# Package 'Kurt'

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**Type** Package

**Title** Performs Kurtosis-Based Statistical Analyses

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**Description** Computes measures of multivariate kurtosis, matrices of fourth-order moments and cumulants, kurtosis-based projection pursuit. Franceschini, C. and Loperfido, N. (2018, ISBN:978-3-319-73905-2). ``An Algorithm for Finding Projections with Extreme Kurtosis''. Loperfido, N. (2017,ISSN:0024-3795). ``A New Kurtosis Matrix, with Statistical Applications''.

**License** GPL (>= 2)

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**Depends** labstatR (>= 1.0.11), matrixcalc (>= 1.0.5), polynom (>= 1.4-0), expm (>= 0.999-6)

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**Description**

Computes measures of multivariate kurtosis, matrices of fourth-order moments and cumulants, kurtosis-based projection pursuit

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**Author(s)**

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**References**

Franceschini, C. and Loperfido, N. (2012). On some Inequalities between Measures of Multivariate Kurtosis, with Application to Financial Returns. In "Mathematical and Statistical Methods for Actuarial Sciences and Finance", Perna, C. and Sibillo, M. (Eds.), Springer, 211-218.

Franceschini, C. and Loperfido, N. (2018). An Algorithm for Finding Projections with Extreme Kurtosis. In "Studies in Theoretical and Applied Statistics: SIS2016-48th Meeting of the Italian Statistical Society, Salerno 8-10 June 2016", Perna C., Pratesi M. and Ruiz-Gazen A. (Eds.), Springer.

Henze, N. (1994). On Mardia's kurtosis test for multivariate normality. Communications in statistics-Theory and Methods 23:4, 1031-1045.

Kollo, T. (2008). Multivariate skewness and kurtosis measures with an application in ICA. Journal of Multivariate Analysis 99, 2328-2338.

Kollo, T. and Srivastava, M.S. (2005). Estimation and testing of parameters in multivariate Laplace distribution. Comm. Statist. 33, 2363–2687.

Koziol, J.A. (1987). An alternative formulation of Neyman's smooth goodness of fit tests under composite alternatives. Metrika 34, 17-24.

Koziol, J.A. (1989). A note on measures of multivariate kurtosis. Biometrical Journal 31, 619-624.

Loperfido, N. (2011). Spectral Analysis of the Fourth Moment Matrix. Linear Algebra and its Applications 435, 1837-1844.

Loperfido, N. (2014). A Note on the Fourth Cumulant of a Finite Mixture Distribution. Journal of Multivariate Analysis 123, 386-394.

Loperfido, N. (2017). A New Kurtosis Matrix, with Statistical Applications. Linear Algebra and its Applications 512, 1-17.

Loperfido N. (2019). Kurtosis-Based Projection Pursuit for Outlier Detection in Financial Time Series. The European Journal of Finance, to appear.

Loperfido, N. (2020). Some Remarks on Koziol's Kurtosis. Journal of Multivariate Analysis 175, to appear.

Malkovich, J.F. and Afifi, A.A. (1973). On Tests for Multivariate Normality. J. Amer. Statist. Ass. 68, 176-179.

Mardia, K.V. (1970). Measures of multivariate skewness and kurtosis with applications. Biometrika 57, 519-530.

Mardia, K. V. and Kent, J. T. (1991). Rao Score Tests for Goodness of Fit and Independence. Biometrika 78, 355-36.

Miettinen J., Taskinen S., Nordhausen K. and Oja H. (2015). Fourth Moments and Independent Component Analysis. Statistical Science 30, 372-390.

Mori T.F., Rohatgi V.K. and Szekely G.J. (1993). On multivariate skewness and kurtosis. Theory Probab. Appl. 38, 547-551.

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| Cum4 | *Cum4: fourth multivariate cumulant* |
|------|--------------------------------------|

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## Description

returns a matrix containing the fourth cumulants of the given data

## Usage

```
Cum4(data, type, shape)
```

## Arguments

| | |
|------|------|
| data | data matrix |
| type | type=0 uses original data, type=1 uses centered data, type=2 uses standardized data |
| shape | if shape="square" the output is a d^2 x d^2 matrix. If shape="rectangular", the output is a d x d^3 matrix. Where d is the number of variables |

## Value

K4 is the matrix containing the fourth cumulants of the given data

## Author(s)

Cinzia Franceschini and Nicola Loperfido

## Examples

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode
Cum4(iris[,1:4], 1, "square") # returns a matrix containing the fourth cumulants of the given data
```

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ExtKur                      *ExtKur: kurtosis based projection pursuit*

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## Description

Returns a data projection with either maximal or minimal kurtosis.

## Usage

```
ExtKur(data, iterations, maxmin)
```

## Arguments

| | |
|---|---|
| data | data matrix |
| iterations | number of required iterations |
| maxmin | is the choice to either maximise ("MAX") or minimise ("MIN") kurtosis |

## Value

| | |
|---|---|
| linear | vector of coefficients |
| projection | vector of projected data |
| kurt | extreme kurtosis attainable by a data projection |

## Author(s)

Cinzia Franceschini and Nicola Loperfido

## Examples

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode
ExtKur(iris[,1:4],10,"MAX") #returns a data projection with maximal kurtosis
```

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| ExtKurBiv | *ExtKurBiv: kurtosis-based projection pursuit for bivariate random vectors* |
|---|---|

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## Description

Returns a projection of bivariate data with either maximal or minimal kurtosis.

## Usage

```
ExtKurBiv(data, maxmin)
```

## Arguments

| | |
|---|---|
| data | data matrix |
| maxmin | choice between maximal ("MAX") and minimal ("MIN") kurtosis |

## Value

| | |
|---|---|
| linearMAX | coefficients of the projections maximising kurtosis |
| projectionMAX | projection with maximal kurtosis |
| kurtMAX | maximal kurtosis |
| linearMIN | coefficients of the projections minimising kurtosis |
| projectionMIN | projection with minimal kurtosis |
| kurtMIN | minimal kurtosis |

## Author(s)

Cinzia Franceschini and Nicola Loperfido

## Examples

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode

ExtKurBiv(iris[,1:2],"MAX")# returns a projection of bivariate data with maximal kurtosis
```

| Fourth | *Fourth: matrices of fourth moments or fourth cumulants* |
|--------|-----------------------------------------------------------|

### Description

Returns a matrix containing either the fourth moments or the fourth cumulants. It recalls the functions Fourth4 and Cum4.

### Usage

```
Fourth(data, type, shape, feature)
```

### Arguments

| | |
|---------|---|
| data | data matrix |
| type | type =0 is the ordinary fourth moment / cumulant; type =1 is the centered fourth moment / cumulant; type =2 is the standardized fourth moment / cumulant |
| shape | "square" or "rectangular" |
| feature | "moment" or "cumulant". If feature is "moment", the function computes the fourth moment of a data matrix. The function recalls the function Fourth4. If feature is "cumulant", the function computes the fourth multivariate cumulant. The function recalls the function Cum4. |

### Value

| | |
|----|---|
| M | Fourth square moment matrix |
| MM | Fourth rectangular moment matrix |
| K4 | Fourth cumulants of the given data |

### Author(s)

Cinzia Franceschini and Nicola Loperfido

### Examples

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode
Fourth(iris[,1:4], 1,"square", "moment")#returns a matrix containing the fourth moments
```

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Fourth4            *Fourth4: fourth moment of a data matrix*

---

### Description

Returns a matrix containing the fourth moments.

### Usage

```
Fourth4(data, type, shape)
```

### Arguments

| | |
|---|---|
| data | data matrix |
| type | type=0 is the ordinary fourth moment, type=1 is the centered fourth moment, type=2 is the standardized fourh moment |
| shape | "square" or "rectangular" |

### Value

| | |
|---|---|
| M | Fourth square moment matrix |
| MM | Fourth rectangular moment matrix |

### Author(s)

Cinzia Franceschini and Nicola Loperfido

### Examples

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode
Fourth4(iris[,1:4], 1,"square") #returns a matrix containing the fourth moments
```

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NoKurt            *NoKurt: data projections whose excess kurtosis is as close to zero as possible*

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### Description

Data projections whose excess kurtosis is as close to zero as possible. Excess kurtosis is the fourth standardized cumulant , that is the fourth standardized moment minus three.

### Usage

```
NoKurt(data, number)
```

## Arguments

| | |
|---|---|
| `data` | data matrix |
| `number` | number of required projections. It must be greater than one and less than the number of variables |

## Value

| | |
|---|---|
| `Nkurtoses` | kurtoses of Nprojections |
| `Nprojections` | data projections ordered according to the absolute values of their excess kurtoses |
| `MATRIX` | matrix characterizing the projection |

## Author(s)

Cinzia Franceschini and Nicola Loperfido

## Examples

```
data(iris)
iris<-data.matrix(iris[,1:4])
NoKurt(iris[,1:4],3)#returns data projections whose excess kurtosis is as close to zero as possible
```

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| optik | *optik* |
|---|---|

---

## Description

It computes the matrix containing the smallest and largest kurtoses of data projections as well as the corresponding directions.

## Usage

```
optik(data)
```

## Arguments

| | |
|---|---|
| `data` | data matrix |

## Value

| | |
|---|---|
| `kurMAX` | kurtosis of the projection maximizing kurtosis |
| `pMAX` | projection maximizing kurtosis |
| `dMAX` | direction maximizing kurtosis |
| `kurMINbis` | kurtosis of the projection minimizing kurtosis |
| `pMINbis` | projection minimizing kurtosis |
| `dMINbis` | direction minimizing kurtosis |

## Author(s)

Cinzia Franceschini and Nicola Loperfido

## Examples

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode
optik(iris[,1:4])#starting values of the projections with extreme kurtosis
```

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| ScalarKurt | *ScalarKurt* |
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## Description

Returns the statistic and the p-value of either Mardia's kurtosis or Koziol's kurtosis tests for normality .

## Usage

```
ScalarKurt(data, feature, type, prob)
```

## Arguments

| | |
| --- | --- |
| data | data matrix |
| feature | "moment" or "cumulant" |
| type | "Mardia" or "Koziol" |
| prob | "lower" if probability is P[X <= x], "upper" if probability is P[X > x], "twoside" if probability is computed on both tails |

## Details

For Koziol kurtosis only the upper tail is meaningful

## Value

statistic

pvalue

## Author(s)

Cinzia Franceschini and Nicola Loperfido

## Examples

```
data(iris)
iris<-data.matrix(iris)#returns the matrix obtained by converting the data frame to numeric mode
ScalarKurt(iris[,1:4],"moment","Mardia","upper")#returns the statistic and the p-value
#of Mardia's kurtosis test for normality
```

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