

# Package ‘ICSsmoothing’

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**Type** Package

**Title** Data Smoothing by Interpolating Cubic Splines

**Version** 1.2.8

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**Description** We construct the explicit form of clamped cubic interpolating spline (both uniform - knots are equidistant and non-uniform - knots are arbitrary). Using this form, we propose a linear regression model suitable for real data smoothing.

**Depends** R (>= 3.5.0), polynom, ggplot2

**License** GPL-2

**Encoding** UTF-8

**LazyData** true

**RoxygenNote** 7.2.3

**Suggests** knitr, rmarkdown, testthat

**VignetteBuilder** knitr

**NeedsCompilation** no

**Repository** CRAN

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CERN	<i>277 measurements of the cross sections for <math>\pi^-p</math> collision (nuclear physics).</i>
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**Description**

277 measurements of the cross sections for  $\pi^-p$  collision (nuclear physics).

**Usage**

CERN

**Format**

A data frame with 277 elements.

**Source**

<https://link.springer.com/article/10.1007/BF02683433>

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cics_explicit	<i>Construct the explicit form of non-uniform clamped interpolating cubic spline (NcICS).</i>
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**Description**

cics\_explicit constructs the explicit form of non-uniform clamped interpolating cubic spline (via Hermite cubic spline) for nodes uu, function values yy and exterior-node derivatives d.

**Usage**

```
cics_explicit(  
  uu,  
  yy,  
  d,  
  clr = c("blue", "red"),  
  xlab = NULL,  
  ylab = NULL,  
  title = NULL  
)
```

**Arguments**

uu	a vector of arbitrary nodes (ordered ascendingly), with magnitude $n+2$ , $n \geq 1$ .
yy	a vector of function values pertaining to nodes in uu.
d	a vector of two values of derivative, in the first and the last node of uu.
clrs	a vector of colours that are used alternately to plot the graph of spline's components.
xlab	a title (optional parameter) for the x axis.
ylab	a title (optional parameter) for the y axis.
title	a title (optional parameter) for the plot.

**Value**

	a list with components
spline_coeffs	matrix, whose i-th row contains coefficients of non-uniform ICS's i-th component.
spline_polynomials	list of NcICS's components string representations.
B	4-element array of $(n+1) \times (n+4)$ matrices, whereas element in i-th row and j-th column of l-th matrix contains coefficient by $x^{\{l-1\}}$ of cubic polynomial that is in i-th row and j-th column of matrix B from spline's explicit form
	$S = B.\gamma.$
gamma	$\gamma$ = vector of spline coefficients - function values and exterior-node derivatives that takes part in the explicit form $S = B.\gamma.$
aux_BF	A basis function of the spline
aux_tridiag_inverse	An inverse of the tridiagonal matrix used for spline derivatives construction

**Examples**

```

cics_explicit(
  uu = c(1, 2.2, 3, 3.8, 7),
  CERN$y[1:5],
  d=c(0,-2),
  xlab="X axis",
  ylab="Y axis"
)

uu <- c(0, 1, 4, 6);
yy <- c(4, 5, 2, 1.8);
sp <- cics_explicit(uu, yy, c(1,0))
sp$spline_polynomials
### <~~~>
### Spline components' coefficients
explicit_spline(sp$B, sp$gamma)
sp$spline_coeffs == .Last.value

```

---

`cics_explicit_smooth` *Smooth given data set by k-component non-uniform clamped interpolating spline (NcICS).*

---

### Description

`cics_explicit_smooth` constructs the non-uniform clamped interpolating spline with  $k$  components that smoothes given data set  $\{(xx[i], yy[i]), i=1..length(xx)\}$ .

### Usage

```
cics_explicit_smooth(
  xx,
  yy,
  uu,
  clr = c("blue", "red"),
  d,
  xlab = NULL,
  ylab = NULL,
  title = NULL
)
```

### Arguments

<code>xx</code>	a vector of data set's x-coordinates (that are in increasing order).
<code>yy</code>	a vector of data set's y-coordinates.
<code>uu</code>	a vector of arbitrary nodes, based on which we construct the smoothing spline. <code>uu[1]</code> and <code>uu[length(uu)]</code> must be equal to <code>xx[1]</code> and <code>xx[length(xx)]</code> , respectively.
<code>clr</code>	a vector of colours that are used alternately to plot the graph of spline's components.
<code>d</code>	a vector (optional parameter) that contains two values of derivative, in the first and the last node from <code>uu</code> . If missing, values of derivative are estimated by given linear regression model. If present, their contribution is removed from linear model and only function values are estimated.
<code>xlab</code>	a title (optional parameter) for the x axis.
<code>ylab</code>	a title (optional parameter) for the y axis.
<code>title</code>	a title (optional parameter) for the plot.

### Value

a list with components

`est_spline_coeffs` 4-element array of  $(k) \times (k+3)$  matrices, whereas element in  $i$ -th row and  $j$ -th of 1-th matrix contains coefficient by  $x^{1-1}$  of cubic polynomial, which is in  $i$ -th row and  $j$ -th column of matrix  $B$  from smoothing spline's explicit form

$$S = B \cdot \gamma.$$

`est_spline_polynomials` list of string representations of smoothing NcICS.

`est_gamma` vector of estimated smoothing spline's coefficients (function values and exterior-node derivatives).

`aux_BF` A basis function of the spline

`aux_tridiag_inverse` An inverse of the tridiagonal matrix used for spline derivatives construction

`aux_M` An estimation matrix used to compute `est_gamma`

### Examples

```
cics_explicit_smooth(
  xx = CERN$x,
  yy = CERN$y,
  d = c(0, 1),
  uu = c(1, sort(runif(20,1,277)), 277),
  xlab = "X axis",
  ylab = "Y axis"
)

yy <- c(1, 2, 3, 4, 3, 2, 2, 3, 5, 6, 7, 6, 5, 5, 4, 3, 2, 1, 0)
xx <- c(1:length(yy))
uu <- c(1,7,10,19)
sp <- cics_explicit_smooth(xx,yy,uu)
### We can change the derivatives at the end nodes:
sp <- cics_explicit_smooth(xx,yy, uu, d=c(3,-7/10))

### CERN:
uu <- c(1, 15, 26, 63, 73, 88, 103, 117, 132, 200, 203, 219, 258, 277)
sp <- cics_explicit_smooth(
  xx = CERN$x,
  yy = CERN$y,
  d = c(1, 0),
  uu
)
```

---

`cics_unif_explicit`      *Construct the explicit form of uniform clamped interpolating cubic spline (UcICS).*

---

**Description**

cics\_unif\_explicit constructs the explicit form of uniform clamped interpolating cubic spline (via Hermite cubic spline) for nodes uu, function values yy and exterior-node derivatives d.

**Usage**

```
cics_unif_explicit(
    uumin,
    uumax,
    yy,
    d,
    clr = c("blue", "red"),
    xlab = NULL,
    ylab = NULL,
    title = NULL
)
```

**Arguments**

uumin	a starting node.
uumax	an ending node.
yy	a vector of function values pertaining to nodes in uu.
d	a vector of two values of derivative, in the first and the last node of uu.
clr	a vector (optional parameter) of colours that are used alternately to plot the graph of spline's components.
xlab	a title (optional parameter) for the x axis.
ylab	a title (optional parameter) for the y axis.
title	a title (optional parameter) for the plot.

**Value**

A list of spline components

spline\_coeffs matrix, whose i-th row contains coefficients of uniform ICS's i-th component.  
spline\_polynomials

list of UclCS's components string representations.

B 4-element array of (n+1)x(n+4) matrices, whereas element in i-th row and j-th column of l-th matrix contains coefficient by  $x^{l-1}$  of cubic polynomial that is in i-th row and j-th column of matrix B from spline's explicit form

$$S = B.\gamma.$$

gamma  $\gamma$  = vector of spline coefficients - function values and exterior-node derivatives that takes part in the explicit form  $S = B.\gamma$ .

aux\_BF A basis function of the spline

aux\_tridiag\_inverse

An inverse of the tridiagonal matrix used for spline derivatives construction

**Examples**

```
yy <- c(4, 5, 2, 1.8);
sp <- cics_unif_explicit(0, 6, yy, c(2, 0.9))
sp$spline_polynomials
### <~~>
### Spline components' coefficients
explicit_spline(sp$B, sp$gamma)
sp$spline_coeffs == .Last.value
```

---

```
cics_unif_explicit_smooth
```

*Smooth given data set by k-component uniform clamped interpolating spline (UcICS).*

---

**Description**

`cics_unif_explicit_smooth` constructs the uniform clamped interpolating spline with  $k$  components that smoothes given data set  $\{(xx[i], yy[i]), i=1..length(xx)\}$ .

**Usage**

```
cics_unif_explicit_smooth(
  xx,
  yy,
  k,
  clr = c("blue", "red"),
  d,
  xlab = NULL,
  ylab = NULL,
  title = NULL,
  plotTF = TRUE
)
```

**Arguments**

<code>xx</code>	a vector of data set's x-coordinates (that are in increasing order).
<code>yy</code>	a vector of data set's y-coordinates.
<code>k</code>	a chosen number of components of smoothing UcICS (integer $\geq 2$ ).
<code>clr</code>	a vector of colours that are used alternately to plot the graph of spline's components.
<code>d</code>	a vector (optional parameter) that contains two values of derivative, in the first and the last computed node. If missing, values of derivative are estimated by given linear regression model. If present, their contribution is removed from linear model and only function values are estimated.
<code>xlab</code>	a title (optional parameter) for the x axis.

ylab	a title (optional parameter) for the y axis.
title	a title (optional parameter) for the plot.
plotTF	a boolean value (optional parameter), if TRUE then plot.

**Value**

a list with components

nodes	vector of equidistant nodes, based on which we construct the smoothing spline.
est_spline_coeffs	4-element array of $(k) \times (k+3)$ matrices, whereas element in $i$ -th row and $j$ -th of 1-th matrix contains coefficient by $x^{\{1-1\}}$ of cubic polynomial, which is in $i$ -th row and $j$ -th column of matrix $B$ from smoothing spline's explicit form

$$S = B \cdot \gamma.$$

est_spline_polynomials	list of string representations of smoothing UcICS.
est_gamma	vector of estimated smoothing spline's coefficients (function values and exterior-node derivatives).
aux_BF	A basis function of the spline
aux_tridiag_inverse	An inverse of the tridiagonal matrix used for spline derivatives construction
aux_M	An estimation matrix used to compute est_gamma

**Examples**

```
cp <- cics_unif_explicit_smooth(
xx = CERN$x,
yy = CERN$y,
k = 19, #23,
d = c(1, 0),
xlab = "X axis",
ylab = "Y axis"
)
```

---

explicit_spline	<i>The function computes the coefficients of the cubic polynomials as spline components of the clamped interpolating cubic spline of class <math>C^2</math> in its explicit form <math>S=B * \gamma</math>.</i>
-----------------	---

---

**Description**

The function computes the coefficients of the cubic polynomials as spline components of the clamped interpolating cubic spline of class  $C^2$  in its explicit form  $S=B * \gamma$ .



**Usage**

```
explicit_spline(B, gamma)
```

**Arguments**

**B** a 4-element array of  $(n+1) \times (n+4)$  matrices, whereas element in  $i$ -th row and  $j$ -th column of  $l$ -th matrix contains coefficient by  $x^{l-1}$  of cubic polynomial that is in  $i$ -th row and  $j$ -th column of matrix  $B$  from spline's explicit form  $S=B \cdot \text{gamma}$ .

**gamma** a vector of spline coefficients - function values and exterior-node derivatives that takes part in the explicit form  $S = B \cdot \gamma$ .

**Value**

a matrix with four columns, whose  $i$ -th row contains the coefficients of the splines's  $i$ -th component.

**Examples**

```
# See functions cics_explicit, cics_unif_explicit and the vignette.
```

---

forecast_demo	<i>Forecasting demo using cics_unif_explicit_smooth.</i>
---------------	--

---

**Description**

Forecasting demo using cics\_unif\_explicit\_smooth.

**Usage**

```
forecast_demo()
```

**Value**

a forecast result

**Examples**

```
# Plots as well as the process of computation of future derivatives and values using extrapolation.
ud <- forecast_demo()
```

---

hermite\_bf\_matrix      *Construct 4 Hermite basis functions.*

---

**Description**

hermite\_bf\_matrix constructs matrix of Hermite basis functions' coefficients on [u,v], that is the matrix of 4 cubic polynomials' coefficients of one-component Hermite cubic spline.

**Usage**

```
hermite_bf_matrix(u, v)
```

**Arguments**

u                    a left border of interval [u,v].  
v                    a right border of interval [u,v],  $u \leq v$ .

**Value**

The matrix of 4 Hermite basis functions' coefficients.

**Examples**

```
hermite_bf_matrix(0,1)  
hermite_bf_matrix(-2,3)
```

---

tridiag\_inv\_general      *Construct inverse of a general tridiagonal matrix.*

---

**Description**

tridiag\_inv\_general constructs inverse of a general tridiagonal matrix T of order n, using Usmani's theorem.

**Usage**

```
tridiag_inv_general(T, n)
```

**Arguments**

T                    a tridiagonal matrix.  
n                    an order of given tridiagonal matrix.

**Value**

The inverse of matrix T.

**Examples**

```
tridiag_inv_general(matrix(c(1, 4, 0, -9), 2, 2), 2)
tridiag_inv_general(matrix(c(1, 3, 5, -2, 0, 8, 7, 6, 6), 3, 3), 3)
```

---

```
tridiag_inv_unif_by_sums
```

*Construct inverse of a tridiagonal matrix  $T_n(a,b,a)$ .*

---

**Description**

tridiag\_inv\_unif\_by\_sums constructs inverse of a regular tridiagonal matrix  $T_n(a,b,a)$  with constant entries by a special algorithm using sums of matrix elements.

**Usage**

```
tridiag_inv_unif_by_sums(n, a, b)
```

**Arguments**

n	an order of given tridiagonal matrix.
a	a value of tridiagonal matrix elements that are off-diagonal.
b	a value of tridiagonal matrix diagonal elements.

**Value**

The inverse of matrix  $T_n(a,b,a)$ .

**Examples**

```
tridiag_inv_unif_by_sums(5, 1, 4)
tridiag_inv_unif_by_sums(9, 10, -1)
```

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