

# Package ‘OSDR’

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**Type** Package

**Title** Finds an Optimal System of Distinct Representatives

**Version** 1.1.4

**Date** 2022-03-07

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**Description** Provides routines for finding an Optimal System of Distinct Representatives (OSDR), as defined by D.Gale (1968) <[doi:10.1016/S0021-9800\(68\)80039-0](https://doi.org/10.1016/S0021-9800(68)80039-0)>.

**License** GPL-3

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OSDR-package

*Finds an Optimal System of Distinct Representatives***Description**

Provides routines for finding an Optimal System of Distinct Representatives (OSDR), as defined by D.Gale (1968) <doi:10.1016/S0021-9800(68)80039-0>.

**Details**

Package: OSDR  
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 Description: Provides routines for finding an Optimal System of Distinct Representatives (OSDR), as defined by D.Gale (1968)  
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 URL: <https://cran.r-project.org/package=OSDR>

Index of help topics:

OSDR	Finds an Optimal System of Distinct Representatives from a family of subsets.
OSDR-package	Finds an Optimal System of Distinct Representatives
exdata	Executive dataset (subset of)
listmat	From cost matrix to list of feasible assignments.
matlist	From list of feasible assignments to cost matrix.

The aim of this small package is to provide a function solving the assignment problem on an *ordered* set. This matching problem can be viewed in two ways. First, as the original ordered assignment problem, where a set of jobs, ordered by importance, must be filled by suitable applicants in the best possible way. Second, as an ordered matching problem where treated unit should be matched in order of importance with suitable controls. Suitability can be obtained by a "caliper": units within the caliper having zero matching cost and units outside the caliper having infinity cost. The main function OSDR exploits an algorithm suggested by I.Anderson to find an *order optimal* matching, as defined by D.Gale (see [OSDR](#) for details). The package includes some utilities and examples (both combinatorial and statistically oriented) to illustrate the use of OSDR.

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**References**

Gale, D. (1968) Optimal matching in an ordered set: an application of matroid theory. *Journal of Combinatorial Theory* 4, pp. 176-180.

Anderson, I. (1989) A first course in Combinatorial Mathematics. *Oxford University Press*.

Rosenbaum, P. R. (1989). Optimal matching for observational studies. *Journal of the American Statistical Association* 84(408): pp. 1024-1032.

**Examples**

#See OSDR help for the examples.

---

exdata	<i>Executive dataset (subset of)</i>
--------	--------------------------------------

---

**Description**

Contains executives data for an italian firm operating in the energy sector (year 2015). The complete dataset contains 302 rows (10 firms) and 180 columns (it will be released in next package version).

**Usage**

```
data("exdata")
```

**Format**

A data frame with 28 observations on the following 8 variables.

firm a character vector

position a numeric vector

education a numeric vector

year\_born a numeric vector

contract a numeric vector

part\_fulltime a numeric vector

seniority a numeric vector

sex a numeric vector

**Details**

position is coded: 4 = top manager, 3 = medium/first line manager, 2 = supervisor  
sex is coded: 0 = M, 1 = F  
education is coded: Post-graduate = 5, Graduate = 4, High school = 3  
contract type is coded: fixed term = 4; permanent = 3  
seniority is measured by number of years in the current position

**Examples**

```
# load executive data
data(exdata)

# case study: matched samples for comparing women and men executives

table(exdata$sex)
table(exdata$position,exdata$sex)
# There are more women and more in apical position.

# A complete matching is not possible for several choices of the caliper.
# Gap differences tend to be higher for higher ranks

# e.g. Lynn and Thompson(1997), Above the glass ceiling? A comparison of
# matched Samples of Men and Women Executives. J. of Appl. Psych. 82(3)

# so we would give higher matching priority to women in higher position.
# We can use OSDR to find a minimum cost matching
# performing matching by decreasing hierarchical position.
```

---

listmat

*From cost matrix to list of feasible assignments.*


---

**Description**

Transforms the cost matrix of the assignment problem in the corresponding list of suitable applicants for each job.

**Usage**

```
listmat(x)
```

**Arguments**

x                    A cost matrix where entry  $(i, j)$  is the cost of assigning job  $i$  to applicant  $j$ .

**Details**

In statistical matching problems the input is usually the cost matrix while in assignment problems is the list of assignable elements. Functions `matlist` and `listmat` go back and forth these two representations of the input.

**Value**

A list of suitable applicants. The  $i^{th}$  element of the list contains the suitable applicants for job  $i$ . These are all applicants with *finite* assignment cost. Note that if the cost matrix contains finite non zero values there is a loss of information in the transformation.

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**See Also**

See [matlist](#), the reverse function.

**Examples**

```
# a list of feasible applicants for five jobs
M1<-c("A","B","C")
M2<-c("A","C")
M3<-c("B")
M4<-c("A","C")
M5<-c("A","D")
M <-list(M1,M2,M3,M4,M5)
M

# list --> cost matrix
m <- listmat(M)

# cost matrix --> list
l <- matlist(m)
```

---

matlist

*From list of feasible assignments to cost matrix.*


---

**Description**

Transforms the list of suitable applicants into the equivalent cost matrix.

**Usage**

```
matlist(x)
```

**Arguments**

**x** An ordered list of applicants for a set of jobs. The  $i^{th}$  element of the list contains applicants suitable for job  $i$ . Equivalently, an ordered list of controls where the  $i^{th}$  element of the list contains controls matchable with the  $i^{th}$  treated unit.

**Details**

In statistical matching problems the input is usually the cost matrix while in assignment problems is the list of assignable elements. Functions `matlist` and `listmat` go back and forth these two representations of the input.

**Value**

The cost matrix of the assignment problem. The  $(i, j)$  entry of the matrix is zero if the  $j^{\text{th}}$  applicant is assignable to the  $i^{\text{th}}$  job and  $\infty$  otherwise. Equivalently, the  $(i, j)$  entry of the matrix is zero if the  $j^{\text{th}}$  control unit can be matched to the  $i^{\text{th}}$  treated unit and  $\infty$  otherwise.

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**See Also**

See also the reverse function [listmat](#).

**Examples**

```
# a list of feasible applicants for five jobs
M1<-c("A", "B", "C")
M2<-c("A", "C")
M3<-c("B")
M4<-c("A", "C")
M5<-c("A", "D")
M <-list(M1,M2,M3,M4,M5)
M

# the corresponding cost matrix
m<-listmat(M)

# back to the list
l<-matlist(m)
```

---

OSDR

*Finds an Optimal System of Distinct Representatives from a family of subsets.*

---

**Description**

The function finds an Optimal Set of Distinct Representatives (OSDR) from an ordered family of subsets. Optimality is order-wise in the sense specified by Gale: any other SDR cannot be larger and its elements cannot be chosen from more important sets (see details).

**Usage**

OSDR(M)

**Arguments**

M An ordered family of subsets. In assignment problems the list specifying the feasible applicants for each job. In matching problems the list specifying the controls matchable with each treated unit. The list is assumed *ordered* by decreasing assignment/matching priority.

**Details**

Consider a set of jobs and a set of suitable applicants for each job. It was shown by D.Gale that there exists an *optimally assignable* subset of the jobs in the sense that if  $a_1, \dots, a_k$  is optimal and  $b_1, \dots, b_h$  is another assignable set of jobs then: a)  $k \geq h$  and b)  $a_i \leq b_i$  for all  $i$ . The latter means that job  $a_i$  has as higher a priority as job  $b_i$ . From a graph perspective, if  $J$  is the set of jobs and  $A$  is the set of applicants the OSDR is an *order wise* maximum size matching of the bipartite graph with vertex set  $\text{Union}A$ .

Function OSDR finds the optimal matching using an algorithmic proof of Hall's theorem due to logician D.J. Shoesmith. The algorithm assigns greedily until possible and correct backward when necessary. A message is given when a correction is attempted (when it fails another message warns that Hall's condition is not satisfied).

**Value**

A list containing three elements: the OSDR (a zero in position  $i$  indicates impossibility of assigning job  $i$ ), the index of optimally assignable jobs and the index of unassignable jobs. If the latter is not empty M does not meet Hall's condition, i.e., a complete SDR is not possible.

**Note**

In statistical matching problems the sets  $J$  and  $A$  are the sets of treated and control units and it is usually required to find the minimum covariate distance matching of treated versus control units. When a complete matching does not exist it can be convenient to find either an order optimal matching or a minimum cost matching on the OSDR subset (see the gender gap example).

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**References**

Gale, D. (1968) Optimal matching in an ordered set: an application of matroid theory. *Journal of Combinatorial Theory* 4: 176-180.

Anderson, I. (1989) A first course in Combinatorial Mathematics. *Oxford University Press*.

**See Also**

See also [matlist](#), [listmat](#)

**Examples**

```

### example 1
# M is the list of suitable applicants for five jobs
M1 <- c("A" , "B")
M2 <- c("B" , "C")
M3 <- c("B")
M4 <- c("A" , "C")
M5 <- c("B" , "C" , "D")
M <- list(M1 , M2 , M3 , M4 , M5)

OSDR(M)

# $OSDR
# [1] "A" "C" "B" "0" "D"
# $matched
# [1] 1 2 3 5
# $unmatched
# [1] 4

# job 4 unmatched so Hall's condition is not satisfied: it's impossible to fill all the jobs
# note that there are (order-\emph{suboptimal}) assignments of the same length of the optimal:
# eg: 0CBAD , BC0AD

#### example 2: slightly modified: more than one order optimal matching
M1<-c("A","B","C")
M2<-c("A","C")
M3<-c("B")
M4<-c("A","C")
M5<-c("A","D")
M <-list(M1,M2,M3,M4,M5)

OSDR(M)

# note there are other order optimal matchings: ACB0D or CAB0D
# note there are also other maximum size matchings (not order optimal):
# e.g. 0CBAD or BC0AD

```



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