Package ‘GRS.test’

January 20, 2025

Type Package

Title GRS Test for Portfolio Efficiency, Its Statistical Power Analysis, and Optimal Significance Level Calculation

Version 1.2

Date 2022-06-29

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Description Computational resources for test proposed by Gibbons, Ross, Shanken (1989)<DOI:10.2307/1913625>.
It also has the functions for the power analysis and the choice of the optimal level of significance.
The optimal level is determined by minimizing the expected loss from hypothesis testing.

License GPL-2

NeedsCompilation no

Repository CRAN

Date/Publication 2022-07-01 21:30:21 UTC

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`GRS.test-package` *GRS Test for Portfolio Efficiency, Its Statistical Power Analysis, and Optimal Significance Level Calculation*

Description

Computational resources for test proposed by Gibbons, Ross, Shanken (1989)<DOI:10.2307/1913625>. It also has the functions for the power analysis and the choice of the optimal level of significance. The optimal level is determined by minimizing the expected loss from hypothesis testing.

Details

The DESCRIPTION file:

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Type:	Package
Title:	GRS Test for Portfolio Efficiency, Its Statistical Power Analysis, and Optimal Significance Level Calculation
Version:	1.2
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License:	GPL-2

Index of help topics:

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                        Likelihood Estimator for Covariance matrix
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GRS.Powerfunc           Power functions for the GRS test
GRS.T                   Sample Size Selection for the GRS test
GRS.optimal             Optimal Level of Significance for the GRS test:
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GRS.optimalboot         Optimal Level of Significance for the GRS test:
                        Bootstrapping
GRS.optimalbootweight   Weighted Optimal Level of Significance for the
                        GRS test: Bootstrapping
GRS.optimalweight       Weighted Optimal Level of Significance for the
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                        Statistical Power Analysis, and Optimal
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                        risk factors, obtained from French's library
```

The package accompanies the working paper:

Kim and Shamsuddin, 2017, Empirical Validity of Asset-pricing Models: Application of Optimal Significance Level and Equal Probability Test

The function GRS.test returns the GRS test statistics with model estimation results.

The function GRS.MLtest provides an alternative test statistic with theta and theta* estimation results.

Additional functions for the power analysis and calculation of optimal level of significance are also included.

Author(s)

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References

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>

Fama and French, 1993, Common risk factors in the returns on stocks and bonds, Journal of Financial Economics, 33, 3-56. <DOI:10.1016/0304-405X(93)90023-5>

Fama and French, 2015, A five-factor asset-pricing model, Journal of Financial Economics, 1-22. <DOI:http://dx.doi.org/10.1016/j.jfineco.2014.10.010>

See Also

The examples replicate the results reported in Fama and French (1993) and Kim and Shamsuddin (2016)

Examples

```
data(data)
factor.mat = data[1:342,2:4]           # Fama-French 3-factor model
ret.mat = data[1:342,8:ncol(data)]     # 25 size-BM portfolio returns
GRS.test(ret.mat,factor.mat)$GRS.stat   # Table 9C of Fama-French (1993)
```

`data` *Fama-French Data: 25 size-B/M portfolio and risk factors, obtained from French's library*

Description

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Usage

```
data("data")
```

Format

A data frame with 630 observations on the following 32 variables.

`date` date

`RM_RF` Market Excess Return

`SMB` SMB

`HML` HML

`RMW` RMW

`CMA` CMA

`MOM` MOM

`P11` Portfolio Returns

`P12` Portfolio Returns

`P13` Portfolio Returns

`P14` Portfolio Returns

`P15` Portfolio Returns

`P21` Portfolio Returns

`P22` Portfolio Returns

`P23` Portfolio Returns

`P24` Portfolio Returns

`P25` Portfolio Returns

`P31` Portfolio Returns

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`P43` Portfolio Returns

`P44` Portfolio Returns

`P45` Portfolio Returns

`P51` Portfolio Returns

`P52` Portfolio Returns

`P53` Portfolio Returns

`P54` Portfolio Returns

`P55` Portfolio Returns

Details

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Source

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Monthly from 1963 to 2015

References

Fama and French, 1993, Common risk factors in the returns on stocks and bonds, Journal of Financial Economics, 33, 3-56. <DOI:10.1016/0304-405X(93)90023-5>

Fama and French, 2015, A five-factor asset-pricing model, Journal of Financial Economics, 116-1-22. <DOI:http://dx.doi.org/10.1016/j.jfineco.2014.10.010>

Examples

```
data(data)
y=ts(data[,2],frequency=12,start=c(1950,1))
plot.ts(y)
```

`GRS.MLtest` *GRS Test Statistic and p-value based on Maximum Likelihood Estimator for Covariance matrix*

Description

W statistic given in (7) of GRS (1989) <DOI:10.2307/1913625>

Usage

```
GRS.MLtest(ret.mat, factor.mat)
```

Arguments

`ret.mat`	portfolio return matrix, T by N
`factor.mat`	matrix of risk factors, T by K

Details

T: sample size, N: number of portfolio returns, K: number of risk factors

Value

`GRS.stat`	GRS test statistic
`GRS.pval`	its p-value
`theta`	maximum Sharpe ratio of the K factor portfolios
`thetas`	slope of the efficient frontier based on all assets
`ratio`	theta/thetas, proportion of the potential efficiency

Note

Applicable to CAPM as well as a multi-factor model

Author(s)

Jae H. Kim

References

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>

See Also

Fama and French, 1993, Common risk factors in the returns on stocks and bonds, Journal of Financial Economics, 33, 3-56. <DOI:10.1016/0304-405X(93)90023-5>

Fama and French, 2015, A five-factor asset-pricing model, Journal of Financial Economics, 116-1-22. <DOI:http://dx.doi.org/10.1016/j.jfineco.2014.10.010>

Examples

```
data(data)
factor.mat = data[1:342,2:4]          # Fama-French 3-factor model
ret.mat = data[1:342,8:ncol(data)]    # 25 size-BM portfolio returns
GRS.MLtest(ret.mat,factor.mat)       # See column (iv), Table 9C of Fama-French (1993)
```

`GRS.optimal` *Optimal Level of Significance for the GRS test: Normality Assumption*

Description

The optimal level is calculated by minimizing expected loss from hypothesis testing

The F-distributions are used to calculate the power, under the normality assumption

Usage

```
GRS.optimal(T, N, K, theta, ratio, p = 0.5, k = 1, Graph = TRUE)
```

Arguments

`T`	sample size
`N`	the number of portfolio returns
`K`	the number of risk factors
`theta`	maximum Sharpe ratio of the K factor portfolios
`ratio`	theta/thetas, proportion of the potential efficiency

p	prior probability for H0, default is p = 0.5
k	relative loss, k = L2/L1, default is k = 1
Graph	show graph if TRUE. No graph otherwise

Details

Based on the power calculation of the GRS test, as in GRS (1989) <DOI:10.2307/1913625>.

The blue square in the plot is the point where the expected loss is mimimized.

The red horizontal line in the plot indicates the point of the covnentional level of significance (alpha = 0.05).

Value

opt.sig	Optimal level of significance
opt.crit	Critical value corresponding to opt.sig
opt.beta	Type II error probability corresponding to opt.sig

Note

ratio = theta/thetas

thetas = maximum Sharpe ratio of the K factor portfolios: GRS (1989) <DOI:10.2307/1913625>

Author(s)

Jae H. Kim

References

Leamer, E. 1978, Specification Searches: Ad Hoc Inference with Nonexperimental Data, Wiley, New York.

Kim, JH and Ji, P. 2015, Significance Testing in Empirical Finance: A Critical Review and Assessment, Journal of Empirical Finance 34, 1-14. <DOI:http://dx.doi.org/10.1016/j.jempfin.2015.08.006>

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>

Kim and Shamsuddin, 2017, Empirical Validity of Asset-pricing Models: Application of Optimal Significance Level and Equal Probability Test

See Also

Kim and Choi, 2017, Choosing the Level of Significance: A Decision-theoretic Approach

Examples

```
GRS.optimal(T=90, N=25, K=3, theta=0.25, ratio=0.4) # Figure 3 of Kim and Shamsuddin (2017)
```

`GRS.optimalboot` *Optimal Level of Significance for the GRS test: Bootstrapping*

Description

The optimal level is calculated by minimizing expected loss from hypothesis testing

The bootstrap is used to calculate the power.

The power is calculated at the estimated values (unrestricted) of parameters under H1.

Usage

```
GRS.optimalboot(ret.mat,factor.mat,p=0.5,k=1,nboot=3000,wild=FALSE,Graph=TRUE)
```

Arguments

`ret.mat`	portfolio return matrix, T by N
`factor.mat`	matrix of risk factors, T by K
`p`	prior probability for H0, default is p = 0.5
`k`	relative loss, k = L2/L1, default is k = 1
`nboot`	the number of bootstrap iterations, the default is 3000
`wild`	if TRUE, wild bootsrap is conducted; if FALSE (default), bootstrap is based on iid residual resampling
`Graph`	show graph if TRUE (default). No graph otherwise

Details

The blue square in the plot is the point where the expected loss is mimimized.

The red horizontal line in the plot indicates the point of the covnentional level of significance (alpha = 0.05).

The function also returns the density functions under H0 and H1 (black and red curves, with vertical line the critical value at the optimal level).

Value

`opt.sig`	Optimal level of significance
`opt.crit`	Critical value corresponding to opt.sig
`opt.beta`	Type II error probability corresponding to opt.sig

Note

The example below sets nboot=500 for faster execution, but a higher number is recommended.

Author(s)

Jae H. Kim

References

Leamer, E. 1978, Specification Searches: Ad Hoc Inference with Nonexperimental Data, Wiley, New York.

Kim, JH and Ji, P. 2015, Significance Testing in Empirical Finance: A Critical Review and Assessment, Journal of Empirical Finance 34, 1-14. <DOI:http://dx.doi.org/10.1016/j.jempfin.2015.08.006>

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>

Kim and Shamsuddin, 2017, Empirical Validity of Asset-pricing Models: Application of Optimal Significance Level and Equal Probability Test

See Also

Kim and Choi, 2017, Choosing the Level of Significance: A Decision-theoretic Approach

Examples

```
data(data)
n=60; m1=nrow(data)-n+1; m2=nrow(data)  # Choose the last n observations from the data set
factor.mat = data[m1:m2,2:6]            # Fama-French 5-factors
ret.mat = data[m1:m2,8:ncol(data)]      # 25 size-BM portfolio returns
GRS.optimalboot(ret.mat,factor.mat,p=0.5,k=1,nboot=500,wild=TRUE,Graph=TRUE)
```

`GRS.optimalbootweight` *Weighted Optimal Level of Significance for the GRS test: Bootstrapping*

Description

The optimal level is calculated by minimizing expected loss from hypothesis testing

The bootstrap is used to calculate the power.

The non-centrality paramter is estimated and its bootstrap distribution is obtained.

9 percentiles from this distribution is used to caculate the power and the optimal level.

These optimal levesl ared weighted using the weights from the density of the bootstrap distribution of lambda.

Usage

```
GRS.optimalbootweight(ret.mat,factor.mat,p=0.5,k=1,nboot=3000,wild=FALSE,Graph=TRUE)
```

Arguments

`ret.mat`	portfolio return matrix, T by N
`factor.mat`	matrix of risk factors, T by K
`p`	prior probability for H0, default is p = 0.5
`k`	relative loss, k = L2/L1, default is k = 1
`nboot`	the number of bootstrap iterations, the default is 3000
`wild`	if TRUE, wild bootsrap is conducted; if FALSE (default), bootstrap is based on iid residual resampling
`Graph`	show graph if TRUE (default). No graph otherwise

Details

Power is calculated based on the bootstrap

The plot shows the bootstrap distribution of lambda (non-centrality parameter)

See Kim and Choi, 2017, Choosing the Level of Significance: A Decision-theoretic Approach

Value

`opt.sig`	Optimal level of significance
`opt.crit`	Critical value corresponding to opt.sig

Note

The example below sets nboot=500 for faster execution, but a higher number is recommended.

Author(s)

Jae H. Kim

References

Leamer, E. 1978, Specification Searches: Ad Hoc Inference with Nonexperimental Data, Wiley, New York.

Kim, JH and Ji, P. 2015, Significance Testing in Empirical Finance: A Critical Review and Assessment, Journal of Empirical Finance 34, 1-14. <DOI:http://dx.doi.org/10.1016/j.jempfin.2015.08.006>

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>

Kim and Shamsuddin, 2017, Empirical Validity of Asset-pricing Models: Application of Optimal Significance Level and Equal Probability Test

See Also

Kim and Choi, 2017, Choosing the Level of Significance: A Decision-theoretic Approach

Examples

```
data(data)
n=60; m1=nrow(data)-n+1; m2=nrow(data)  # Choose the last n observations from the data set
factor.mat = data[m1:m2,2:6]            # Fama-French 5-factors
ret.mat = data[m1:m2,8:ncol(data)]      # 25 size-BM portfolio returns
GRS.optimalboot(ret.mat,factor.mat,p=0.5,k=1,nboot=500,wild=TRUE,Graph=TRUE)
```

`GRS.optimalweight` *Weighted Optimal Level of Significance for the GRS test: Normality Assumption*

Description

The optimal level is calculated by minimizing expected loss from hypothesis testing.

The F-distributions are used to calculate the power, under the normality assumption

The power is calculated using a range of non-centrality parameters (lamdba), folloing a folded-normal distribution.

The weights are obtained from the density function of folded-normal distribution.

See, for details, Kim and Choi, 2017, Choosing the Level of Significance: A Decision-theoretic Approach.

Usage

```
GRS.optimalweight(T, N, K, theta, ratio, delta = 3, p = 0.5, k = 1, Graph = TRUE)
```

Arguments

`T`	sample size
`N`	the number of portfolio returns
`K`	the number of risk factors
`theta`	maximum Sharpe ratio of the K factor portfolios
`ratio`	theta/thetas, proportion of the potential efficiency
`delta`	the standard deviation of the folded-normal distribution, default is 3
`p`	prior probability for H0, default is p = 0.5
`k`	relative loss, k = L2/L1, default is k = 1
`Graph`	show graph if TRUE. No graph otherwise

Details

Based on the power calculation of the GRS test, as in GRS (1989) <DOI:10.2307/1913625>.

The plot shows the folded-normal distribution.

Value

`opt.sig`	Optimal level of significance
`opt.crit`	Critical value corresponding to opt.sig

Note

ratio = theta/thetas

thetas = maximum Sharpe ratio of the K factor portfolios: GRS (1989) <DOI:10.2307/1913625>

Author(s)

Jae H. Kim

References

Leamer, E. 1978, Specification Searches: Ad Hoc Inference with Nonexperimental Data, Wiley, New York.

Kim, JH and Ji, P. 2015, Significance Testing in Empirical Finance: A Critical Review and Assessment, Journal of Empirical Finance 34, 1-14. <DOI:http://dx.doi.org/10.1016/j.jempfin.2015.08.006>

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>

Kim and Shamsuddin, 2017, Empirical Validity of Asset-pricing Models: Application of Optimal Significance Level and Equal Probability Test

See Also

Kim and Choi, 2017, Choosing the Level of Significance: A Decision-theoretic Approach

Examples

```
GRS.optimalweight(T=90, N=25, K=3, theta=0.25, ratio=0.4)
```

`GRS.Power` *Statistical Power of the GRS test*

Description

Calculates the power of the GRS test with density functions under H0 and H1

Usage

```
GRS.Power(T, N, K, theta, ratio, alpha = 0.05, xmax = 10, Graph = "TRUE")
```

Arguments

`T`	sample size
`N`	the number of portfolio returns
`K`	the number of risk factors
`theta`	maximum Sharpe ratio of the K factor portfolios
`ratio`	theta/thetas, proportion of the potential efficiency
`alpha`	the level of significance, default is 0.05
`xmax`	the support of the desnity is from 0 to xmax, default is 10
`Graph`	show graph if TRUE. No graph otherwise

Details

Calculate the power following GRS (1989) <DOI:10.2307/1913625>

The distribution under H1 is based on the value of theta and ratio

Under H0: ratio = 1; under H1: ratio < 1

Value

`Power`	power of the test
`Critical.value`	critical value at alpha

Note

The graph option plots the density functions of the GRS test under H0 and H1.

The blue vertical line represents the critical value at alpha level of significance

The black density function is the one under H0, and the gray-shaded area is level of significance.

The red one is the one under H1, and the red-shaded area is the power.

Author(s)

Jae H. Kim

References

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>

See Also

GRS(1989) <DOI:10.2307/1913625>

Examples

```
GRS.Power(T=120, N=25, K=3, theta=0.3, ratio=0.5)  # Figure 1 of Kim and Shamsuddin (2016)
```

GRS.Powerfunc — *Power functions for the GRS test*

Description

The function plots the power functions for a range of sample size (T), given the other parameter values

Usage

```
GRS.Powerfunc(Tvec, N, K, theta, alpha = 0.05)
```

Arguments

`Tvec`	a vector of sample sizes
`N`	the number of portfolio returns
`K`	the number of risk factors
`theta`	maximum Sharpe ratio of the K factor portfolios
`alpha`	the level of significance, default is 0.05

Details

The power is plotted against the ratio=theta/thetas, the proprotion of potential efficiency

Value

Power: Matrix of power values plotted

Note

Under H0: ratio = 1, so the power = alpha when ratio = 1.

The power increases as the ratio declines from 1.

The power increases with sample size, so the upper power function is associated with larger sample size.

Author(s)

Jae H. Kim

References

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>

See Also

GRS (1989) <DOI:10.2307/1913625>

Examples

```
GRS.Powerfunc(Tvec=c(60,120),N=25, K=3,theta=0.3)  # Figure 2 of Kim and Shamsuddin (2016)
```

`GRS.T` *Sample Size Selection for the GRS test*

Description

Given the desired level of Type I and II error probabilities, the function returns the sample size required.

Usage

```
GRS.T(N, K, theta, ratio, alpha, beta, Tmax = 10000)
```

Arguments

`N`	the number of portfolio returns
`K`	the number of risk factors
`theta`	maximum Sharpe ratio of the K factor portfolios
`ratio`	theta/thetas, proportion of the potential efficiency
`alpha`	the desried level of significance, or Type I error probability
`beta`	the desried level of Type II error probability
`Tmax`	the maximum number of sample size, default is 10000

Details

the desired level of power = 1 - beta

Value

`Required.T`	required sample size
`Critical.value`	the corresponding critical value

Note

Critical.value is from the F-distribution with df1=N and df2=Required.T-N-K degrees of freedom, at the alpha level of significance.

Author(s)

Jae H. Kim

References

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>

See Also

Kim and Shamsuddin, 2017, Empirical Validity of Asset-pricing Models: Application of Optimal Significance Level and Equal Probability Test

Examples

```
GRS.T(N=25,K=3,theta=0.25,ratio=0.4,alpha=0.05, beta=0.05, Tmax=5000)
```

`GRS.test` *GRS test and Model Estimation Results*

Description

Wu statistic given in (5) of GRS (1989) <DOI:10.2307/1913625>

The function also provide estimation results for asset pricing models

Usage

```
GRS.test(ret.mat, factor.mat)
```

Arguments

`ret.mat`	portfolio return matrix, T by N
`factor.mat`	matrix of risk factors, T by K

Details

T: sample size, N: number of portfolio returns, K: number of risk factors

Value

`GRS.stat`	GRS test statistic
`GRS.pval`	its p-value
`coef`	matrix of coefficient estimates from N equations, N by (K+1)
`resid`	matrix of residuals from N equations, T by N
`tstat`	matrix of t-statistics for coefficients, N by (K+1)
`se`	matrix of standard errors for coefficients, N by (K+1)
`R2`	matrix of R-squares for N equations, N by 1

Note

Applicable to CAPM as well as a multi-factor model

Author(s)

Jae H. Kim

References

Gibbons, Ross, Shanken, 1989. A test of the efficiency of a given portfolio, Econometrica, 57,1121-1152. <DOI:10.2307/1913625>

See Also

Fama and French, 1993, Common risk factors in the returns on stocks and bonds, Journal of Financial Economics, 33, 3-56. <DOI:10.1016/0304-405X(93)90023-5>

Fama and French, 2015, A five-factor asset-pricing model, Journal of Financial Economics, 1-22. <DOI:http://dx.doi.org/10.1016/j.jfineco.2014.10.010>

Examples

```
data(data)
factor.mat = data[1:342,2:4]          # Fama-French 3-factor model
ret.mat = data[1:342,8:ncol(data)]    # 25 size-BM portfolio returns
GRS.test(ret.mat,factor.mat)$GRS.stat  # See Table 9C of Fama-French (1993)
```

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