# Package 'fromo'

November 30, 2024

·
Type Package
Maintainer Steven E. Pav <shabbychef@gmail.com></shabbychef@gmail.com>
Version 0.2.4
<b>Date</b> 2024-11-29
License LGPL-3
Title Fast Robust Moments
<pre>BugReports https://github.com/shabbychef/fromo/issues</pre>
Description Fast, numerically robust computation of weighted moments via 'Rcpp'.  Supports computation on vectors and matrices, and Monoidal append of moments.  Moments and cumulants over running fixed length windows can be computed, as well as over time-based windows.  Moment computations are via a generalization of Welford's method, as described by Bennett et. (2009) <doi:10.1109 clustr.2009.5289161="">.</doi:10.1109>
Imports Rcpp (>= 0.12.3), methods
LinkingTo Rcpp
Suggests knitr, testthat, moments, PDQutils, e1071, microbenchmark
RoxygenNote 7.3.2
<pre>URL https://github.com/shabbychef/fromo</pre>
VignetteBuilder knitr
Collate 'fromo.r' 'RcppExports.R' 'zzz_centsums.R'
NeedsCompilation yes
<b>Author</b> Steven E. Pav [aut, cre] ( <a href="https://orcid.org/0000-0002-4197-6195">https://orcid.org/0000-0002-4197-6195</a> )
Repository CRAN
<b>Date/Publication</b> 2024-11-30 05:00:03 UTC
Contents
fromo-package

2 fromo-package

8	as.centcosums	4
ä	as.centsums	5
(	c.centcosums	7
(	c.centsums	7
(	cent2raw	8
(	centcosums-accessor	8
(	centcosums-class	9
(	centsums-class	10
(	cent_cosums	12
(	cent_sums	13
f	fromo-NEWS	15
1	running_apx_quantiles	16
1	running_centered	19
1	running_correlation	24
1	running_sd3	28
1	unning_sum	34
S	sd3	36
S	show	41
t	_running_apx_quantiles	42
t	_running_centered	46
t	_running_correlation	52
t	_running_sd3	58
t	_running_sum	65
	%-%,centcosums,centcosums-method	
(	%-%	69
		<b>70</b>

fromo-package

Fast Robust Moments.

# Description

Fast, numerically robust moments computations, along with computation of cumulants, running means, etc.

# **Robust Moments**

Welford described a method for 'robust' one-pass computation of the standard deviation. By 'robust', we mean robust to round-off caused by a large shift in the mean. This method was generalized by Terriberry, and Bennett *et. al.* to the case of higher-order moments. This package provides those algorithms for computing moments.

Generally we should find that the stock implementations of sd, skewness and so on are *already* robust and likely using these algorithms under the hood. This package was written for a few reasons:

1. As an exercise to learn Rcpp.

fromo-package 3

2. Often I found I needed the first *k* moments. For example, when computing the Z-score, the standard deviation and mean must be computed separately, which is inefficient. Similarly Merten's correction for the standard error of the Sharpe ratio uses the first four moments. These are all computed as a side effect of computation of the kurtosis, but discarded by the standard methods.

## Legal Mumbo Jumbo

fromo is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the GNU Lesser General Public License for more details.

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

This package was developed as an exercise in learning Rcpp.

## Author(s)

Steven E. Pav <shabbychef@gmail.com>

Maintainer: Steven E. Pav <shabbychef@gmail.com> (ORCID)

## References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

## See Also

Useful links:

- https://github.com/shabbychef/fromo
- Report bugs at https://github.com/shabbychef/fromo/issues

4 as.centcosums

accessor

Accessor methods.

## **Description**

Access slot data from a centsums object.

#### Usage

```
sums(x)
## S4 method for signature 'centsums'
sums(x)

moments(x, type = c("central", "raw", "standardized"))
## S4 method for signature 'centsums'
moments(x, type = c("central", "raw", "standardized"))
```

# **Arguments**

x a centsums object.

type the type of moment to compute.

# Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

## Author(s)

Steven E. Pav <shabbychef@gmail.com>

as.centcosums

Coerce to a centcosums object.

# Description

Convert data to a centcosums object.

## Usage

```
as.centcosums(x, order=2, na.omit=TRUE)
## Default S3 method:
as.centcosums(x, order = 2, na.omit = TRUE)
```

as.centsums 5

# **Arguments**

x a matrix.

order the order, defaulting to 2.

na.omit whether to remove rows with NA.

## **Details**

Computes the raw cosums on data, and stuffs the results into a centcosums object.

## Value

A centcosums object.

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

## Author(s)

Steven E. Pav <shabbychef@gmail.com>

## **Examples**

```
set.seed(123)
x <- matrix(rnorm(100*3),ncol=3)
cs <- as.centcosums(x, order=2)</pre>
```

as.centsums

Coerce to a centsums object.

# **Description**

Convert data to a centsums object.

# Usage

```
as.centsums(
    x,
    order = 3,
    na.rm = TRUE,
    wts = NULL,
    check_wts = FALSE,
    normalize_wts = FALSE
```

6 as.centsums

```
## Default S3 method:
as.centsums(
    x,
    order = 3,
    na.rm = TRUE,
    wts = NULL,
    check_wts = FALSE,
    normalize_wts = FALSE)
```

## **Arguments**

x a numeric, array, or matrix.

order the order, defaulting to length(sums)+1.

na.rm whether to remove NA.

wts an optional vector of weights. Weights are 'replication' weights, meaning a

value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na\_rm is true. When there is no checking, an NA value will

cause the output to be NA.

check\_wts a boolean for whether the code shall check for negative weights, and throw an

error when they are found. Default false for speed.

normalize\_wts a boolean for whether the weights should be renormalized to have a mean value

of 1. This mean is computed over elements which contribute to the moments, so if na\_rm is set, that means non-NA elements of wts that correspond to non-NA

elements of the data vector.

#### **Details**

Computes the raw sums on data, and stuffs the results into a centsums object.

## Value

A centsums object.

## Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

c.centcosums 7

## Author(s)

```
Steven E. Pav <shabbychef@gmail.com>
```

# Examples

```
set.seed(123)
x <- rnorm(1000)
cs <- as.centsums(x, order=5)</pre>
```

c.centcosums

concatenate centcosums objects.

# Description

Concatenate centcosums objects.

# Usage

```
\mbox{method}\{c\}\{\mbox{centcosums}\}(...)
```

# Arguments

... centcosums objects

## See Also

```
join_cent_cosums
```

c.centsums

concatenate centsums objects.

# Description

Concatenate centsums objects.

# Usage

```
\mbox{method} c}{centsums}(...)
```

# **Arguments**

... centsums objects

# See Also

```
join_cent_sums
```

8 centcosums-accessor

cent2raw Convert between different types of moments, raw, central, standard- ized.	cent2raw	Convert between different types of moments, raw, central, standard-ized.
---	----------	--

# **Description**

Given raw or central or standardized moments, convert to another type.

# Usage

```
cent2raw(input)
```

## **Arguments**

input

a vector of the count, then the mean, then the 2 through k raw or central mo-

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

## Author(s)

Steven E. Pav <shabbychef@gmail.com>

```
centcosums-accessor Accessor methods.
```

# **Description**

Access slot data from a centcosums object.

## Usage

```
cosums(x)
## S4 method for signature 'centcosums'
cosums(x)

comoments(x, type = c("central", "raw"))
## S4 method for signature 'centcosums'
comoments(x, type = c("central", "raw"))
```

centcosums-class 9

# **Arguments**

x a centcosums object.

type the type of moment to compute.

## Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

centcosums-class

centcosums Class.

# Description

An S4 class to store (centered) cosums of data, and to support operations on the same.

## Usage

```
## S4 method for signature 'centcosums'
initialize(.Object, cosums, order = NA_real_)
centcosums(cosums, order = NULL)
```

## **Arguments**

. Object a centcosums object, or proto-object. cosums the output of cent\_cosums, say. order the order, defaulting to 2.

# **Details**

A centcosums object contains a multidimensional array (now only 2-diemnsional), as output by cent\_cosums.

#### Value

An object of class centcosums.

# **Slots**

```
cosums a multidimensional array of the cosums. order the maximum order, ignored for now.
```

10 centsums-class

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

#### Author(s)

```
Steven E. Pav <shabbychef@gmail.com>
```

#### References

```
Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html
```

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

 $Cook, J. \, D. \, "Comparing three methods of computing standard deviation." \, https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/$ 

#### See Also

```
cent_cosums
```

# **Examples**

```
obj <- new("centcosums",cosums=cent_cosums(matrix(rnorm(100*3),ncol=3),max_order=2),order=2)
```

centsums-class

centsums Class.

#### **Description**

An S4 class to store (centered) sums of data, and to support operations on the same.

## Usage

```
## $4 method for signature 'centsums'
initialize(.Object, sums, order = NA_real_)
centsums(sums, order = NULL)
```

# Arguments

.Object a centsums object, or proto-object.

sums a numeric vector.

order the order, defaulting to length(sums)+1.

centsums-class 11

#### **Details**

A centsums object contains a vector value of the data count, the mean, and the kth centered sum, for k up to some maximum order.

#### Value

An object of class centsums.

## **Slots**

sums a numeric vector of the sums.

order the maximum order.

## Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

## Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

## **Examples**

```
obj <- new("centsums", sums=c(1000, 1.234, 0.235), order=2)
```

12 cent\_cosums

cent_cosums	Multivariate centerea	l sums; join an	d unjoined.
-------------	-----------------------	-----------------	-------------

# Description

Compute, join, or unjoin multivariate centered (co-) sums.

## Usage

```
cent_cosums(v, max_order = 2L, na_omit = FALSE)
cent_comoments(v, max_order = 2L, used_df = 0L, na_omit = FALSE)
join_cent_cosums(ret1, ret2)
unjoin_cent_cosums(ret3, ret2)
```

## **Arguments**

V	an $m$ by $n$ matrix, each row an independent observation of some $n$ variate variable.
max_order	the maximum order of cosum to compute. For now this can only be 2; in the future higher order cosums should be possible.
na_omit	a boolean; if TRUE, then only rows of $\boldsymbol{v}$ with complete observations will be used.
used_df	the number of degrees of freedom consumed, used in the denominator of the centered moments computation. These are subtracted from the number of observations.
ret1	a multdimensional array as output by cent_cosums.
ret2	a multdimensional array as output by cent_cosums.
ret3	a multdimensional array as output by cent_cosums.

## Value

a multidimensional arry of dimension max\_order, each side of length 1+n. For the case currently implemented where max\_order must be 2, the output is a symmetric matrix, where the element in the 1,1 position is the count of complete) rows of v, the 2:(n+1),1 column is the mean, and the 2:(n+1),2:(n+1) is the co *sums* matrix, which is the covariance up to scaling by the count. cent\_comments performs this normalization for you.

## Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

cent\_sums 13

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

#### See Also

cent\_sums

## **Examples**

```
set.seed(1234)
x1 <- matrix(rnorm(1e3*5,mean=1),ncol=5)
x2 <- matrix(rnorm(1e3*5,mean=1),ncol=5)
max_ord <- 2L
rs1 <- cent_cosums(x1,max_ord)
rs2 <- cent_cosums(x2,max_ord)
rs3 <- cent_cosums(rbind(x1,x2),max_ord)
rs3alt <- join_cent_cosums(rs1,rs2)
stopifnot(max(abs(rs3 - rs3alt)) < 1e-7)
rs1alt <- unjoin_cent_cosums(rs3,rs2)
rs2alt <- unjoin_cent_cosums(rs3,rs1)
stopifnot(max(abs(rs1 - rs1alt)) < 1e-7)
stopifnot(max(abs(rs2 - rs2alt)) < 1e-7)</pre>
```

cent\_sums

Centered sums; join and unjoined.

# **Description**

Compute, join, or unjoin centered sums.

14 cent\_sums

## Usage

```
cent_sums(
   v,
   max_order = 5L,
   na_rm = FALSE,
   wts = NULL,
   check_wts = FALSE,
   normalize_wts = TRUE
)

join_cent_sums(ret1, ret2)

unjoin_cent_sums(ret3, ret2)
```

#### **Arguments**

v a vector

max\_order the maximum order of the centered moment to be computed.

na\_rm whether to remove NA, false by default.

wts an optional vector of weights. Weights are 'replication' weights, meaning a

value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na\_rm is true. When there is no checking, an NA value will

cause the output to be NA.

check\_wts a boolean for whether the code shall check for negative weights, and throw an

error when they are found. Default false for speed.

normalize\_wts a boolean for whether the weights should be renormalized to have a mean value

of 1. This mean is computed over elements which contribute to the moments, so if na\_rm is set, that means non-NA elements of wts that correspond to non-NA

elements of the data vector.

ret1 an ord + 1 vector as output by cent\_sums consisting of the count, the mean,

then the k through ordth centered sum of some observations.

ret2 an ord + 1 vector as output by cent\_sums consisting of the count, the mean,

then the k through ordth centered sum of some observations.

ret3 an ord + 1 vector as output by cent\_sums consisting of the count, the mean,

then the k through ordth centered sum of some observations.

## Value

a vector the same size as the input consisting of the adjusted version of the input. When there are not sufficient (non-nan) elements for the computation, NaN are returned.

fromo-NEWS 15

## Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

 $Cook, J. \, D. \, "Comparing three \, methods \, of \, computing \, standard \, deviation." \, https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/$ 

# **Examples**

```
set.seed(1234)
x1 <- rnorm(1e3,mean=1)
x2 <- rnorm(1e3,mean=1)
max_ord <- 6L
rs1 <- cent_sums(x1,max_ord)
rs2 <- cent_sums(c(x1,x2),max_ord)
rs3 <- cent_sums(c(x1,x2),max_ord)
rs3alt <- join_cent_sums(rs1,rs2)
stopifnot(max(abs(rs3 - rs3alt)) < 1e-7)
rs1alt <- unjoin_cent_sums(rs3,rs2)
rs2alt <- unjoin_cent_sums(rs3,rs1)
stopifnot(max(abs(rs1 - rs1alt)) < 1e-7)
stopifnot(max(abs(rs2 - rs2alt)) < 1e-7)</pre>
```

fromo-NEWS

News for package 'fromo':

# **Description**

News for package 'fromo'

# Version 0.2.4 (2024-11-29)

- adding running correlation, covariance, regression coefficients.
- adding t-running correlation, covariance, regression coefficients.

## Version 0.2.3 (2024-11-07)

- · adding flag and checking for negative even moments due to roundoff.
- fixing divide by zero which would corrupt results when input was constant.

# Version 0.2.2 (2024-11-03)

• fix t\_running\_sum and others to act as documented when variable\_win is flagged.

## Version 0.2.1 (2019-01-29)

• fix memory leak for case where the mean only need be computed via a Welford object.

# Version 0.2.0 (2019-01-12)

- add std\_cumulants
- add running\_sum, running\_mean.
- Kahan compensated summation for these.
- Welford object under the hood.
- add weighted moments computation.
- add time-based running window computations.
- some speedups for obviously fast cases: no checking of NA, etc.
- move github figures to location CRAN understands.

# Version 0.1.3 (2016-04-04)

• submit to CRAN

# Initial Version 0.1.0 (2016-03-25)

· start work

# **Description**

Computes cumulants up to some given order, then employs the Cornish-Fisher approximation to compute approximate quantiles using a Gaussian basis.

## Usage

```
running_apx_quantiles(
  ٧,
 р,
 window = NULL,
 wts = NULL,
 max\_order = 5L,
 na_rm = FALSE,
 min_df = 0L,
  used_df = 0,
  restart_period = 100L,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_apx_median(
 window = NULL,
 wts = NULL,
 max\_order = 5L,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 0,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

## **Arguments**

v a vector

p the probability points at which to compute the quantiles. Should be in the range

(0,1).

window

the window size. If given as finite integer or double, passed through. If NULL, NA\_integer\_, NA\_real\_ or Inf are given, equivalent to an infinite window size. If negative, an error will be thrown.

wts

an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the

weight is NA when na\_rm is true. When there is no checking, an NA value will

cause the output to be NA.

max\_order the maximum order of the centered moment to be computed.

whether to remove NA, false by default. na\_rm

min\_df the minimum df to return a value, otherwise NaN is returned. This can be used

to prevent moments from being computed on too few observations. Defaults to

zero, meaning no restriction.

used\_df the number of degrees of freedom consumed, used in the denominator of the

centered moments computation. These are subtracted from the number of ob-

servations.

restart\_period the recompute period. because subtraction of elements can cause loss of preci-

> sion, the computation of moments is restarted periodically based on this parameter. Larger values mean fewer restarts and faster, though less accurate results.

a boolean for whether the code shall check for negative weights, and throw an check\_wts

error when they are found. Default false for speed.

normalize wts a boolean for whether the weights should be renormalized to have a mean value

> of 1. This mean is computed over elements which contribute to the moments, so if na\_rm is set, that means non-NA elements of wts that correspond to non-NA

elements of the data vector.

check\_negative\_moments

a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only

if you know what you are doing.

#### **Details**

Computes the cumulants, then approximates quantiles using AS269 of Lee & Lin.

# Value

A matrix, with one row for each element of x, and one column for each element of q.

## Note

The current implementation is not as space-efficient as it could be, as it first computes the cumulants for each row, then performs the Cornish-Fisher approximation on a row-by-row basis. In the future, this computation may be moved earlier into the pipeline to be more space efficient. File an issue if the memory footprint is an issue for you.

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

## Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

```
Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html
```

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

## See Also

```
t_running_apx_quantiles, running_cumulants, PDQutils::qapx_cf, PDQutils::AS269.
```

# **Examples**

```
x <- rnorm(1e5)
xq <- running_apx_quantiles(x,c(0.1,0.25,0.5,0.75,0.9))
xm <- running_apx_median(x)</pre>
```

running\_centered

Compare data to moments computed over a sliding window.

#### **Description**

Computes moments over a sliding window, then adjusts the data accordingly, centering, or scaling, or z-scoring, and so on.

# Usage

```
running_centered(
  ٧,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
  used_df = 1,
  lookahead = 0L,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = FALSE,
  check_negative_moments = TRUE
)
running_scaled(
  ٧,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  lookahead = 0L,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
running_zscored(
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
 lookahead = 0L,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_sharpe(
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
```

```
compute_se = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_tstat(
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

#### **Arguments**

v a vector

window

the window size. if given as finite integer or double, passed through. If NULL, NA\_integer\_, NA\_real\_ or Inf are given, equivalent to an infinite window size. If negative, an error will be thrown.

wts

an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na\_rm is true. When there is no checking, an NA value will cause the output to be NA.

na\_rm

whether to remove NA, false by default.

min\_df

the minimum df to return a value, otherwise NaN is returned. This can be used to prevent *e.g.* Z-scores from being computed on only 3 observations. Defaults to zero, meaning no restriction, which can result in infinite Z-scores during the burn-in period.

used\_df

the number of degrees of freedom consumed, used in the denominator of the centered moments computation. These are subtracted from the number of observations.

lookahead for some of the operations, the value is compared to mean and standard deviation

possibly using 'future' or 'past' information by means of a non-zero lookahead.

Positive values mean data are taken from the future.

restart\_period the recompute period. because subtraction of elements can cause loss of preci-

sion, the computation of moments is restarted periodically based on this parameter. Larger values mean fewer restarts and faster, though less accurate results.

check\_wts a boolean for whether the code shall check for negative weights, and throw an

error when they are found. Default false for speed.

normalize\_wts a boolean for whether the weights should be renormalized to have a mean value

of 1. This mean is computed over elements which contribute to the moments, so if na\_rm is set, that means non-NA elements of wts that correspond to non-NA

elements of the data vector.

check\_negative\_moments

a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only

if you know what you are doing.

 ${\tt compute\_se} \qquad \qquad {\tt for \ running\_sharpe, \ return \ an \ extra \ column \ of \ the \ standard \ error, \ as \ computed}$ 

by Mertens' correction.

#### **Details**

Given the length n vector x, for a given index i, define  $x^{(i)}$  as the vector of  $x_{i-window+1}, x_{i-window+2}, ..., x_i$ , where we do not run over the 'edge' of the vector. In code, this is essentially x[(max(1,i-window+1)):i]. Then define  $\mu_i$ ,  $\sigma_i$  and  $n_i$  as, respectively, the sample mean, standard deviation and number of non-NA elements in  $x^{(i)}$ .

We compute output vector m the same size as x. For the 'centered' version of x, we have  $m_i = x_i - \mu_i$ . For the 'scaled' version of x, we have  $m_i = x_i/\sigma_i$ . For the 'z-scored' version of x, we have  $m_i = (x_i - \mu_i)/\sigma_i$ . For the 't-scored' version of x, we have  $m_i = \sqrt{n_i \mu_i/\sigma_i}$ .

We also allow a 'lookahead' for some of these operations. If positive, the moments are computed using data from larger indices; if negative, from smaller indices. Letting j=i+lookahead: For the 'centered' version of x, we have  $m_i=x_i-\mu_j$ . For the 'scaled' version of x, we have  $m_i=x_i/\sigma_j$ . For the 'z-scored' version of x, we have  $m_i=(x_i-\mu_j)/\sigma_j$ .

#### Value

a vector the same size as the input consisting of the adjusted version of the input. When there are not sufficient (non-nan) elements for the computation, NaN are returned.

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

```
Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html
```

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

## See Also

t\_running\_centered, scale

#### **Examples**

```
if (require(moments)) {
    set.seed(123)
    x <- rnorm(5e1)
   window <- 10L
    rm1 <- t(sapply(seq_len(length(x)),function(iii) {</pre>
                   xrang <- x[max(1,iii-window+1):iii]</pre>
                   c(sd(xrang), mean(xrang), length(xrang)) },
                   simplify=TRUE))
    rcent <- running_centered(x,window=window)</pre>
    rscal <- running_scaled(x,window=window)</pre>
    rzsco <- running_zscored(x,window=window)</pre>
    rshrp <- running_sharpe(x,window=window)</pre>
    rtsco <- running_tstat(x,window=window)</pre>
    rsrse <- running_sharpe(x,window=window,compute_se=TRUE)</pre>
    stopifnot(max(abs(rcent - (x - rm1[,2])), na.rm=TRUE) < 1e-12)
    stopifnot(max(abs(rscal - (x / rm1[,1])), na.rm=TRUE) < 1e-12)
    stopifnot(max(abs(rzsco - ((x - rm1[,2]) / rm1[,1])), na.rm=TRUE) < 1e-12)
    stopifnot(max(abs(rshrp - (rm1[,2] / rm1[,1])), na.rm=TRUE) < 1e-12)
```

running\_correlation

Compute covariance, correlation, regression over a sliding window

# Description

Computes 2nd moments and comoments, as well as the means, over an infinite or finite sliding window, returning a matrix with the correlation, covariance, regression coefficient, and so on.

# Usage

```
running_correlation(
  Х,
 у,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 restart_period = 100L,
  check_wts = FALSE,
  check_negative_moments = TRUE
)
running_covariance(
 х,
 у,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
```

```
running_covariance_3(
 у,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
running_regression_slope(
 х,
 у,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 restart_period = 100L,
 check_wts = FALSE,
  check_negative_moments = TRUE
)
running_regression_intercept(
 Х,
 у,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
  restart_period = 100L,
  check_wts = FALSE,
 check_negative_moments = TRUE
)
running_regression_fit(
 х,
 у,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
  restart_period = 100L,
  check_wts = FALSE,
  check_negative_moments = TRUE
```

```
)
running_regression_diagnostics(
 у,
 window = NULL,
 wts = NULL,
  na_rm = FALSE,
 min_df = 0L,
  used_df = 2,
  restart_period = 100L,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

#### **Arguments**

a vector Х y a vector

window the window size. if given as finite integer or double, passed through. If NULL,

NA\_integer\_, NA\_real\_ or Inf are given, equivalent to an infinite window size.

If negative, an error will be thrown.

wts an optional vector of weights. Weights are 'replication' weights, meaning a

> value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na\_rm is true. When there is no checking, an NA value will

cause the output to be NA.

whether to remove NA, false by default. na\_rm

the minimum df to return a value, otherwise NaN is returned. This can be used min\_df to prevent moments from being computed on too few observations. Defaults to

zero, meaning no restriction.

restart\_period the recompute period. because subtraction of elements can cause loss of preci-

sion, the computation of moments is restarted periodically based on this parameter. Larger values mean fewer restarts and faster, though less accurate results.

check\_wts a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.

a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this

check\_negative\_moments

flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only if you know what you are doing.

used\_df

the number of degrees of freedom consumed, used in the denominator of the standard errors computation. These are subtracted from the number of observations.

normalize wts

a boolean for whether the weights should be renormalized to have a mean value of 1. This mean is computed over elements which contribute to the moments, so if na\_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector.

#### **Details**

Computes the correlation or covariance, or OLS regression coefficients and standard errors. These are computed via the numerically robust one-pass method of Bennett *et. al.* 

#### Value

Typically a matrix, usually only one row of the output value. More specifically:

**running\_covariance** Returns a single column of the covariance of x and y.

**running\_correlation** Returns a single column of the correlation of x and y.

**running\_covariance\_3** Returns three columns: the variance of x, the covariance of x and y, and the variance of y, in that order.

running\_regression\_slope Returns a single column of the slope of the OLS regression.

running\_regression\_intercept Returns a single column of the intercept of the OLS regression.

**running\_regression\_fit** Returns two columns: the regression intercept and the regression slope of the OLS regression.

**running\_regression\_diagnostics** Returns five columns: the regression intercept, the regression slope, the regression standard error, the standard error of the intercept, the standard error of the slope of the OLS regression.

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

28 running\_sd3

As this code may add and remove observations, numerical imprecision may result in negative estimates of squared quantities, like the second or fourth moments. By default we check for this condition in running computations. It may also be mitigated somewhat by setting a smaller restart\_period. Post an issue if you experience this bug.

## Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

```
Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html
```

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

# **Examples**

```
x <- rnorm(1e5)
y <- rnorm(1e5) + x
rho <- running_correlation(x, y, window=100L)</pre>
```

running\_sd3

Compute first K moments over a sliding window

# **Description**

Compute the (standardized) 2nd through kth moments, the mean, and the number of elements over an infinite or finite sliding window, returning a matrix.

## Usage

```
running_sd3(
    V,
    window = NULL,
    wts = NULL,
    na_rm = FALSE,
    min_df = 0L,
    used_df = 1,
    restart_period = 100L,
    check_wts = FALSE,
    normalize_wts = TRUE,
```

running\_sd3 29

```
check_negative_moments = TRUE
running_skew4(
 ٧,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
 restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
 check_negative_moments = TRUE
)
running_kurt5(
  ٧,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
 check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_sd(
  ٧,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
 check_negative_moments = TRUE
)
running_skew(
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
```

running\_sd3

```
used_df = 1,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_kurt(
  ٧,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_cent_moments(
 window = NULL,
 wts = NULL,
 max\_order = 5L,
 na_rm = FALSE,
 max_order_only = FALSE,
 min_df = 0L,
 used_df = 0,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
running_std_moments(
 window = NULL,
 wts = NULL,
 max\_order = 5L,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 0,
  restart_period = 100L,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

running\_sd3 31

```
running_cumulants(
    v,
    window = NULL,
    wts = NULL,
    max_order = 5L,
    na_rm = FALSE,
    min_df = 0L,
    used_df = 0,
    restart_period = 100L,
    check_wts = FALSE,
    normalize_wts = TRUE,
    check_negative_moments = TRUE
)
```

#### **Arguments**

v a vector

window the window size. if given as finite integer or double, passed through. If NULL,

 ${\tt NA\_integer\_, NA\_real\_ or \ Inf \ are \ given, \ equivalent \ to \ an \ infinite \ window \ size.}$ 

If negative, an error will be thrown.

wts an optional vector of weights. Weights are 'replication' weights, meaning a

value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na\_rm is true. When there is no checking, an NA value will

cause the output to be NA.

na\_rm whether to remove NA, false by default.

min\_df the minimum df to return a value, otherwise NaN is returned. This can be used

to prevent moments from being computed on too few observations. Defaults to

zero, meaning no restriction.

used\_df the number of degrees of freedom consumed, used in the denominator of the

centered moments computation. These are subtracted from the number of ob-

servations.

restart\_period the recompute period. because subtraction of elements can cause loss of preci-

sion, the computation of moments is restarted periodically based on this parameter. Larger values mean fewer restarts and faster, though less accurate results.

check\_wts a boolean for whether the code shall check for negative weights, and throw an

error when they are found. Default false for speed.

normalize\_wts a boolean for whether the weights should be renormalized to have a mean value

of 1. This mean is computed over elements which contribute to the moments, so

32 running\_sd3

> if na\_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector.

check\_negative\_moments

a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only if you know what you are doing.

max\_order

the maximum order of the centered moment to be computed.

max\_order\_only for running\_cent\_moments, if this flag is set, only compute the maximum order centered moment, and return in a vector.

#### **Details**

Computes the number of elements, the mean, and the 2nd through kth centered (and typically standardized) moments, for k=2,3,4. These are computed via the numerically robust one-pass method of Bennett et. al.

Given the length n vector x, we output matrix M where  $M_{i,j}$  is the order-j+1 moment (i.e. excess kurtosis, skewness, standard deviation, mean or number of elements) of  $x_{i-window+1}, x_{i-window+2}, ..., x_i$ . Barring NA or NaN, this is over a window of size window. During the 'burn-in' phase, we take fewer elements.

## Value

Typically a matrix, where the first columns are the kth, k-1th through 2nd standardized, centered moments, then a column of the mean, then a column of the number of (non-nan) elements in the input, with the following exceptions:

running\_cent\_moments Computes arbitrary order centered moments. When max\_order\_only is set, only a column of the maximum order centered moment is returned.

running\_std\_moments Computes arbitrary order standardized moments, then the standard deviation, the mean, and the count. There is not yet an option for max\_order\_only, but probably should be.

running\_cumulants Computes arbitrary order cumulants, and returns the kth, k-1th, through the second (which is the variance) cumulant, then the mean, and the count.

## Note

the kurtosis is excess kurtosis, with a 3 subtracted, and should be nearly zero for Gaussian input.

The moment computations provided by from are numerically robust, but will often not provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

running\_sd3 33

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

As this code may add and remove observations, numerical imprecision may result in negative estimates of squared quantities, like the second or fourth moments. By default we check for this condition in running computations. It may also be mitigated somewhat by setting a smaller restart\_period. Post an issue if you experience this bug.

## Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

 $Cook, J. \, D. \, "Comparing three \, methods \, of \, computing \, standard \, deviation." \, https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/$ 

## **Examples**

```
x <- rnorm(1e5)
xs3 <- running_sd3(x,10)
xs4 <- running_skew4(x,10)</pre>
if (require(moments)) {
    set.seed(123)
    x <- rnorm(5e1)
    window <- 10L
    kt5 <- running_kurt5(x,window=window)</pre>
    rm1 <- t(sapply(seq_len(length(x)),function(iii) {</pre>
                 xrang <- x[max(1,iii-window+1):iii]</pre>
                 c(moments::kurtosis(xrang)-3.0,moments::skewness(xrang),
                 sd(xrang),mean(xrang),length(xrang)) },
              simplify=TRUE))
    stopifnot(max(abs(kt5 - rm1),na.rm=TRUE) < 1e-12)</pre>
}
xc6 <- running_cent_moments(x,window=100L,max_order=6L)</pre>
```

running\_sum

running\_sum

Compute sums or means over a sliding window.

## **Description**

Compute the mean or sum over an infinite or finite sliding window, returning a vector the same size as the input.

## Usage

```
running_sum(
  ٧,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
  restart_period = 10000L,
  check_wts = FALSE
)
running_mean(
  ٧,
 window = NULL,
 wts = NULL,
 na_rm = FALSE,
 min_df = 0L,
 restart_period = 10000L,
  check_wts = FALSE
)
```

#### **Arguments**

v window a vector.

the window size. if given as finite integer or double, passed through. If NULL, NA\_integer\_, NA\_real\_ or Inf are given, equivalent to an infinite window size. If negative, an error will be thrown.

wts

an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na\_rm is true. When there is no checking, an NA value will cause the output to be NA.

running\_sum 35

na\_rm whether to remove NA, false by default.

restart\_period the recompute period. because subtraction of elements can cause loss of preci-

sion, the computation of moments is restarted periodically based on this parameter. Larger values mean fewer restarts and faster, though potentially less accurate results. Unlike in the computation of even order moments, loss of precision is

unlikely to be disastrous, so the default value is rather large.

check\_wts a boolean for whether the code shall check for negative weights, and throw an

error when they are found. Default false for speed.

min\_df the minimum df to return a value, otherwise NaN is returned, only for the means

computation. This can be used to prevent moments from being computed on too

few observations. Defaults to zero, meaning no restriction.

#### **Details**

Computes the mean or sum of the elements, using a Kahan's Compensated Summation Algorithm, a numerically robust one-pass method.

Given the length n vector x, we output matrix M where  $M_{i,1}$  is the sum or mean of  $x_{i-window+1}, x_{i-window+2}, ..., x_i$ . Barring NA or NaN, this is over a window of size window. During the 'burn-in' phase, we take fewer elements. If fewer than min\_df for running\_mean, returns NA.

#### Value

A vector the same size as the input.

# Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

## Author(s)

Steven E. Pav <shabbychef@gmail.com>

## References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

Kahan, W. "Further remarks on reducing truncation errors," Communications of the ACM, 8 (1), 1965. doi:10.1145/363707.363723

36 sd3

Wikipedia contributors "Kahan summation algorithm," Wikipedia, The Free Encyclopedia, https://en.wikipedia.org/w/index.php?title=Kahan\_summation\_algorithm&oldid=777164752 (accessed May 31, 2017).

# **Examples**

```
x <- rnorm(1e5)
xs <- running_sum(x,10)
xm <- running_mean(x,100)</pre>
```

sd3

Compute first K moments

# Description

Compute the (standardized) 2nd through kth moments, the mean, and the number of elements.

# Usage

```
sd3(
  ٧,
  na_rm = FALSE,
 wts = NULL,
  sg_df = 1,
  check_wts = FALSE,
  normalize\_wts = TRUE
)
skew4(
 na_rm = FALSE,
 wts = NULL,
  sg_df = 1,
  check_wts = FALSE,
  normalize_wts = TRUE
)
kurt5(
  ٧,
  na_rm = FALSE,
 wts = NULL,
  sg_df = 1,
  check_wts = FALSE,
  normalize\_wts = TRUE
)
cent_moments(
```

sd3 37

```
٧,
 max\_order = 5L,
 used_df = 0L,
 na_rm = FALSE,
 wts = NULL,
  check_wts = FALSE,
  normalize_wts = TRUE
)
std_moments(
  ٧,
 max_order = 5L,
 used_df = 0L,
  na_rm = FALSE,
 wts = NULL,
  check_wts = FALSE,
  normalize_wts = TRUE
)
cent_cumulants(
  ٧,
 max_order = 5L,
 used_df = 0L,
 na_rm = FALSE,
 wts = NULL,
  check_wts = FALSE,
  normalize_wts = TRUE
)
std_cumulants(
 max\_order = 5L,
 used_df = 0L,
  na_rm = FALSE,
 wts = NULL,
  check_wts = FALSE,
  normalize_wts = TRUE
)
```

# **Arguments**

wts

v a vector
na\_rm whether to remove NA, false by default.

an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check

38 sd3

for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na\_rm is true. When there is no checking, an NA value will cause the output to be NA.

sg\_df the number of degrees of freedom consumed in the computation of the variance

or standard deviation. This defaults to 1 to match the 'Bessel correction'.

check\_wts a boolean for whether the code shall check for negative weights, and throw an

error when they are found. Default false for speed.

normalize\_wts a boolean for whether the weights should be renormalized to have a mean value

of 1. This mean is computed over elements which contribute to the moments, so if na\_rm is set, that means non-NA elements of wts that correspond to non-NA

elements of the data vector.

max\_order the maximum order of the centered moment to be computed.

used\_df the number of degrees of freedom consumed, used in the denominator of the centered moments computation. These are subtracted from the number of ob-

centered moments computation. These are subtracted

servations.

## **Details**

Computes the number of elements, the mean, and the 2nd through kth centered standardized moment, for k = 2, 3, 4. These are computed via the numerically robust one-pass method of Bennett *et. al.* In general they will *not* match exactly with the 'standard' implementations, due to differences in roundoff.

These methods are reasonably fast, on par with the 'standard' implementations. However, they will usually be faster than calling the various standard implementations more than once.

Moments are computed as follows, given some values  $x_i$  and optional weights  $w_i$ , defaulting to 1, the weighted mean is computed as

$$\mu = \frac{\sum_{i} x_i w_i}{\sum_{i} w_i}.$$

The weighted kth central sum is computed as

$$S_k = \sum_i \left( x_i - \mu \right)^k w_i.$$

Let  $n = \sum_i w_i$  be the sum of weights (or number of observations in the unweighted case). Then the weighted kth central moment is computed as that weighted sum divided by the adjusted sum weights:

$$\mu_k = \frac{\sum_i (x_i - \mu)^k w_i}{n - \nu},$$

where  $\nu$  is the 'used df', provided by the user to adjust the denominator. (Typical values are 0 or 1.) The weighted kth standardized moment is the central moment divided by the second central moment to the k/2 power:

$$\tilde{\mu}_k = \frac{\mu_k}{\mu_2^{k/2}}.$$

The (centered) rth cumulant, for  $r \ge 2$  is then computed using the formula of Willink, namely

$$\kappa_r = \mu_r - \sum_{j=1}^{r-2} \binom{r-1}{j} \kappa_{r-j} \mu_j,$$

39

That is

$$\kappa_2 = \mu_2,$$
 $\kappa_3 = \mu_3,$ 
 $\kappa_4 = \mu_4 - 3\mu_2^2,$ 
 $\kappa_5 = \mu_5 - 10\mu_3\mu_2,$ 

and so on.

The standardized rth cumulant is the rth centered cumulant divided by  $\mu_2^{r/2}$ .

## Value

a vector, filled out as follows:

- **sd3** A vector of the (sample) standard devation, mean, and number of elements (or the total weight when wts are given).
- **skew4** A vector of the (sample) skewness, standard devation, mean, and number of elements (or the total weight when wts are given).
- **kurt5** A vector of the (sample) excess kurtosis, skewness, standard devation, mean, and number of elements (or the total weight when wts are given).
- **cent\_moments** A vector of the (sample) kth centered moment, then k-1th centered moment, ..., then the *variance*, the mean, and number of elements (total weight when wts are given).
- **std\_moments** A vector of the (sample) kth standardized (and centered) moment, then k-1th, ..., then standard devation, mean, and number of elements (total weight).
- **cent\_cumulants** A vector of the (sample) kth (centered, but this is redundant) cumulant, then the k-1th, ..., then the variance (which is the second cumulant), then the mean, then the number of elements (total weight).
- **std\_cumulants** A vector of the (sample) kth standardized (and centered, but this is redundant) cumulant, then the k-1th, ..., down to the third, then the variance, the mean, then the number of elements (total weight).

#### Note

The first centered (and standardized) moment is often defined to be identically 0. Instead cent\_moments and std\_moments returns the mean. Similarly, the second standardized moments defined to be identically 1; std\_moments instead returns the standard deviation. The reason is that a user can always decide to ignore the results and fill in a 0 or 1 as they need, but could not efficiently compute the mean and standard deviation from scratch if we discard it. The antepenultimate element of the output of std\_cumulants is not a one, even though that 'should' be the standardized second cumulant.

The antepenultimate element of the output of cent\_moments, cent\_cumulants and std\_cumulants is the *variance*, not the standard deviation. All other code return the standard deviation in that place.

40 sd3

The kurtosis is *excess kurtosis*, with a 3 subtracted, and should be nearly zero for Gaussian input.

The term 'centered cumulants' is redundant. The intent was to avoid possible collision with existing code named 'cumulants'.

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

## Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

Willink, R. "Relationships Between Central Moments and Cumulants, with Formulae for the Central Moments of Gamma Distributions." Communications in Statistics - Theory and Methods, 32 no 4 (2003): 701-704. doi:10.1081/STA120018823

## **Examples**

```
x <- rnorm(1e5)
sd3(x)[1] - sd(x)
skew4(x)[4] - length(x)
skew4(x)[3] - mean(x)
skew4(x)[2] - sd(x)
if (require(moments)) {
    skew4(x)[1] - skewness(x)
}

# check 'robustness'; only the mean should change:
kurt5(x + 1e12) - kurt5(x)
# check speed
if (require(microbenchmark) && require(moments)) {
    dumbk <- function(x) { c(kurtosis(x) - 3.0,skewness(x),sd(x),mean(x),length(x)) }
    set.seed(1234)</pre>
```

show 41

```
x <- rnorm(1e6)</pre>
  print(kurt5(x) - dumbk(x))
  microbenchmark(dumbk(x), kurt5(x), times=10L)
}
y <- std_moments(x,6)</pre>
cml <- cent_cumulants(x,6)</pre>
std <- std_cumulants(x,6)</pre>
# check that skew matches moments::skewness
if (require(moments)) {
    set.seed(1234)
    x <- rnorm(1000)
    resu <- fromo::skew4(x)
    msku <- moments::skewness(x)</pre>
    stopifnot(abs(msku - resu[1]) < 1e-14)</pre>
}
# check skew vs e1071 skewness, which has a different denominator
if (require(e1071)) {
    set.seed(1234)
    x <- rnorm(1000)
    resu <- fromo::skew4(x)</pre>
    esku <- e1071::skewness(x,type=3)
    nobs <- resu[4]</pre>
    stopifnot(abs(esku - resu[1] * ((nobs-1)/nobs)^(3/2)) < 1e-14)
    # similarly:
    resu <- fromo::std_moments(x,max_order=3,used_df=0)</pre>
    stopifnot(abs(esku - resu[1] * ((nobs-1)/nobs)^(3/2)) < 1e-14)
}
```

show

Show a centsums object.

# Description

Displays the centsums object.

# Usage

```
show(object)
## S4 method for signature 'centsums'
show(object)
```

## **Arguments**

object

a centsums object.

# Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

## Author(s)

Steven E. Pav <shabbychef@gmail.com>

# **Examples**

```
set.seed(123)
x <- rnorm(1000)
obj <- as.centsums(x, order=5)
obj</pre>
```

t\_running\_apx\_quantiles

Compute approximate quantiles over a sliding time window

# Description

Computes cumulants up to some given order, then employs the Cornish-Fisher approximation to compute approximate quantiles using a Gaussian basis.

# Usage

```
t_running_apx_quantiles(
  ٧,
 р,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
 max_order = 5L,
 na_rm = FALSE,
 min_df = 0L,
  used_df = 0,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

```
t_running_apx_median(
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
 max_order = 5L,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 0,
 restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

## **Arguments**

v a vector of data.

p the probability points at which to compute the quantiles. Should be in the range

(0,1).

time an optional vector of the timestamps of v. If given, must be the same length as

v. If not given, we try to infer it by summing the  $time\_deltas$ .

an optional vector of the deltas of timestamps. If given, must be the same length as v. If not given, and wts are given and wts\_as\_delta is true, we take the wts as the time deltas. The deltas must be positive. We sum them to arrive at the

times.

the window size, in time units. if given as finite integer or double, passed through. If NULL, NA\_integer\_, NA\_real\_ or Inf are given, and variable\_win is true, then we infer the window from the lookback times: the first window is infinite, but the remaining is the deltas between lookback times. If variable\_win is false, then these undefined values are equivalent to an infinite window. If

negative, an error will be thrown.

an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na\_rm is true. When there is no checking, an NA value will cause the output to be NA.

window

wts

lb\_time a vector of the times from which lookback will be performed. The output should

be the same size as this vector. If not given, defaults to time.

max\_order the maximum order of the centered moment to be computed.

na\_rm whether to remove NA, false by default.

min\_df the minimum df to return a value, otherwise NaN is returned. This can be used

to prevent moments from being computed on too few observations. Defaults to

zero, meaning no restriction.

used\_df the number of degrees of freedom consumed, used in the denominator of the

centered moments computation. These are subtracted from the number of ob-

servations.

restart\_period the recompute period. because subtraction of elements can cause loss of preci-

sion, the computation of moments is restarted periodically based on this parameter. Larger values mean fewer restarts and faster, though less accurate results.

variable\_win if true, and the window is not a concrete number, the computation window be-

comes the time between lookback times.

wts\_as\_delta if true and the time and time\_deltas are not given, but wts are given, we take

wts as the time\_deltas.

check\_wts a boolean for whether the code shall check for negative weights, and throw an

error when they are found. Default false for speed.

normalize\_wts a boolean for whether the weights should be renormalized to have a mean value

of 1. This mean is computed over elements which contribute to the moments, so if na\_rm is set, that means non-NA elements of wts that correspond to non-NA

elements of the data vector.

check\_negative\_moments

a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only

if you know what you are doing.

# Details

Computes the cumulants, then approximates quantiles using AS269 of Lee & Lin.

# Value

A matrix, with one row for each element of x, and one column for each element of q.

## **Time Windowing**

This function supports time (or other counter) based running computation. Here the input are the data  $x_i$ , and optional weights vectors,  $w_i$ , defaulting to 1, and a vector of time indices,  $t_i$  of the same length as x. The times must be non-decreasing:

$$t_1 \leq t_2 \leq \dots$$

It is assumed that  $t_0 = -\infty$ . The window, W is now a time-based window. An optional set of lookback times are also given,  $b_j$ , which may have different length than the x and w. The output will correspond to the lookback times, and should be the same length. The jth output is computed over indices i such that

$$b_i - W < t_i \le b_i$$
.

For comparison functions (like Z-score, rescaling, centering), which compare values of  $x_i$  to local moments, the lookbacks may not be given, but a lookahead L is admitted. In this case, the jth output is computed over indices i such that

$$t_i - W + L < t_i \le t_i + L.$$

If the times are not given, 'deltas' may be given instead. If  $\delta_i$  are the deltas, then we compute the times as

$$t_i = \sum_{1 < j < i} \delta_j.$$

The deltas must be the same length as x. If times and deltas are not given, but weights are given and the 'weights as deltas' flag is set true, then the weights are used as the deltas.

Some times it makes sense to have the computational window be the space between lookback times. That is, the jth output is to be computed over indices i such that

$$b_{j-1} - W < t_i \le b_j.$$

This can be achieved by setting the 'variable window' flag true and setting the window to null. This will not make much sense if the lookback times are equal to the times, since each moment computation is over a set of a single index, and most moments are underdefined.

## Note

The current implementation is not as space-efficient as it could be, as it first computes the cumulants for each row, then performs the Cornish-Fisher approximation on a row-by-row basis. In the future, this computation may be moved earlier into the pipeline to be more space efficient. File an issue if the memory footprint is an issue for you.

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

## Author(s)

```
Steven E. Pav <shabbychef@gmail.com>
```

## References

```
Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html
```

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

## See Also

```
running_apx_quantiles, t_running_cumulants, PDQutils::qapx_cf, PDQutils::AS269.
```

## **Examples**

t\_running\_centered

Compare data to moments computed over a time sliding window.

# Description

Computes moments over a sliding window, then adjusts the data accordingly, centering, or scaling, or z-scoring, and so on.

# Usage

```
t_running_centered(
  time = NULL,
  time_deltas = NULL,
  window = NULL,
 wts = NULL,
  na_rm = FALSE,
  min_df = 0L,
  used_df = 1,
  lookahead = 0,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_scaled(
  ٧,
  time = NULL,
  time_deltas = NULL,
  window = NULL,
 wts = NULL,
  na_rm = FALSE,
 min_df = 0L,
  used_df = 1,
  lookahead = 0,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_zscored(
  ٧,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
  wts = NULL,
  na_rm = FALSE,
  min_df = 0L,
  used_df = 1,
  lookahead = 0,
  restart_period = 100L,
```

```
variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_sharpe(
  ٧,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
  na_rm = FALSE,
  compute_se = FALSE,
 min_df = 0L,
  used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_tstat(
  ٧,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
  na_rm = FALSE,
  compute_se = FALSE,
 min_df = 0L,
  used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

# **Arguments**

v a vector of data.

time an optional vector of the timestamps of v. If given, must be the same length as

v. If not given, we try to infer it by summing the time\_deltas.

time\_deltas an optional vector of the deltas of timestamps. If given, must be the same length

as v. If not given, and wts are given and  $wts\_as\_delta$  is true, we take the wts as the time deltas. The deltas must be positive. We sum them to arrive at the

times.

window the window size, in time units. if given as finite integer or double, passed

through. If NULL, NA\_integer\_, NA\_real\_ or Inf are given, and variable\_win is true, then we infer the window from the lookback times: the first window is infinite, but the remaining is the deltas between lookback times. If variable\_win is false, then these undefined values are equivalent to an infinite window. If

negative, an error will be thrown.

wts an optional vector of weights. Weights are 'replication' weights, meaning a

value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na\_rm is true. When there is no checking, an NA value will

cause the output to be NA.

na\_rm whether to remove NA, false by default.

min\_df the minimum df to return a value, otherwise NaN is returned. This can be used

to prevent e.g. Z-scores from being computed on only 3 observations. Defaults to zero, meaning no restriction, which can result in infinite Z-scores during the

burn-in period.

used\_df the number of degrees of freedom consumed, used in the denominator of the

centered moments computation. These are subtracted from the number of ob-

servations.

lookahead for some of the operations, the value is compared to mean and standard deviation

possibly using 'future' or 'past' information by means of a non-zero lookahead. Positive values mean data are taken from the future. This is in time units, and so

should be a real.

restart\_period the recompute period. because subtraction of elements can cause loss of preci-

sion, the computation of moments is restarted periodically based on this parameter. Larger values mean fewer restarts and faster, though less accurate results.

variable\_win if true, and the window is not a concrete number, the computation window be-

comes the time between lookback times.

wts\_as\_delta if true and the time and time\_deltas are not given, but wts are given, we take

wts as the time\_deltas.

check\_wts a boolean for whether the code shall check for negative weights, and throw an

error when they are found. Default false for speed.

normalize\_wts

a boolean for whether the weights should be renormalized to have a mean value of 1. This mean is computed over elements which contribute to the moments, so if na\_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector.

check\_negative\_moments

a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only if you know what you are doing.

lb\_time

a vector of the times from which lookback will be performed. The output should be the same size as this vector. If not given, defaults to time.

compute\_se

for running\_sharpe, return an extra column of the standard error, as computed by Mertens' correction.

## **Details**

Given the length n vector x, for a given index i, define  $x^{(i)}$  as the elements of x defined by the sliding time window (see the section on time windowing). Then define  $\mu_i$ ,  $\sigma_i$  and  $n_i$  as, respectively, the sample mean, standard deviation and number of non-NA elements in  $x^{(i)}$ .

We compute output vector m the same size as x. For the 'centered' version of x, we have  $m_i = x_i - \mu_i$ . For the 'scaled' version of x, we have  $m_i = x_i/\sigma_i$ . For the 'z-scored' version of x, we have  $m_i = (x_i - \mu_i)/\sigma_i$ . For the 't-scored' version of x, we have  $m_i = \sqrt{n_i \mu_i/\sigma_i}$ .

We also allow a 'lookahead' for some of these operations. If positive, the moments are computed using data from larger indices; if negative, from smaller indices.

## Value

a vector the same size as the input consisting of the adjusted version of the input. When there are not sufficient (non-nan) elements for the computation, NaN are returned.

# **Time Windowing**

This function supports time (or other counter) based running computation. Here the input are the data  $x_i$ , and optional weights vectors,  $w_i$ , defaulting to 1, and a vector of time indices,  $t_i$  of the same length as x. The times must be non-decreasing:

$$t_1 \leq t_2 \leq \dots$$

It is assumed that  $t_0 = -\infty$ . The window, W is now a time-based window. An optional set of lookback times are also given,  $b_j$ , which may have different length than the x and w. The output will correspond to the lookback times, and should be the same length. The jth output is computed over indices i such that

$$b_i - W < t_i \le b_i$$
.

For comparison functions (like Z-score, rescaling, centering), which compare values of  $x_i$  to local moments, the lookbacks may not be given, but a lookahead L is admitted. In this case, the jth output is computed over indices i such that

$$t_i - W + L < t_i \le t_i + L$$
.

If the times are not given, 'deltas' may be given instead. If  $\delta_i$  are the deltas, then we compute the times as

$$t_i = \sum_{1 \le i \le i} \delta_j.$$

The deltas must be the same length as x. If times and deltas are not given, but weights are given and the 'weights as deltas' flag is set true, then the weights are used as the deltas.

Some times it makes sense to have the computational window be the space between lookback times. That is, the jth output is to be computed over indices i such that

$$b_{i-1} - W < t_i \le b_i.$$

This can be achieved by setting the 'variable window' flag true and setting the window to null. This will not make much sense if the lookback times are equal to the times, since each moment computation is over a set of a single index, and most moments are underdefined.

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

## Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

 $Cook, J.\ D.\ "Comparing three methods of computing standard deviation." \ https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/$ 

## See Also

running\_centered, scale

# **Description**

Computes 2nd moments and comoments, as well as the means, over an infinite or finite sliding time based window, returning a matrix with the correlation, covariance, regression coefficient, and so on.

## Usage

```
t_running_correlation(
 х,
 у,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
 na_rm = FALSE,
 min_df = 0L,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  check_negative_moments = TRUE
t_running_covariance(
 у,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
  used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
 normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

```
t_running_covariance_3(
 у,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_regression_slope(
 х,
 у,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
 na_rm = FALSE,
 min_df = 0L,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  check_negative_moments = TRUE
)
t_running_regression_intercept(
 х,
 у,
 time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
  restart_period = 100L,
  variable_win = FALSE,
```

```
wts_as_delta = TRUE,
  check_wts = FALSE,
  check_negative_moments = TRUE
)
t_running_regression_fit(
  х,
 у,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  check_negative_moments = TRUE
)
t_running_regression_diagnostics(
 х,
 у,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
  used_df = 2,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

# **Arguments**

as x and y. If not given, and wts are given and wts\_as\_delta is true, we take the wts as the time deltas. The deltas must be positive. We sum them to arrive at the times.

window

the window size, in time units. if given as finite integer or double, passed through. If NULL, NA\_integer\_, NA\_real\_ or Inf are given, and variable\_win is true, then we infer the window from the lookback times: the first window is infinite, but the remaining is the deltas between lookback times. If variable\_win is false, then these undefined values are equivalent to an infinite window. If negative, an error will be thrown.

wts

an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na\_rm is true. When there is no checking, an NA value will cause the output to be NA.

1b\_time

a vector of the times from which lookback will be performed. The output should be the same size as this vector. If not given, defaults to time.

na\_rm

whether to remove NA, false by default.

min\_df

the minimum df to return a value, otherwise NaN is returned. This can be used to prevent moments from being computed on too few observations. Defaults to zero, meaning no restriction.

restart\_period

the recompute period. because subtraction of elements can cause loss of precision, the computation of moments is restarted periodically based on this parameter. Larger values mean fewer restarts and faster, though less accurate results.

variable\_win

if true, and the window is not a concrete number, the computation window becomes the time between lookback times.

wts\_as\_delta

if true and the time and time\_deltas are not given, but wts are given, we take wts as the time\_deltas.

check\_wts

a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.

## check\_negative\_moments

a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only if you know what you are doing.

used\_df the number of degrees of freedom consumed, used in the denominator of the

standard errors computation. These are subtracted from the number of observa-

tions.

normalize\_wts a boolean for whether the weights should be renormalized to have a mean value

of 1. This mean is computed over elements which contribute to the moments, so if na\_rm is set, that means non-NA elements of wts that correspond to non-NA

elements of the data vector.

#### **Details**

Computes the correlation or covariance, or OLS regression coefficients and standard errors. These are computed via the numerically robust one-pass method of Bennett *et. al.* 

## Value

Typically a matrix, usually only one row of the output value. More specifically:

**running\_covariance** Returns a single column of the covariance of x and y.

running\_correlation Returns a single column of the correlation of x and y.

**running\_covariance\_3** Returns three columns: the variance of x, the covariance of x and y, and the variance of y, in that order.

running\_regression\_slope Returns a single column of the slope of the OLS regression.

running\_regression\_intercept Returns a single column of the intercept of the OLS regression.

**running\_regression\_fit** Returns two columns: the regression intercept and the regression slope of the OLS regression.

**running\_regression\_diagnostics** Returns five columns: the regression intercept, the regression slope, the regression standard error, the standard error of the intercept, the standard error of the slope of the OLS regression.

## **Time Windowing**

This function supports time (or other counter) based running computation. Here the input are the data  $x_i$ , and optional weights vectors,  $w_i$ , defaulting to 1, and a vector of time indices,  $t_i$  of the same length as x. The times must be non-decreasing:

$$t_1 \leq t_2 \leq \dots$$

It is assumed that  $t_0 = -\infty$ . The window, W is now a time-based window. An optional set of lookback times are also given,  $b_j$ , which may have different length than the x and w. The output will correspond to the lookback times, and should be the same length. The jth output is computed over indices i such that

$$b_j - W < t_i \le b_j.$$

For comparison functions (like Z-score, rescaling, centering), which compare values of  $x_i$  to local moments, the lookbacks may not be given, but a lookahead L is admitted. In this case, the jth output is computed over indices i such that

$$t_i - W + L < t_i \le t_i + L.$$

If the times are not given, 'deltas' may be given instead. If  $\delta_i$  are the deltas, then we compute the times as

$$t_i = \sum_{1 \le j \le i} \delta_j.$$

The deltas must be the same length as x. If times and deltas are not given, but weights are given and the 'weights as deltas' flag is set true, then the weights are used as the deltas.

Some times it makes sense to have the computational window be the space between lookback times. That is, the *j*th output is to be computed over indices *i* such that

$$b_{j-1} - W < t_i \le b_j.$$

This can be achieved by setting the 'variable window' flag true and setting the window to null. This will not make much sense if the lookback times are equal to the times, since each moment computation is over a set of a single index, and most moments are underdefined.

#### Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

As this code may add and remove observations, numerical imprecision may result in negative estimates of squared quantities, like the second or fourth moments. By default we check for this condition in running computations. It may also be mitigated somewhat by setting a smaller restart\_period. Post an issue if you experience this bug.

# Author(s)

Steven E. Pav <shabbychef@gmail.com>

#### References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

58 t\_running\_sd3

## **Examples**

```
x <- rnorm(1e5)
y <- rnorm(1e5) + x
rho <- t_running_correlation(x, y, time=seq_along(x), window=100L)</pre>
```

t\_running\_sd3

Compute first K moments over a sliding time-based window

# **Description**

Compute the (standardized) 2nd through kth moments, the mean, and the number of elements over an infinite or finite sliding time based window, returning a matrix.

## Usage

```
t_running_sd3(
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
 na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_skew4(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
  used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
```

t\_running\_sd3 59

```
check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_kurt5(
  ٧,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
  used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_sd(
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
  used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_skew(
  ٧,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
```

t\_running\_sd3

```
na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_kurt(
  ٧,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
  na_rm = FALSE,
 min_df = 0L,
 used_df = 1,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_cent_moments(
  ν,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
 max\_order = 5L,
 na_rm = FALSE,
 max_order_only = FALSE,
 min_df = 0L,
 used_df = 0,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

t\_running\_sd3 61

```
t_running_std_moments(
  ٧,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
 max_order = 5L,
 na_rm = FALSE,
 min_df = 0L,
  used_df = 0,
  restart_period = 100L,
  variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
t_running_cumulants(
  ٧,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
 max_order = 5L,
  na_rm = FALSE,
 min_df = 0L,
  used_df = 0,
  restart_period = 100L,
  variable_win = FALSE,
  wts_as_delta = TRUE,
  check_wts = FALSE,
  normalize_wts = TRUE,
  check_negative_moments = TRUE
)
```

# Arguments

a vector of data.

time an optional vector of the timestamps of v. If given, must be the same length as

v. If not given, we try to infer it by summing the time\_deltas.

time\_deltas an optional vector of the deltas of timestamps. If given, must be the same length as v. If not given, and wts are given and wts\_as\_delta is true, we take the wts

as v. If not given, and wes are given and wes\_as\_as\_astreams true, we take the wes as the time deltas. The deltas must be positive. We sum them to arrive at the

times.

62 t\_running\_sd3

window

the window size, in time units. if given as finite integer or double, passed through. If NULL, NA\_integer\_, NA\_real\_ or Inf are given, and variable\_win is true, then we infer the window from the lookback times: the first window is infinite, but the remaining is the deltas between lookback times. If variable\_win is false, then these undefined values are equivalent to an infinite window. If negative, an error will be thrown.

wts

an optional vector of weights. Weights are 'replication' weights, meaning a value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na\_rm is true. When there is no checking, an NA value will cause the output to be NA.

lb\_time

a vector of the times from which lookback will be performed. The output should be the same size as this vector. If not given, defaults to time.

na\_rm

whether to remove NA, false by default.

min\_df

the minimum df to return a value, otherwise NaN is returned. This can be used to prevent moments from being computed on too few observations. Defaults to zero, meaning no restriction.

used\_df

the number of degrees of freedom consumed, used in the denominator of the centered moments computation. These are subtracted from the number of observations.

restart\_period

the recompute period. because subtraction of elements can cause loss of precision, the computation of moments is restarted periodically based on this parameter. Larger values mean fewer restarts and faster, though less accurate results.

variable\_win

if true, and the window is not a concrete number, the computation window becomes the time between lookback times.

wts\_as\_delta

if true and the time and time\_deltas are not given, but wts are given, we take wts as the time\_deltas.

check\_wts

a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.

normalize wts

a boolean for whether the weights should be renormalized to have a mean value of 1. This mean is computed over elements which contribute to the moments, so if na\_rm is set, that means non-NA elements of wts that correspond to non-NA elements of the data vector.

# check\_negative\_moments

a boolean flag. Normal computation of running moments can result in negative estimates of even order moments due to loss of numerical precision. With this flag active, the computation checks for negative even order moments and restarts the computation when one is detected. This should eliminate the possibility of negative even order moments. The downside is the speed hit of checking on

 $t_{running\_sd3}$  63

every output step. Note also the code checks for negative moments of every even order tracked, even if they are not output; that is if the kurtosis, say, is being computed, and a negative variance is detected, then the computation is restarted. Defaults to TRUE to avoid negative even moments. Set to FALSE only if you know what you are doing.

max\_order

the maximum order of the centered moment to be computed.

max\_order\_only for running\_cent\_moments, if this flag is set, only compute the maximum order centered moment, and return in a vector.

## **Details**

Computes the number of elements, the mean, and the 2nd through kth centered (and typically standardized) moments, for k = 2, 3, 4. These are computed via the numerically robust one-pass method of Bennett *et. al.* 

Given the length n vector x, we output matrix M where  $M_{i,j}$  is the order-j+1 moment (i.e. excess kurtosis, skewness, standard deviation, mean or number of elements) of some elements  $x_i$  defined by the sliding time window. Barring NA or NaN, this is over a window of time width window.

#### Value

Typically a matrix, where the first columns are the kth, k-1th through 2nd standardized, centered moments, then a column of the mean, then a column of the number of (non-nan) elements in the input, with the following exceptions:

**t\_running\_cent\_moments** Computes arbitrary order centered moments. When max\_order\_only is set, only a column of the maximum order centered moment is returned.

t\_running\_std\_moments Computes arbitrary order standardized moments, then the standard deviation, the mean, and the count. There is not yet an option for max\_order\_only, but probably should be.

**t\_running\_cumulants** Computes arbitrary order cumulants, and returns the kth, k-1th, through the second (which is the variance) cumulant, then the mean, and the count.

## **Time Windowing**

This function supports time (or other counter) based running computation. Here the input are the data  $x_i$ , and optional weights vectors,  $w_i$ , defaulting to 1, and a vector of time indices,  $t_i$  of the same length as x. The times must be non-decreasing:

$$t_1 \leq t_2 \leq \dots$$

It is assumed that  $t_0 = -\infty$ . The window, W is now a time-based window. An optional set of lookback times are also given,  $b_j$ , which may have different length than the x and w. The output will correspond to the lookback times, and should be the same length. The jth output is computed over indices i such that

$$b_i - W < t_i \le b_i$$
.

For comparison functions (like Z-score, rescaling, centering), which compare values of  $x_i$  to local moments, the lookbacks may not be given, but a lookahead L is admitted. In this case, the jth output is computed over indices i such that

$$t_i - W + L < t_i \le t_i + L.$$

64 t\_running\_sd3

If the times are not given, 'deltas' may be given instead. If  $\delta_i$  are the deltas, then we compute the times as

$$t_i = \sum_{1 \le j \le i} \delta_j.$$

The deltas must be the same length as x. If times and deltas are not given, but weights are given and the 'weights as deltas' flag is set true, then the weights are used as the deltas.

Some times it makes sense to have the computational window be the space between lookback times. That is, the jth output is to be computed over indices i such that

$$b_{i-1} - W < t_i \le b_i.$$

This can be achieved by setting the 'variable window' flag true and setting the window to null. This will not make much sense if the lookback times are equal to the times, since each moment computation is over a set of a single index, and most moments are underdefined.

#### Note

the kurtosis is excess kurtosis, with a 3 subtracted, and should be nearly zero for Gaussian input.

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

As this code may add and remove observations, numerical imprecision may result in negative estimates of squared quantities, like the second or fourth moments. By default we check for this condition in running computations. It may also be mitigated somewhat by setting a smaller restart\_period. Post an issue if you experience this bug.

## Author(s)

Steven E. Pav <shabbychef@gmail.com>

## References

Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

t\_running\_sum 65

## See Also

```
running_sd3.
```

# **Examples**

```
x <- rnorm(1e5)
xs3 <- t_running_sd3(x,time=seq_along(x),window=10)
xs4 <- t_running_skew4(x,time=seq_along(x),window=10)
# but what if you only cared about some middle values?
xs4 <- t_running_skew4(x,time=seq_along(x),lb_time=(length(x) / 2) + 0:10,window=20)</pre>
```

t\_running\_sum

Compute sums or means over a sliding time window.

# **Description**

Compute the mean or sum over an infinite or finite sliding time window, returning a vector the same size as the lookback times.

# Usage

```
t_running_sum(
  ٧,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
 lb_time = NULL,
 na_rm = FALSE,
 min_df = 0L,
 restart_period = 10000L,
 variable_win = FALSE,
 wts_as_delta = TRUE,
  check_wts = FALSE
)
t_running_mean(
  ٧,
  time = NULL,
  time_deltas = NULL,
 window = NULL,
 wts = NULL,
  lb_time = NULL,
 na_rm = FALSE,
 min_df = 0L,
  restart_period = 10000L,
```

66 t\_running\_sum

```
variable_win = FALSE,
wts_as_delta = TRUE,
check_wts = FALSE
)
```

## **Arguments**

v a vector.

time an optional vector of the timestamps of v. If given, must be the same length as

v. If not given, we try to infer it by summing the time\_deltas.

time\_deltas an optional vector of the deltas of timestamps. If given, must be the same length

as v. If not given, and wts are given and wts\_as\_delta is true, we take the wts as the time deltas. The deltas must be positive. We sum them to arrive at the

times.

window the window size, in time units. if given as finite integer or double, passed

through. If NULL, NA\_integer\_, NA\_real\_ or Inf are given, and variable\_win is true, then we infer the window from the lookback times: the first window is infinite, but the remaining is the deltas between lookback times. If variable\_win is false, then these undefined values are equivalent to an infinite window. If

negative, an error will be thrown.

wts an optional vector of weights. Weights are 'replication' weights, meaning a

value of 2 is shorthand for having two observations with the corresponding v value. If NULL, corresponds to equal unit weights, the default. Note that weights are typically only meaningfully defined up to a multiplicative constant, meaning the units of weights are immaterial, with the exception that methods which check for minimum df will, in the weighted case, check against the sum of weights. For this reason, weights less than 1 could cause NA to be returned unexpectedly due to the minimum condition. When weights are NA, the same rules for checking v are applied. That is, the observation will not contribute to the moment if the weight is NA when na\_rm is true. When there is no checking, an NA value will

cause the output to be NA.

lb\_time a vector of the times from which lookback will be performed. The output should

be the same size as this vector. If not given, defaults to time.

na\_rm whether to remove NA, false by default.

min\_df the minimum df to return a value, otherwise NaN is returned, only for the means

computation. This can be used to prevent moments from being computed on too

few observations. Defaults to zero, meaning no restriction.

restart\_period the recompute period. because subtraction of elements can cause loss of preci-

sion, the computation of moments is restarted periodically based on this parameter. Larger values mean fewer restarts and faster, though potentially less accurate results. Unlike in the computation of even order moments, loss of precision is

unlikely to be disastrous, so the default value is rather large.

variable\_win if true, and the window is not a concrete number, the computation window be-

comes the time between lookback times.

wts\_as\_delta if true and the time and time\_deltas are not given, but wts are given, we take

wts as the time\_deltas.

t\_running\_sum 67

check\_wts

a boolean for whether the code shall check for negative weights, and throw an error when they are found. Default false for speed.

## **Details**

Computes the mean or sum of the elements, using a Kahan's Compensated Summation Algorithm, a numerically robust one-pass method.

Given the length n vector x, we output matrix M where  $M_{i,1}$  is the sum or mean of some elements  $x_i$  defined by the sliding time window. Barring NA or NaN, this is over a window of time width window.

## Value

A vector the same size as the lookback times.

## **Time Windowing**

This function supports time (or other counter) based running computation. Here the input are the data  $x_i$ , and optional weights vectors,  $w_i$ , defaulting to 1, and a vector of time indices,  $t_i$  of the same length as x. The times must be non-decreasing:

$$t_1 \leq t_2 \leq \dots$$

It is assumed that  $t_0 = -\infty$ . The window, W is now a time-based window. An optional set of lookback times are also given,  $b_j$ , which may have different length than the x and w. The output will correspond to the lookback times, and should be the same length. The jth output is computed over indices i such that

$$b_j - W < t_i \le b_j$$
.

For comparison functions (like Z-score, rescaling, centering), which compare values of  $x_i$  to local moments, the lookbacks may not be given, but a lookahead L is admitted. In this case, the jth output is computed over indices i such that

$$t_j - W + L < t_i \le t_j + L.$$

If the times are not given, 'deltas' may be given instead. If  $\delta_i$  are the deltas, then we compute the times as

$$t_i = \sum_{1 < j < i} \delta_j.$$

The deltas must be the same length as x. If times and deltas are not given, but weights are given and the 'weights as deltas' flag is set true, then the weights are used as the deltas.

Some times it makes sense to have the computational window be the space between lookback times. That is, the jth output is to be computed over indices i such that

$$b_{i-1} - W < t_i < b_i$$
.

This can be achieved by setting the 'variable window' flag true and setting the window to null. This will not make much sense if the lookback times are equal to the times, since each moment computation is over a set of a single index, and most moments are underdefined.

## Note

The moment computations provided by fromo are numerically robust, but will often *not* provide the same results as the 'standard' implementations, due to differences in roundoff. We make every attempt to balance speed and robustness. User assumes all risk from using the fromo package.

Note that when weights are given, they are treated as replication weights. This can have subtle effects on computations which require minimum degrees of freedom, since the sum of weights will be compared to that minimum, not the number of data points. Weight values (much) less than 1 can cause computations to return NA somewhat unexpectedly due to this condition, while values greater than one might cause the computation to spuriously return a value with little precision.

#### Author(s)

Steven E. Pav <shabbychef@gmail.com>

## References

```
Terriberry, T. "Computing Higher-Order Moments Online." https://web.archive.org/web/20140423031833/http://people.xiph.org/~tterribe/notes/homs.html
```

J. Bennett, et. al., "Numerically Stable, Single-Pass, Parallel Statistics Algorithms," Proceedings of IEEE International Conference on Cluster Computing, 2009. doi:10.1109/CLUSTR.2009.5289161

Cook, J. D. "Accurately computing running variance." https://www.johndcook.com/standard\_deviation/

Cook, J. D. "Comparing three methods of computing standard deviation." https://www.johndcook.com/blog/2008/09/26/comparing-three-methods-of-computing-standard-deviation/

Kahan, W. "Further remarks on reducing truncation errors," Communications of the ACM, 8 (1), 1965. doi:10.1145/363707.363723

Wikipedia contributors "Kahan summation algorithm," Wikipedia, The Free Encyclopedia, https://en.wikipedia.org/w/index.php?title=Kahan\_summation\_algorithm&oldid=777164752 (accessed May 31, 2017).

# **Examples**

```
x <- rnorm(1e5)
xs <- t_running_sum(x,time=seq_along(x),window=10)
xm <- t_running_mean(x,time=cumsum(runif(length(x))),window=7.3)</pre>
```

%-%, centcosums, centcosums-method

unconcatenate centcosums objects.

# Description

Unconcatenate centcosums objects.

%-%

# Usage

```
## S4 method for signature 'centcosums,centcosums' x %-% y
```

# Arguments

```
x a centcosums objectsy a centcosums objects
```

# See Also

```
unjoin_cent_cosums
```

%-%

unconcatenate centsums objects.

# Description

Unconcatenate centsums objects.

# Usage

# **Arguments**

```
x a centsums objectsy a centsums objects
```

# See Also

```
unjoin_cent_sums
```

# **Index**

centcosums-class, 9 centsums-class, 10  * package fromo-package, 2 %-%, centsums, centsums-method (%-%), 69 %-%, 69 %-%, centcosums, centcosums-method, 68 accessor, 4 as.centcosums, 5  c.centsums, 7 cent2raw, 8  (centsums-class), 10 initialize, centsums-method (centsums-class), 10 join_cent_cosums (cent_cosums), 12 join_cent_sums (cent_sums), 13 kurt5 (sd3), 36  kurt5 (sd3), 36  running_apx_median (running_apx_quantiles), 16 running_apx_quantiles), 16 running_apx_quantiles, 16, 46
* package fromo-package, 2 %-%,centsums,centsums-method (%-%), 69 %-%, 69 %-%,centcosums,centcosums-method, 68 accessor, 4 as.centcosums, 4 as.centsums, 5  c.centcosums, 7 c.centcosums, 7 cent2raw, 8  (centsums-class), 10 join_cent_cosums (cent_cosums), 12 join_cent_sums (cent_sums), 13  kurt5 (sd3), 36  moments (accessor), 4 moments,centsums-method (accessor), 4 running_apx_median (running_apx_quantiles), 16 running_apx_quantiles, 16, 46
fromo-package, 2 %-%, centsums, centsums-method (%-%), 69 %-%, 69 %-%, centcosums, centcosums-method, 68 accessor, 4 as.centcosums, 4 as.centsums, 5  c.centcosums, 7 c.centcosums, 7 c.cent2raw, 8  join_cent_cosums (cent_cosums), 12 join_cent_sums (cent_sums), 13 kurt5 (sd3), 36  kurt5 (sd3), 36  running_apx_median (running_apx_median (running_apx_quantiles), 16 running_apx_quantiles, 16, 46
%-%, centsums, centsums-method (%-%), 69 %-%, 69 %-%, centcosums, centcosums-method, 68 %-%, centcosums, centcosums-method, 68 accessor, 4 as.centcosums, 4 as.centsums, 5 moments (accessor), 4 moments, centsums-method (accessor), 4 c.centcosums, 7 c.centsums, 7 cent2raw, 8  join_cent_cosums (cent_cosums), 12 join_cent_sums (cent_sums), 13 kurt5 (sd3), 36  running_apx_median (running_apx_median (running_apx_quantiles), 16 running_apx_quantiles), 16 running_apx_quantiles, 16, 46
%-%, 69  %-%, centcosums, centcosums-method, 68  accessor, 4 as.centcosums, 4 as.centsums, 5  c.centcosums, 7 c.centsums, 7 cent2raw, 8  join_cent_sums (cent_sums), 13  kurt5 (sd3), 36  moments (accessor), 4 moments, centsums-method (accessor), 4  running_apx_median
%-%, centcosums, centcosums-method, 68  accessor, 4  as.centcosums, 4  as.centsums, 5  moments (accessor), 4  moments, centsums-method (accessor), 4  c.centcosums, 7  c.centsums, 7  c.centsums, 7  cent2raw, 8  kurt5 (sd3), 36  moments (accessor), 4  moments, centsums-method (accessor), 4  running_apx_median
kurt5 (sd3), 36  accessor, 4  as.centcosums, 4  as.centsums, 5  moments (accessor), 4  moments, centsums-method (accessor), 4  c.centcosums, 7  c.centsums, 7  c.centsums, 7  cent2raw, 8  kurt5 (sd3), 36  moments (accessor), 4  moments, centsums-method (accessor), 4  running_apx_median  (running_apx_quantiles), 16  running_apx quantiles, 16, 46
accessor, 4 as.centcosums, 4 as.centsums, 5  moments (accessor), 4 moments, centsums-method (accessor), 4  c.centcosums, 7 c.centsums, 7 c.centsums, 7 cent2raw, 8  moments (accessor), 4  running_apx_median (running_apx_quantiles), 16 running_apx_quantiles, 16, 46
as.centcosums, 4 as.centsums, 5  moments (accessor), 4 moments, centsums-method (accessor), 4  c.centcosums, 7 c.centsums, 7 c.centsums, 7 cent2raw, 8  moments (accessor), 4 mo
as.centsums, 5 moments, centsums-method (accessor), 4  c.centcosums, 7 c.centsums, 7 c.centsums, 7 cent2raw, 8 running_apx_median (running_apx_quantiles), 16 running_apx_quantiles, 16, 46
c.centcosums, 7 c.centsums, 7 cent2raw, 8  running_apx_median (running_apx_quantiles), 16 running_apx_quantiles, 16, 46
c.centsums, 7 (running_apx_quantiles), 16 cent2raw, 8 running_apx_quantiles, 16, 46
c.centsums, 7 (running_apx_quantiles), 16 cent2raw, 8 running apx quantiles. 16, 46
cent2raw, 8 running apx quantiles, 16, 46
cent_comoments (cent_cosums), 12 running_cent_moments (running_sd3), 28
cent_cosums, 9, 12, 12 running centered, 19, 51
cent_cumulants (sd3), 36 running correlation, 24
cent_moments(sd3), 36 running covariance
cent_sums, 13, 14 (running correlation), 24
centcosums (centcosums-class), 9 running covariance 3
centcosums-accessor, 8 (running correlation), 24
centcosums-class, 9 running cumulants, 19
centsums (centsums-class), 10 running cumulants (running sd3), 28
centsums-class, 10 running kurt (running sd3), 28
comoments (centcosums-accessor), 8 running kurt5 (running sd3), 28
comoments, centcosums-method running mean 16
(centcosums-accessor), 8 running mean(running sum), 34
cosums (centcosums-accessor), 8 running regression diagnostics
cosums, centcosums-method (running correlation). 24
(centcosums-accessor), 8 running_regression_fit
fromo (fromo-package), 2 (running_correlation), 24
fromo-NEWS, 15 running_regression_intercept
fromo-package, 2 (running_correlation), 24
running_regression_slope
initialize, centcosums-class (running_correlation), 24
(centcosums-class), 9 running_scaled (running_centered), 19
initialize, centcosums-method running_sd (running_sd3), 28
(centcosums-class), 9 running_sd3, 28, 65

INDEX 71

running_sharpe (running_centered), 19	t_running_sharpe(t_running_centered),
running_skew(running_sd3), 28	46
running_skew4 (running_sd3), 28	t_running_skew(t_running_sd3), 58
running_std_moments(running_sd3), 28	t_running_skew4 (t_running_sd3), 58
running_sum, <i>16</i> , 34	t_running_std_moments(t_running_sd3),
running_tstat(running_centered),19	58
running_zscored(running_centered),19	t_running_sum, 65
	t_running_tstat (t_running_centered), 46
scale, 23, 51	<pre>t_running_zscored(t_running_centered),</pre>
sd3, 36	46
show, 41	
show, centsums-method (show), 41	unjoin_cent_cosums (cent_cosums), 12
skew4 (sd3), 36	<pre>unjoin_cent_sums (cent_sums), 13</pre>
std_cumulants(sd3),36	
std_moments (sd3), 36	
sums (accessor), 4	
sums, centcosums-method	
(centcosums-accessor), 8	
sums, centsums-method (accessor), 4	
t_running_apx_median	
<pre>(t_running_apx_quantiles), 42</pre>	
t_running_apx_quantiles, 19,42	
t_running_cent_moments(t_running_sd3),	
58	
t_running_centered, 23, 46	
t_running_correlation, 52	
t_running_covariance	
<pre>(t_running_correlation), 52</pre>	
t_running_covariance_3	
<pre>(t_running_correlation), 52</pre>	
t_running_cumulants, 46	
t_running_cumulants(t_running_sd3),58	
t_running_kurt(t_running_sd3),58	
t_running_kurt5(t_running_sd3),58	
t_running_mean(t_running_sum),65	
t_running_regression_diagnostics	
(t_running_correlation), 52	
t_running_regression_fit	
(t_running_correlation), 52	
t_running_regression_intercept	
(t_running_correlation), 52	
t_running_regression_slope	
(t_running_correlation), 52	
t_running_scaled(t_running_centered),	
46	
t_running_sd(t_running_sd3),58	
t_running_sd3,58	