlovina and Hubbard (2019) <doi:10.1080/25742558.2019.1701917>.

Depends R (>= 3.4.0)

Imports stats, utils

License GPL (>= 2)

Encoding UTF-8

RoxygenNote 7.3.1

Suggests knitr, rmarkdown, testthat

VignetteBuilder knitr

NeedsCompilation no

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Contents

`one_combination` *Generate symbolic expression for expectation*

Description

Generate a string with symbolic expression for expectation of powers and products of non-central (raw) sample moments of an arbitrary order.

Usage

```
one_combination(powvect, smpsize = "n")
```

Arguments

`powvect`	vector of non-negative integers representing exponents $j_1, \ldots, j_m$ of non-central moments in expectation (see "Details"). The position (index) of an element of this vector indicates a corresponding moment, e.g. for $E(\overline{X}^5 \overline{X^4})$, `powvect = c(5, 0, 0, 1)`. Thus the vector will have `m` elements if `m`'th is the highest moment.
`smpsize`	symbol to be used for sample size. Defaults to `"n"`.

Details

For a zero-mean random variable `X` and a sample $X_1, \ldots, X_n$, find $E(\bar{X}^{j_1} \overline{X^2}^{j_2} \overline{X^3}^{j_3} \cdots \overline{X^m}^{j_m})$, where $overline X^k = 1/n \sum_{i=1}^{n} X_i^k$ is a k'th non-central sample moment. The expression is given in terms of sample size and true moments μ_k of X. These expectations can subsequently be used for generating unbiased central moment estimators of an arbitrary order, Edgeworth expansions, and possibly solving other higher-order problems.

Value

A string representing a symbolic expression for further processing using computer algebra (e.g. with *Sage* or *SymPy*), for calculating numeric values, or to be rendered with *Latex*.

Examples

```
one_combination(c(5, 0, 2, 1))
```

---

uM *Unbiased central moment estimates*

Description

Calculate unbiased estimates of central moments and their powers and products up to specified order.

Usage

```
uM(smp, order)
```

Arguments

`smp`	sample.
`order`	highest order of the estimates to calclulate. Estimates of lower orders will be included.

Details

Unbiased estimates up to the 6th order can be calculated. Second and third orders contain estimates of the variance and third central moment, fourth order includes estimates of fourth moment and squared variance (μ_2^2), fifth order - of fifth moment and a product of second and third moments ($\mu_2\mu_3$), sixth order - of sixth moment, a product of second and fourth moments ($\mu_2\mu_4$), squared third moment (μ_3^2), and cubed variance (μ_2^3).

Value

A named vector of estimates of central moments and their powers and products up to `order`. The highest order available is 6th. The names of the elements are `"M2", "M3", "M4", "M5", "M6"` for corresponding central moments, `"M2M3", "M2M4"` for products of the moments (second and third, second and fourth), and `"M2pow2", "M2pow3", "M3pow2"` for powers of the moments - corresponding to estimates of squared variance, cubed variance, and squared third moment.

References

Gerlovina, I. and Hubbard, A.E. (2019). *Computer algebra and algorithms for unbiased moment estimation of arbitrary order.* Cogent Mathematics & Statistics, 6(1).

See Also

uMpool for two-sample pooled estimates.

Examples

```
smp <- rgamma(10, shape = 3)
uM(smp, 6)
```

---

uM2 *Unbiased central moment estimates*

Description

Calculate unbiased estimates of central moments and their powers and products.

Usage

```
uM2(m2, n)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^2$ for a vector X.
n	sample size.

Value

Unbiased variance estimate.

See Also

Other unbiased estimates (one-sample): uM2M3(), uM2M4(), uM2pow2(), uM2pow3(), uM3(), uM3pow2(), uM4(), uM5(), uM6()

Examples

```
n <- 10
smp <- rgamma(n, shape = 3)
m <- mean(smp)
m <- c(m, mean((smp - m[1])^2))
uM2(m[2], n) - var(smp)
```

uM2M3 *Unbiased central moment estimates*

Description

Calculate unbiased estimates of central moments and their powers and products.

Usage

```
uM2M3(m2, m3, m5, n)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/n \sum_{i=1}^n ((X_i - \bar{X})^2$ for a vector X.
m3	naive biased third central moment estimate $m_3 = 1/n \sum_{i=1}^n ((X_i - \bar{X})^3$ for a vector X.
m5	naive biased fifth central moment estimate $m_5 = \sum_{i=1}^n ((X_i - \bar{X})^5$ for a vector X.
n	sample size.

Value

Unbiased estimate of a product of second and third central moments $\mu_2 \mu_3$, where μ_2 and μ_3 are second and third central moments respectively.

See Also

Other unbiased estimates (one-sample): uM2(), uM2M4(), uM2pow2(), uM2pow3(), uM3(), uM3pow2(), uM4(), uM5(), uM6()

Examples

```
n <- 10
smp <- rgamma(n, shape = 3)
m <- mean(smp)
for (j in 2:5) {
  m <- c(m, mean((smp - m[1])^j))
}
uM2M3(m[2], m[3], m[5], n)
```

uM2M3pool *Pooled central moment estimates - two-sample*

Description

Calculate pooled unbiased estimates of central moments and their powers and products.

Usage

```
uM2M3pool(m2, m3, m5, n_x, n_y)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^2+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^2$ for vectors X and Y.
m3	naive biased third central moment estimate $m_3 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^3+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^3$ for vectors X and Y.
m5	naive biased fifth central moment estimate $m_5 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^5+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^5$ for vectors X and Y.
n_x	number of observations in the first group.
n_y	number of observations in the second group.

Value

Pooled estimate of a product of second and third central moments $\mu_2\mu_3$, where μ_2 and μ_3 are second and third central moments respectively.

See Also

Other pooled estimates (two-sample): uM2M4pool(), uM2pool(), uM2pow2pool(), uM2pow3pool(), uM3pool(), uM3pow2pool(), uM4pool(), uM5pool(), uM6pool()

Examples

```
nx <- 10
ny <- 8
shp <- 3
smpx <- rgamma(nx, shape = shp) - shp
smpy <- rgamma(ny, shape = shp)
mx <- mean(smpx)
my <- mean(smpy)
m  <- numeric(5)
for (j in 2:5) {
  m[j] <- mean(c((smpx - mx)^j, (smpy - my)^j))
}
uM2M3pool(m[2], m[3], m[5], nx, ny)
```

uM2M4 — *Unbiased central moment estimates*

Description

Calculate unbiased estimates of central moments and their powers and products.

Usage

```
uM2M4(m2, m3, m4, m6, n)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^2$ for a vector X.
m3	naive biased third central moment estimate $m_3 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^3$ for a vector X.
m4	naive biased fourth central moment estimate $m_4 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^4$ for a vector X.
m6	naive biased sixth central moment estimate $m_6 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^6$ for a vector X.
n	sample size.

Value

Unbiased estimate of a product of second and fourth central moments $\mu_2\mu_4$, where μ_2 and μ_4 are second and fourth central moments respectively.

See Also

Other unbiased estimates (one-sample): uM2(), uM2M3(), uM2pow2(), uM2pow3(), uM3(), uM3pow2(), uM4(), uM5(), uM6()

Examples

```
n <- 10
smp <- rgamma(n, shape = 3)
m <- mean(smp)
for (j in 2:6) {
  m <- c(m, mean((smp - m[1])^j))
}
uM2M4(m[2], m[3], m[4], m[6], n)
```

uM2M4pool *Pooled central moment estimates - two-sample*

Description

Calculate pooled unbiased estimates of central moments and their powers and products.

Usage

```
uM2M4pool(m2, m3, m4, m6, n_x, n_y)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^2+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^2$ for vectors X and Y.
m3	naive biased third central moment estimate $m_3 = 1/(n_x + n_y)\sum_{i=1}^{n_x}((X_i - \bar{X})^3 + \sum_{i=1}^{n_y}((Y_i - \bar{Y})^3$ for vectors X and Y.
m4	naive biased fourth central moment estimate $m_4 = 1/(n_x + n_y)\sum_{i=1}^{n_x}((X_i - \bar{X})^4 + \sum_{i=1}^{n_y}((Y_i - \bar{Y})^4$ for vectors X and Y.
m6	naive biased sixth central moment estimate $m_6 = 1/(n_x + n_y)\sum_{i=1}^{n_x}((X_i - \bar{X})^6 + \sum_{i=1}^{n_y}((Y_i - \bar{Y})^6$ for vectors X and Y.
n_x	number of observations in the first group.
n_y	number of observations in the second group.

Value

Pooled estimate of a product of second and fourth central moments $\mu_2\mu_4$, where μ_2 and μ_4 are second and fourth central moments respectively.

See Also

Other pooled estimates (two-sample): uM2M3pool(), uM2pool(), uM2pow2pool(), uM2pow3pool(), uM3pool(), uM3pow2pool(), uM4pool(), uM5pool(), uM6pool()

Examples

```
nx <- 10
ny <- 8
shp <- 3
smpx <- rgamma(nx, shape = shp) - shp
smpy <- rgamma(ny, shape = shp)
mx <- mean(smpx)
my <- mean(smpy)
m  <- numeric(6)
for (j in 2:6) {
  m[j] <- mean(c((smpx - mx)^j, (smpy - my)^j))
}
uM2M4pool(m[2], m[3], m[4], m[6], nx, ny)
```

uM2pool *Pooled central moment estimates - two-sample*

Description

Calculate pooled unbiased estimates of central moments and their powers and products.

Usage

```
uM2pool(m2, n_x, n_y)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/(n_x + n_y) \sum_{i=1}^{n_x}((X_i - \bar{X})^2 + \sum_{i=1}^{n_y}((Y_i - \bar{Y})^2$ for vectors X and Y.
n_x	number of observations in the first group.
n_y	number of observations in the second group.

Value

Pooled variance estimate.

See Also

Other pooled estimates (two-sample): uM2M3pool(), uM2M4pool(), uM2pow2pool(), uM2pow3pool(), uM3pool(), uM3pow2pool(), uM4pool(), uM5pool(), uM6pool()

Examples

```
nx <- 10
ny <- 8
shp <- 3
smpx <- rgamma(nx, shape = shp) - shp
smpy <- rgamma(ny, shape = shp)
m2 <- mean(c((smpx - mean(smpx))^2, (smpy - mean(smpy))^2))
uM2pool(m2, nx, ny)
```

uM2pow2 — *Unbiased central moment estimates*

Description

Calculate unbiased estimates of central moments and their powers and products.

Usage

```
uM2pow2(m2, m4, n)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^2$ for a vector X.
m4	naive biased fourth central moment estimate $m_4 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^4$ for a vector X.
n	sample size.

Value

Unbiased estimate of squared variance μ_2^2, where μ_2 is a variance.

See Also

Other unbiased estimates (one-sample): uM2(), uM2M3(), uM2M4(), uM2pow3(), uM3(), uM3pow2(), uM4(), uM5(), uM6()

Examples

```
n <- 10
smp <- rgamma(n, shape = 3)
m <- mean(smp)
for (j in 2:4) {
  m <- c(m, mean((smp - m[1])^j))
}
uM2pow2(m[2], m[4], n)
```

uM2pow2pool *Pooled central moment estimates - two-sample*

Description

Calculate pooled unbiased estimates of central moments and their powers and products.

Usage

```
uM2pow2pool(m2, m4, n_x, n_y)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^2+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^2$ for vectors X and Y.
m4	naive biased fourth central moment estimate $m_4 = 1/(n_x + n_y)\sum_{i=1}^{n_x}((X_i - \bar{X})^4 + \sum_{i=1}^{n_y}((Y_i - \bar{Y})^4$ for vectors X and Y.
n_x	number of observations in the first group.
n_y	number of observations in the second group.

Value

Pooled estimate of squared variance μ_2^2, where μ_2 is a variance.

See Also

Other pooled estimates (two-sample): uM2M3pool(), uM2M4pool(), uM2pool(), uM2pow3pool(), uM3pool(), uM3pow2pool(), uM4pool(), uM5pool(), uM6pool()

Examples

```
nx <- 10
ny <- 8
shp <- 3
smpx <- rgamma(nx, shape = shp) - shp
smpy <- rgamma(ny, shape = shp)
mx <- mean(smpx)
my <- mean(smpy)
m  <- numeric(4)
for (j in 2:4) {
  m[j] <- mean(c((smpx - mx)^j, (smpy - my)^j))
}
uM2pow2pool(m[2], m[4], nx, ny)
```

uM2pow3 *Unbiased central moment estimates*

Description

Calculate unbiased estimates of central moments and their powers and products.

Usage

```
uM2pow3(m2, m3, m4, m6, n)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/n \sum_{i=1}^n ((X_i - \bar{X})^2$ for a vector X.
m3	naive biased third central moment estimate $m_3 = 1/n \sum_{i=1}^n ((X_i - \bar{X})^3$ for a vector X.
m4	naive biased fourth central moment estimate $m_4 = 1/n \sum_{i=1}^n ((X_i - \bar{X})^4$ for a vector X.
m6	naive biased sixth central moment estimate $m_6 = 1/n \sum_{i=1}^n ((X_i - \bar{X})^6$ for a vector X.
n	sample size.

Value

Unbiased estimate of cubed second central moment μ_2^3, where μ_2 is a variance.

See Also

Other unbiased estimates (one-sample): uM2(), uM2M3(), uM2M4(), uM2pow2(), uM3(), uM3pow2(), uM4(), uM5(), uM6()

Examples

```
n <- 10
smp <- rgamma(n, shape = 3)
m <- mean(smp)
for (j in 2:6) {
  m <- c(m, mean((smp - m[1])^j))
}
uM2pow3(m[2], m[3], m[4], m[6], n)
```

uM2pow3pool	*Pooled central moment estimates - two-sample*

Description

Calculate pooled unbiased estimates of central moments and their powers and products.

Usage

```
uM2pow3pool(m2, m3, m4, m6, n_x, n_y)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/(n_x+n_y) \sum_{i=1}^{n_x}((X_i-\bar{X})^2+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^2$ for vectors X and Y.
m3	naive biased third central moment estimate $m_3 = 1/(n_x+n_y) \sum_{i=1}^{n_x}((X_i-\bar{X})^3+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^3$ for vectors X and Y.
m4	naive biased fourth central moment estimate $m_4 = 1/(n_x+n_y) \sum_{i=1}^{n_x}((X_i-\bar{X})^4+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^4$ for vectors X and Y.
m6	naive biased sixth central moment estimate $m_6 = 1/(n_x+n_y) \sum_{i=1}^{n_x}((X_i-\bar{X})^6+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^6$ for vectors X and Y.
n_x	number of observations in the first group.
n_y	number of observations in the second group.

Value

Pooled estimate of cubed variance central moment μ_2^3, where μ_2 is a variance.

See Also

Other pooled estimates (two-sample): uM2M3pool(), uM2M4pool(), uM2pool(), uM2pow2pool(), uM3pool(), uM3pow2pool(), uM4pool(), uM5pool(), uM6pool()

Examples

```
nx <- 10
ny <- 8
shp <- 3
smpx <- rgamma(nx, shape = shp) - shp
smpy <- rgamma(ny, shape = shp)
mx <- mean(smpx)
my <- mean(smpy)
m  <- numeric(6)
for (j in 2:6) {
  m[j] <- mean(c((smpx - mx)^j, (smpy - my)^j))
}
uM2pow3pool(m[2], m[3], m[4], m[6], nx, ny)
```

uM3 *Unbiased central moment estimates*

Description

Calculate unbiased estimates of central moments and their powers and products.

Usage

```
uM3(m3, n)
```

Arguments

m3 naive biased third central moment estimate $m_3 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^3$ for a vector X.

n sample size.

Value

Unbiased estimate of a third central moment.

See Also

Other unbiased estimates (one-sample): uM2(), uM2M3(), uM2M4(), uM2pow2(), uM2pow3(), uM3pow2(), uM4(), uM5(), uM6()

Examples

```
n <- 10
smp <- rgamma(n, shape = 3)
m <- mean(smp)
for (j in 2:3) {
  m <- c(m, mean((smp - m[1])^j))
}
uM3(m[3], n)
```

uM3pool *Pooled central moment estimates - two-sample*

Description

Calculate pooled unbiased estimates of central moments and their powers and products.

Usage

```
uM3pool(m3, n_x, n_y)
```

Arguments

m3	naive biased third central moment estimate $m_3 = 1/(n_x + n_y) \sum_{i=1}^{n_x}((X_i - \bar{X})^3 + \sum_{i=1}^{n_y}((Y_i - \bar{Y})^3$ for vectors X and Y.
n_x	number of observations in the first group.
n_y	number of observations in the second group.

Value

Pooled estimate of a third central moment.

See Also

Other pooled estimates (two-sample): uM2M3pool(), uM2M4pool(), uM2pool(), uM2pow2pool(), uM2pow3pool(), uM3pow2pool(), uM4pool(), uM5pool(), uM6pool()

Examples

```
nx <- 10
ny <- 8
shp <- 3
smpx <- rgamma(nx, shape = shp) - shp
smpy <- rgamma(ny, shape = shp)
mx <- mean(smpx)
my <- mean(smpy)
m  <- numeric(3)
for (j in 2:3) {
  m[j] <- mean(c((smpx - mx)^j, (smpy - my)^j))
}
uM3pool(m[3], nx, ny)
```

---

uM3pow2 *Unbiased central moment estimates*

Description

Calculate unbiased estimates of central moments and their powers and products.

Usage

```
uM3pow2(m2, m3, m4, m6, n)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^2$ for a vector X.
m3	naive biased third central moment estimate $m_3 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^3$ for a vector X.

m4	naive biased fourth central moment estimate $m_4 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^4$ for a vector X.
m6	naive biased sixth central moment estimate $m_6 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^6$ for a vector X.
n	sample size.

Value

Unbiased estimate of squared third central moment μ_3^2, where μ_3 is a third central moment.

See Also

Other unbiased estimates (one-sample): uM2(), uM2M3(), uM2M4(), uM2pow2(), uM2pow3(), uM3(), uM4(), uM5(), uM6()

Examples

```
n <- 10
smp <- rgamma(n, shape = 3)
m <- mean(smp)
for (j in 2:6) {
  m <- c(m, mean((smp - m[1])^j))
}
uM3pow2(m[2], m[3], m[4], m[6], n)
```

---

uM3pow2pool *Pooled central moment estimates - two-sample*

Description

Calculate pooled unbiased estimates of central moments and their powers and products.

Usage

```
uM3pow2pool(m2, m3, m4, m6, n_x, n_y)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/(n_x+n_y) \sum_{i=1}^{n_x}((X_i-\bar{X})^2+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^2$ for vectors X and Y.
m3	naive biased third central moment estimate $m_3 = 1/(n_x + n_y) \sum_{i=1}^{n_x}((X_i - \bar{X})^3 + \sum_{i=1}^{n_y}((Y_i - \bar{Y})^3$ for vectors X and Y.
m4	naive biased fourth central moment estimate $m_4 = 1/(n_x + n_y) \sum_{i=1}^{n_x}((X_i - \bar{X})^4 + \sum_{i=1}^{n_y}((Y_i - \bar{Y})^4$ for vectors X and Y.
m6	naive biased sixth central moment estimate $m_6 = 1/(n_x + n_y) \sum_{i=1}^{n_x}((X_i - \bar{X})^6 + \sum_{i=1}^{n_y}((Y_i - \bar{Y})^6$ for vectors X and Y.
n_x	number of observations in the first group.
n_y	number of observations in the second group.

Value

Pooled estimate of squared third central moment μ_3^2, where μ_3 is a third central moment.

See Also

Other pooled estimates (two-sample): uM2M3pool(), uM2M4pool(), uM2pool(), uM2pow2pool(), uM2pow3pool(), uM3pool(), uM4pool(), uM5pool(), uM6pool()

Examples

```
nx <- 10
ny <- 8
shp <- 3
smpx <- rgamma(nx, shape = shp) - shp
smpy <- rgamma(ny, shape = shp)
mx <- mean(smpx)
my <- mean(smpy)
m  <- numeric(6)
for (j in 2:6) {
  m[j] <- mean(c((smpx - mx)^j, (smpy - my)^j))
}
uM3pow2pool(m[2], m[3], m[4], m[6], nx, ny)
```

---

uM4 *Unbiased central moment estimates*

Description

Calculate unbiased estimates of central moments and their powers and products.

Usage

```
uM4(m2, m4, n)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/n \sum_{i=1}^n ((X_i - \bar{X})^2$ for a vector X.
m4	naive biased fourth central moment estimate $m_4 = 1/n \sum_{i=1}^n ((X_i - \bar{X})^4$ for a vector X.
n	sample size.

Value

Unbiased estimate of a fourth central moment.

See Also

Other unbiased estimates (one-sample): uM2(), uM2M3(), uM2M4(), uM2pow2(), uM2pow3(), uM3(), uM3pow2(), uM5(), uM6()

Examples

```
n <- 10
smp <- rgamma(n, shape = 3)
m <- mean(smp)
for (j in 2:4) {
  m <- c(m, mean((smp - m[1])^j))
}
uM4(m[2], m[4], n)
```

---

uM4pool *Pooled central moment estimates - two-sample*

Description

Calculate pooled unbiased estimates of central moments and their powers and products.

Usage

```
uM4pool(m2, m4, n_x, n_y)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^2+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^2$ for vectors X and Y.
m4	naive biased fourth central moment estimate $m_4 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^4+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^4$ for vectors X and Y.
n_x	number of observations in the first group.
n_y	number of observations in the second group.

Value

Pooled estimate of a fourth central moment.

See Also

Other pooled estimates (two-sample): uM2M3pool(), uM2M4pool(), uM2pool(), uM2pow2pool(), uM2pow3pool(), uM3pool(), uM3pow2pool(), uM5pool(), uM6pool()

Examples

```
nx <- 10
ny <- 8
shp <- 3
smpx <- rgamma(nx, shape = shp) - shp
smpy <- rgamma(ny, shape = shp)
mx <- mean(smpx)
my <- mean(smpy)
m  <- numeric(4)
```

```
for (j in 2:4) {
  m[j] <- mean(c((smpx - mx)^j, (smpy - my)^j))
}
uM4pool(m[2], m[4], nx, ny)
```

---

uM5 *Unbiased central moment estimates*

Description

Calculate unbiased estimates of central moments and their powers and products.

Usage

```
uM5(m2, m3, m5, n)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^2$ for a vector X.
m3	naive biased third central moment estimate $m_3 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^3$ for a vector X.
m5	naive biased fifth central moment estimate $m_5 = \sum_{i=1}^{n}((X_i - \bar{X})^5$ for a vector X.
n	sample size.

Value

Unbiased estimate of a fifth central moment.

See Also

Other unbiased estimates (one-sample): uM2(), uM2M3(), uM2M4(), uM2pow2(), uM2pow3(), uM3(), uM3pow2(), uM4(), uM6()

Examples

```
n <- 10
smp <- rgamma(n, shape = 3)
m <- mean(smp)
for (j in 2:5) {
  m <- c(m, mean((smp - m[1])^j))
}
uM5(m[2], m[3], m[5], n)
```

uM5pool *Pooled central moment estimates - two-sample*

Description

Calculate pooled unbiased estimates of central moments and their powers and products.

Usage

```
uM5pool(m2, m3, m5, n_x, n_y)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^2+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^2$ for vectors X and Y.
m3	naive biased third central moment estimate $m_3 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^3+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^3$ for vectors X and Y.
m5	naive biased fifth central moment estimate $m_5 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^5+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^5$ for vectors X and Y.
n_x	number of observations in the first group.
n_y	number of observations in the second group.

Value

Pooled estimate of a fifth central moment.

See Also

Other pooled estimates (two-sample): uM2M3pool(), uM2M4pool(), uM2pool(), uM2pow2pool(), uM2pow3pool(), uM3pool(), uM3pow2pool(), uM4pool(), uM6pool()

Examples

```
nx <- 10
ny <- 8
shp <- 3
smpx <- rgamma(nx, shape = shp) - shp
smpy <- rgamma(ny, shape = shp)
mx <- mean(smpx)
my <- mean(smpy)
m  <- numeric(5)
for (j in 2:5) {
  m[j] <- mean(c((smpx - mx)^j, (smpy - my)^j))
}
uM5pool(m[2], m[3], m[5], nx, ny)
```

uM6 *Unbiased central moment estimates*

Description

Calculate unbiased estimates of central moments and their powers and products.

Usage

```
uM6(m2, m3, m4, m6, n)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^2$ for a vector X.
m3	naive biased third central moment estimate $m_3 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^3$ for a vector X.
m4	naive biased fourth central moment estimate $m_4 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^4$ for a vector X.
m6	naive biased sixth central moment estimate $m_6 = 1/n \sum_{i=1}^{n}((X_i - \bar{X})^6$ for a vector X.
n	sample size.

Value

Unbiased estimate of a sixth central moment.

See Also

Other unbiased estimates (one-sample): uM2(), uM2M3(), uM2M4(), uM2pow2(), uM2pow3(), uM3(), uM3pow2(), uM4(), uM5()

Examples

```
n <- 10
smp <- rgamma(n, shape = 3)
m <- mean(smp)
for (j in 2:6) {
  m <- c(m, mean((smp - m[1])^j))
}
uM6(m[2], m[3], m[4], m[6], n)
```

uM6pool *Pooled central moment estimates - two-sample*

Description

Calculate pooled unbiased estimates of central moments and their powers and products.

Usage

```
uM6pool(m2, m3, m4, m6, n_x, n_y)
```

Arguments

m2	naive biased variance estimate $m_2 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^2+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^2$ for vectors X and Y.
m3	naive biased third central moment estimate $m_3 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^3+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^3$ for vectors X and Y.
m4	naive biased fourth central moment estimate $m_4 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^4+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^4$ for vectors X and Y.
m6	naive biased sixth central moment estimate $m_6 = 1/(n_x+n_y)\sum_{i=1}^{n_x}((X_i-\bar{X})^6+\sum_{i=1}^{n_y}((Y_i-\bar{Y})^6$ for vectors X and Y.
n_x	number of observations in the first group.
n_y	number of observations in the second group.

Value

Unbiased estimate of a sixth central moment.

See Also

Other pooled estimates (two-sample): uM2M3pool(), uM2M4pool(), uM2pool(), uM2pow2pool(), uM2pow3pool(), uM3pool(), uM3pow2pool(), uM4pool(), uM5pool()

Examples

```
nx <- 10
ny <- 8
shp <- 3
smpx <- rgamma(nx, shape = shp) - shp
smpy <- rgamma(ny, shape = shp)
mx <- mean(smpx)
my <- mean(smpy)
m  <- numeric(6)
for (j in 2:6) {
  m[j] <- mean(c((smpx - mx)^j, (smpy - my)^j))
}
uM6pool(m[2], m[3], m[4], m[6], nx, ny)
```

uMpool — *Pooled central moment estimates - two-sample*

Description

Calculate unbiased pooled estimates of central moments and their powers and products up to specified order.

Usage

```
uMpool(smp, a, order)
```

Arguments

`smp`	sample.
`a`	vector of the same length as `smp` specifying categories of observations (should contain two unique values).
`order`	highest order of the estimates to calclulate. Estimates of lower orders will be included.

Details

Pooled estimates up to the 6th order can be calculated. Second and third orders contain estimates of the variance and third central moment, fourth order includes estimates of fourth moment and squared variance (μ_2^2), fifth order - of fifth moment and a product of second and third moments ($\mu_2\mu_3$), sixth order - of sixth moment, a product of second and fourth moments ($\mu_2\mu_4$), squared third moment (μ_3^2), and cubed variance (μ_2^3).

Value

A named vector of estimates of central moments and their powers and products up to `order`. The highest order available is 6th. The names of the elements are `"M2", "M3", "M4", "M5", "M6"` for corresponding central moments, `"M2M3", "M2M4"` for products of the moments (second and third, second and fourth), and `"M2pow2", "M2pow3", "M3pow2"` for powers of the moments - corresponding to estimates of squared variance, cubed variance, and squared third moment.

References

Gerlovina, I. and Hubbard, A.E. (2019). *Computer algebra and algorithms for unbiased moment estimation of arbitrary order.* Cogent Mathematics & Statistics, 6(1).

See Also

`uM` for one-sample unbiased estimates.

Examples

```
nsmp <- 23
smp <- rgamma(nsmp, shape = 3)
treatment <- sample(0:1, size = nsmp, replace = TRUE)
uMpool(smp, treatment, 6)
```

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