Package 'sgs'

March 4, 2025

```
Title Sparse-Group SLOPE: Adaptive Bi-Level Selection with FDR Control
Version 0.3.5
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Maintainer Fabio Feser <ff120@ic.ac.uk>
Description Implementation of Sparse-group SLOPE (SGS) (Feser and Evan-
     gelou (2023) <doi:10.48550/arXiv.2305.09467>) models. Linear and logistic regression mod-
     els are supported, both of which can be fit using k-fold cross-validation. Dense and sparse in-
     put matrices are supported. In addition, a general Adaptive Three Operator Splitting (ATOS) (Pe-
     dregosa and Gidel (2018) <doi:10.48550/arXiv.1804.02339>) implementation is pro-
     vided. Group SLOPE (gS-
     LOPE) (Brzyski et al. (2019) <doi:10.1080/01621459.2017.1411269>) and group-based OS-
     CAR models (Feser and Evangelou (2024) <doi:10.48550/arXiv.2405.15357>) are also imple-
     mented. All models are available with strong screening rules (Feser and Evan-
     gelou (2024) <doi:10.48550/arXiv.2405.15357>) for computational speed-up.
Imports Matrix, MASS, caret, grDevices, graphics, methods, stats,
     SLOPE, Rlab, Rcpp (>= 1.0.10)
LinkingTo Rcpp, RcppArmadillo
Suggests SGL, gglasso, glmnet, testthat, knitr, grpSLOPE, rmarkdown
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```

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arma_mv

Matrix Product in RcppArmadillo.

Description

Matrix Product in RcppArmadillo.

Usage

```
arma_mv(m, v)
```

Arguments

m numeric matrix
v numeric vector

Value

matrix product of m and v

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arma_sparse

Matrix Product in RcppArmadillo.

Description

Matrix Product in RcppArmadillo.

Usage

```
arma_sparse(m, v)
```

Arguments

```
m numeric sparse matrix
v numeric vector
```

Value

matrix product of m and v

as_sgs

Fits the adaptively scaled SGS model (AS-SGS).

Description

Fits an SGS model using the noise estimation procedure, termed adaptively scaled SGS (Algorithm 2 from Feser and Evangelou (2023)). This adaptively estimates λ and then fits the model using the estimated value. It is an alternative approach to cross-validation (fit_sgs_cv()). The approach is only compatible with the SGS penalties.

Usage

```
as_sgs(
   X,
   y,
   groups,
   type = "linear",
   pen_method = 2,
   alpha = 0.95,
   vFDR = 0.1,
   gFDR = 0.1,
   standardise = "12",
   intercept = TRUE,
   verbose = FALSE
)
```

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Arguments

X	Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package).
у	Output vector of dimension n . For type="linear" should be continuous and for type="logistic" should be a binary variable.
groups	A grouping structure for the input data. Should take the form of a vector of group indices.
type	The type of regression to perform. Supported values are: "linear" and "logistic".
pen_method	The type of penalty sequences to use.
	• "1" uses the vMean and gMean SGS sequences.
	• "2" uses the vMax and gMax SGS sequences.
alpha	The value of α , which defines the convex balance between SLOPE and gSLOPE. Must be between 0 and 1.
vFDR	Defines the desired variable false discovery rate (FDR) level, which determines the shape of the variable penalties. Must be between 0 and 1.
gFDR	Defines the desired group false discovery rate (FDR) level, which determines the shape of the group penalties. Must be between 0 and 1.
standardise	Type of standardisation to perform on X:
	• "12" standardises the input data to have ℓ_2 norms of one.
	• "11" standardises the input data to have ℓ_1 norms of one.
	• "sd" standardises the input data to have standard deviation of one.
	 "none" no standardisation applied.
intercept	Logical flag for whether to fit an intercept.
verbose	Logical flag for whether to print fitting information.

Value

An object of type "sgs" containing model fit information (see fit_sgs()).

References

Feser, F., Evangelou, M. (2023). *Sparse-group SLOPE: adaptive bi-level selection with FDR-control*, https://arxiv.org/abs/2305.09467

See Also

```
scaled_sgs()
Other model-selection: fit_goscar_cv(), fit_gslope_cv(), fit_sgo_cv(), fit_sgs_cv(),
scaled_sgs()
Other SGS-methods: coef.sgs(), fit_sgo(), fit_sgo_cv(), fit_sgs(), fit_sgs_cv(), plot.sgs(),
predict.sgs(), print.sgs(), scaled_sgs()
```

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atos

Adaptive three operator splitting (ATOS).

Description

Function for fitting adaptive three operator splitting (ATOS) with general convex penalties. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
atos(
 Χ,
 у,
  type = "linear",
  prox_1,
  prox_2,
 pen_prox_1 = 0.5,
  pen_prox_2 = 0.5,
 max_iter = 5000,
  backtracking = 0.7,
 max_iter_backtracking = 100,
  tol = 1e-05,
  prox_1_opts = NULL,
  prox_2_opts = NULL,
  standardise = "12",
  intercept = TRUE,
  x0 = NULL,
  u = NULL
  verbose = FALSE
)
```

Arguments

Χ	Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package)
У	Output vector of dimension n . For type="linear" needs to be continuous and for type="logistic" needs to be a binary variable.
type	The type of regression to perform. Supported values are: "linear" and "logistic".
prox_1	The proximal operator for the first function, $h(x)$.
prox_2	The proximal operator for the second function, $g(x)$.
pen_prox_1	The penalty for the first proximal operator. For the lasso, this would be the sparsity parameter, λ . If operator does not include a penalty, set to 1.
pen_prox_2	The penalty for the second proximal operator.
max_iter	Maximum number of ATOS iterations to perform.
backtracking	The backtracking parameter, τ , as defined in Pedregosa and Gidel (2018).

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max_iter_backtracking

Maximum number of backtracking line search iterations to perform per global

iteration.

tol Convergence tolerance for the stopping criteria.

prox_1_opts Optional argument for first proximal operator. For the group lasso, this would

be the group IDs. Note: this must be inserted as a list.

prox_2_opts Optional argument for second proximal operator.

standardise Type of standardisation to perform on X:

• "12" standardises the input data to have ℓ_2 norms of one.

• "11" standardises the input data to have ℓ_1 norms of one.

• "sd" standardises the input data to have standard deviation of one.

• "none" no standardisation applied.

intercept Logical flag for whether to fit an intercept.

x0 Optional initial vector for x_0 . u Optional initial vector for u.

verbose Logical flag for whether to print fitting information.

Details

atos() solves convex minimization problems of the form

$$f(x) + g(x) + h(x),$$

where f is convex and differentiable with L_f -Lipschitz gradient, and g and h are both convex. The algorithm is not symmetrical, but usually the difference between variations are only small numerical values, which are filtered out. However, both variations should be checked regardless, by looking at x and u. An example for the sparse-group lasso (SGL) is given.

Value

intercept

An object of class "atos" containing:

beta	The fitted values from the regression. Taken to be the more stable fit between x and u, which is usually the former.
x	The solution to the original problem (see Pedregosa and Gidel (2018)).
u	The solution to the dual problem (see Pedregosa and Gidel (2018)).
Z	The updated values from applying the first proximal operator (see Pedregosa and Gidel (2018)).
type	Indicates which type of regression was performed.
success	Logical flag indicating whether ATOS converged, according to tol.
num_it	Number of iterations performed. If convergence is not reached, this will be \max_iter .
certificate	Final value of convergence criteria.

Logical flag indicating whether an intercept was fit.

coef.sgs

References

Pedregosa, F., Gidel, G. (2018). *Adaptive Three Operator Splitting*, https://proceedings.mlr.press/v80/pedregosa18a.html

Description

Print the coefficients using model fitted with one of the following functions: fit_sgs(), fit_sgs_cv(), fit_gslope(), fit_gslope_cv(), fit_sgo(), fit_sgo_cv(), fit_goscar(), fit_goscar_cv(). The predictions are calculated for each "lambda" value in the path.

Usage

```
## S3 method for class 'sgs'
coef(object, ...)
```

Arguments

```
object Object of one of the following classes: "sgs", "sgs_cv", "gslope", "gslope_cv".
... further arguments passed to stats function.
```

Value

The fitted coefficients

See Also

```
fit_sgs(), fit_sgs_cv(), fit_gslope(), fit_gslope_cv()
Other SGS-methods: as_sgs(), fit_sgo(), fit_sgo_cv(), fit_sgs(), fit_sgs_cv(), plot.sgs(),
predict.sgs(), print.sgs(), scaled_sgs()
Other gSLOPE-methods: fit_goscar(), fit_goscar_cv(), fit_gslope(), fit_gslope_cv(),
plot.sgs(), predict.sgs(), print.sgs()
```

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run SGS
model = fit_sgs(X = data$X, y = data$y, groups = groups, type="linear", lambda = 1, alpha=0.95,
vFDR=0.1, gFDR=0.1, standardise = "l2", intercept = TRUE, verbose=FALSE)
# use predict function
model_coef = coef(model)
```

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fit_goscar

Fit a gOSCAR model.

Description

Group OSCAR (gOSCAR) main fitting function. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
fit_goscar(
  Χ,
 у,
  groups,
  type = "linear",
  lambda = "path",
  path_length = 20,
 min_frac = 0.05,
 max_iter = 5000,
  backtracking = 0.7,
 max_iter_backtracking = 100,
  tol = 1e-05,
  standardise = "12",
  intercept = TRUE,
  screen = TRUE,
  verbose = FALSE,
  w_wights = NULL
)
```

Arguments

y

groups

type

lambda

X Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package).

Output vector of dimension n. For type="linear" should be continuous and for type="logistic" should be a binary variable.

A grouping structure for the input data. Should take the form of a vector of group indices.

The type of regression to perform. Supported values are: "linear" and "logistic". The regularisation parameter. Defines the level of sparsity in the model. A higher value leads to sparser models:

- "path" computes a path of regularisation parameters of length "path_length". The path will begin just above the value at which the first predictor enters the model and will terminate at the value determined by min_frac.
- User-specified single value or sequence. Internal scaling is applied based on the type of standardisation. The returned "lambda" value will be the original unscaled value(s).

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path_length The number of λ values to fit the model for. If "lambda" is user-specified, this

is ignored.

min_frac Smallest value of λ as a fraction of the maximum value. That is, the final λ will

be "min_frac" of the first λ value.

max_iter Maximum number of ATOS iterations to perform.

backtracking The backtracking parameter, τ , as defined in Pedregosa and Gidel (2018).

max_iter_backtracking

Maximum number of backtracking line search iterations to perform per global

iteration.

tol Convergence tolerance for the stopping criteria.

standardise Type of standardisation to perform on X:

• "12" standardises the input data to have ℓ_2 norms of one. When using this "1ambda" is scaled internally by $1/\sqrt{n}$.

• "11" standardises the input data to have ℓ_1 norms of one. When using this "lambda" is scaled internally by 1/n.

• "sd" standardises the input data to have standard deviation of one.

• "none" no standardisation applied.

intercept Logical flag for whether to fit an intercept.

screen Logical flag for whether to apply screening rules (see Feser and Evangelou

(2024)). Screening discards irrelevant groups before fitting, greatly improving

speed.

verbose Logical flag for whether to print fitting information.

w_weights Optional vector for the group penalty weights. Overrides the OSCAR penalties

when specified. When entering custom weights, these are multiplied internally

by λ . To void this behaviour, set $\lambda = 1$.

Details

fit_goscar() fits a gOSCAR model (Feser and Evangelou (2024)) using adaptive three operator splitting (ATOS). gOSCAR uses the same model set-up as for gSLOPE, but with different weights (see Bao et al. (2020) and Feser and Evangelou (2024)). The penalties are given by (for a group g with m groups):

$$w_g = \sigma_1 + \sigma_3(m - g),$$

where

$$\sigma_1 = d_i ||X^{\mathsf{T}}y||_{\infty}, \ \sigma_3 = \sigma_1/m.$$

Value

A list containing:

beta

The fitted values from the regression. Taken to be the more stable fit between x and z, which is usually the former. A filter is applied to remove very small values, where ATOS has not been able to shrink exactly to zero. Check this against x and z.

fit_goscar

group_effects	The group values from the regression. Taken by applying the ℓ_2 norm within each group on beta.		
selected_var	A list containing the indicies of the active/selected variables for each "lambda" value. Index 1 corresponds to the first column in X.		
selected_grp	A list containing the indicies of the active/selected groups for each "lambda" value. Index 1 corresponds to the first group in the groups vector. You can see the group order by running unique(groups).		
num_it	Number of iterations performed. If convergence is not reached, this will be ${\tt max_iter}.$		
success	Logical flag indicating whether ATOS converged, according to tol.		
certificate	Final value of convergence criteria.		
x	The solution to the original problem (see Pedregosa and Gidel (2018)).		
u	The solution to the dual problem (see Pedregosa and Gidel (2018)).		
Z	The updated values from applying the first proximal operator (see Pedregosa an Gidel (2018)).		
screen_set	List of groups that were kept after screening step for each "lambda" value. (corresponds to ${\cal S}$ in Feser and Evangelou (2024)).		
epsilon_set	List of groups that were used for fitting after screening for each "lambda" value. (corresponds to $\mathcal E$ in Feser and Evangelou (2024)).		
kkt_violations	List of groups that violated the KKT conditions each "lambda" value. (corresponds to ${\cal K}$ in Feser and Evangelou (2024)).		
pen_gslope	Vector of the group penalty sequence.		
screen	Logical flag indicating whether screening was applied.		
type	Indicates which type of regression was performed.		
intercept	Logical flag indicating whether an intercept was fit.		
standardise	Type of standardisation used.		
lambda	Value(s) of λ used to fit the model.		

References

Bao, R., Gu B., Huang, H. (2020). Fast OSCAR and OWL Regression via Safe Screening Rules, https://proceedings.mlr.press/v119/bao20b

Feser, F., Evangelou, M. (2024). *Strong screening rules for group-based SLOPE models*, https://proceedings.mlr.press/v80/pedregosa18a.html

 $Pedregosa, F., Gidel, G.~(2018). \ \textit{Adaptive Three Operator Splitting}, \\ \textbf{https://proceedings.mlr.press/v80/pedregosa18a.html}$

See Also

```
Other gSLOPE-methods: coef.sgs(), fit_goscar_cv(), fit_gslope(), fit_gslope_cv(), plot.sgs(), predict.sgs(), print.sgs()
```

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Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run gOSCAR
model = fit_goscar(X = data$X, y = data$y, groups = groups, type="linear", path_length = 5,
standardise = "12", intercept = TRUE, verbose=FALSE)
```

fit_goscar_cv

Fit a gOSCAR model using k-fold cross-validation.

Description

Function to fit a pathwise solution of group OSCAR (gOSCAR) models using k-fold cross-validation. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
fit_goscar_cv(
  Χ,
 у,
  groups,
  type = "linear",
  lambda = "path",
 path_length = 20,
 min_frac = 0.05,
 nfolds = 10,
 backtracking = 0.7,
 max_iter = 5000,
 max_iter_backtracking = 100,
  tol = 1e-05,
  standardise = "12",
  intercept = TRUE,
  error_criteria = "mse",
  screen = TRUE,
  verbose = FALSE,
 w_wights = NULL
)
```

Arguments

- X Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package).
- Output vector of dimension n. For type="linear" should be continuous and for type="logistic" should be a binary variable.

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groups A grouping structure for the input data. Should take the form of a vector of

group indices.

type The type of regression to perform. Supported values are: "linear" and "logistic".

1ambda The regularisation parameter. Defines the level of sparsity in the model. A higher value leads to sparser models:

• "path" computes a path of regularisation parameters of length "path_length". The path will begin just above the value at which the first predictor enters the model and will terminate at the value determined by "min_frac".

• User-specified single value or sequence. Internal scaling is applied based on the type of standardisation. The returned "lambda" value will be the original unscaled value(s).

path_length The number of λ values to fit the model for. If "lambda" is user-specified, this

is ignored.

min_frac Smallest value of λ as a fraction of the maximum value. That is, the final λ will

be "min_frac" of the first λ value.

nfolds The number of folds to use in cross-validation.

backtracking The backtracking parameter, τ , as defined in Pedregosa and Gidel (2018).

max_iter Maximum number of ATOS iterations to perform.

max_iter_backtracking

Maximum number of backtracking line search iterations to perform per global

iteration.

tol Convergence tolerance for the stopping criteria.

standardise Type of standardisation to perform on X:

• "12" standardises the input data to have ℓ_2 norms of one.

• "11" standardises the input data to have ℓ_1 norms of one.

• "sd" standardises the input data to have standard deviation of one.

• "none" no standardisation applied.

intercept Logical flag for whether to fit an intercept.

error_criteria The criteria used to discriminate between models along the path. Supported

values are: "mse" (mean squared error) and "mae" (mean absolute error).

screen Logical flag for whether to apply screening rules (see Feser and Evangelou

(2024)). Screening discards irrelevant groups before fitting, greatly improving

speed.

verbose Logical flag for whether to print fitting information.

w_weights Optional vector for the group penalty weights. Overrides the OSCAR penalties

when specified. When entering custom weights, these are multiplied internally

by λ . To void this behaviour, set $\lambda = 1$.

Details

Fits gOSCAR models under a pathwise solution using adaptive three operator splitting (ATOS), picking the 1se model as optimum. Warm starts are implemented.

Value

A list containing:

errors A table containing fitting information about the models on the path.

all_models Fitting information for all models fit on the path, which is a "gslope" object

type.

fit The 1se chosen model, which is a "gslope" object type.

best_lambda The value of λ which generated the chosen model.

best_lambda_id The path index for the chosen model.

References

Bao, R., Gu B., Huang, H. (2020). Fast OSCAR and OWL Regression via Safe Screening Rules, https://proceedings.mlr.press/v119/bao20b

Feser, F., Evangelou, M. (2024). *Strong screening rules for group-based SLOPE models*, https://proceedings.mlr.press/v80/pedregosa18a.html

See Also

```
fit_goscar()
```

```
Other\ gSLOPE-methods:\ coef.sgs(),\ fit\_goscar(),\ fit\_gslope(),\ fit\_gslope\_cv(),\ plot.sgs(),\ predict.sgs(),\ print.sgs()
```

Other model-selection: as_sgs(), fit_gslope_cv(), fit_sgo_cv(), fit_sgs_cv(), scaled_sgs()

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run gOSCAR with cross-validation
cv_model = fit_goscar_cv(X = data$X, y = data$y, groups=groups, type = "linear", path_length = 5,
nfolds=5, min_frac = 0.05, standardise="12",intercept=TRUE,verbose=TRUE)
```

fit_gslope

Fit a gSLOPE model.

Description

Group SLOPE (gSLOPE) main fitting function. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
fit_gslope(
 Χ,
 у,
  groups,
  type = "linear",
  lambda = "path",
  path_length = 20,
 min_frac = 0.05,
  gFDR = 0.1,
  pen_method = 1,
  max_iter = 5000,
  backtracking = 0.7,
  max_iter_backtracking = 100,
  tol = 1e-05,
  standardise = "12",
  intercept = TRUE,
  screen = TRUE,
  verbose = FALSE,
  w_weights = NULL
)
```

Arguments

у

Χ Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package).

Output vector of dimension n. For type="linear" should be continuous and

for type="logistic" should be a binary variable.

A grouping structure for the input data. Should take the form of a vector of groups

group indices.

The type of regression to perform. Supported values are: "linear" and "logistic". type

The regularisation parameter. Defines the level of sparsity in the model. A lambda higher value leads to sparser models:

- "path" computes a path of regularisation parameters of length "path_length". The path will begin just above the value at which the first predictor enters the model and will terminate at the value determined by "min_frac".
- User-specified single value or sequence. Internal scaling is applied based on the type of standardisation. The returned "lambda" value will be the original unscaled value(s).

path_length The number of λ values to fit the model for. If "lambda" is user-specified, this is ignored.

> Smallest value of λ as a fraction of the maximum value. That is, the final λ will be "min_frac" of the first λ value.

Defines the desired group false discovery rate (FDR) level, which determines the shape of the group penalties. Must be between 0 and 1.

min_frac

gFDR

pen_method The type of penalty sequences to use (see Brzyski et al. (2019)):

- "1" uses the gMean gSLOPE sequence.
- "2" uses the gMax gSLOPE sequence.

max_iter Maximum number of ATOS iterations to perform.

backtracking The backtracking parameter, τ , as defined in Pedregosa and Gidel (2018). max_iter_backtracking

Maximum number of backtracking line search iterations to perform per global iteration.

tol Convergence tolerance for the stopping criteria.

standardise Type of standardisation to perform on X:

- "12" standardises the input data to have ℓ_2 norms of one. When using this "lambda" is scaled internally by $1/\sqrt{n}$.
- "11" standardises the input data to have ℓ_1 norms of one. When using this "lambda" is scaled internally by 1/n.
- "sd" standardises the input data to have standard deviation of one.
- "none" no standardisation applied.

intercept Logical flag for whether to fit an intercept.

Logical flag for whether to apply screening rules (see Feser and Evangelou (2024)). Screening discards irrelevant groups before fitting, greatly improving

speed.

verbose Logical flag for whether to print fitting information.

w_weights Optional vector for the group penalty weights. Overrides the penalties from

pen_method if specified. When entering custom weights, these are multiplied

internally by λ . To void this behaviour, set $\lambda = 1$.

Details

fit_gslope() fits a gSLOPE model (Brzyski et al. (2019)) using adaptive three operator splitting (ATOS). gSLOPE is a sparse-group method, so that it selects both variables and groups. Unlike group selection approaches, not every variable within a group is set as active. It solves the convex optimisation problem given by

$$\frac{1}{2n}f(b; y, \mathbf{X}) + \lambda \sum_{g=1}^{m} w_g \sqrt{p_g} ||b^{(g)}||_2,$$

where the penalty sequences are sorted and $f(\cdot)$ is the loss function. In the case of the linear model, the loss function is given by the mean-squared error loss:

$$f(b; y, \mathbf{X}) = \|y - \mathbf{X}b\|_{2}^{2}.$$

In the logistic model, the loss function is given by

$$f(b; y, \mathbf{X}) = -1/n \log(\mathcal{L}(b; y, \mathbf{X})).$$

where the log-likelihood is given by

$$\mathcal{L}(b; y, \mathbf{X}) = \sum_{i=1}^{n} \left\{ y_i b^{\mathsf{T}} x_i - \log(1 + \exp(b^{\mathsf{T}} x_i)) \right\}.$$

The penalty parameters in gSLOPE are sorted so that the largest group effects are matched with the largest penalties, to reduce the group FDR. The gMean sequence (pen_method=1) is given by

$$w_i^{\text{mean}} = \overline{F}_{\chi_{p_j}}^{-1}(1 - q_g i/m), \ i = 1, \dots, m, \text{ where } \overline{F}_{\chi_{p_j}}(x) := \frac{1}{m} \sum_{j=1}^m F_{\chi_{p_j}}(\sqrt{p_j}x),$$

where $F_{\chi_{p_j}}$ is the cumulative distribution function of a χ distribution with p_j degrees of freedom. The gMax sequence (pen_method=2) is given by

$$w_i^{\max} = \max_{j=1,\dots,m} \left\{ \frac{1}{\sqrt{p_j}} F_{\chi_{p_j}}^{-1} \left(1 - \frac{q_g i}{m} \right) \right\},$$

where $F_{\chi_{p_i}}$ is the cumulative distribution function of a χ distribution with p_j degrees of freedom.

Value

A list containing:

beta	The fitted values from the regression. Taken to be the more stable fit between x and z, which is usually the former. A filter is applied to remove very small values, where ATOS has not been able to shrink exactly to zero. Check this against x and z.
group_effects	The group values from the regression. Taken by applying the ℓ_2 norm within each group on beta.
selected_var	A list containing the indicies of the active/selected variables for each "lambda" value. Index 1 corresponds to the first column in X.
selected_grp	A list containing the indicies of the active/selected groups for each "lambda" value. Index 1 corresponds to the first group in the groups vector. You can see the group order by running unique(groups).
num_it	Number of iterations performed. If convergence is not reached, this will be max_iter.
success	Logical flag indicating whether ATOS converged, according to tol.
certificate	Final value of convergence criteria.
x	The solution to the original problem (see Pedregosa and Gidel (2018)).
u	The solution to the dual problem (see Pedregosa and Gidel (2018)).
Z	The updated values from applying the first proximal operator (see Pedregosa and Gidel (2018)).
screen_set	List of groups that were kept after screening step for each "lambda" value. (corresponds to ${\cal S}$ in Feser and Evangelou (2024)).
epsilon_set	List of groups that were used for fitting after screening for each "lambda" value. (corresponds to $\mathcal E$ in Feser and Evangelou (2024)).
kkt_violations	List of groups that violated the KKT conditions each "lambda" value. (corresponds to $\mathcal K$ in Feser and Evangelou (2024)).
pen_gslope	Vector of the group penalty sequence.
screen	Logical flag indicating whether screening was applied.

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type Indicates which type of regression was performed.intercept Logical flag indicating whether an intercept was fit.

standardise Type of standardisation used. lambda Value(s) of λ used to fit the model.

References

```
Brzyski, D., Gossmann, A., Su, W., Bodgan, M. (2019). Group SLOPE – Adaptive Selection of Groups of Predictors, https://www.tandfonline.com/doi/full/10.1080/01621459.2017.1411269
```

Feser, F., Evangelou, M. (2024). *Strong screening rules for group-based SLOPE models*, https://proceedings.mlr.press/v80/pedregosa18a.html

Pedregosa, F., Gidel, G. (2018). *Adaptive Three Operator Splitting*, https://proceedings.mlr.press/v80/pedregosa18a.html

See Also

```
Other gSLOPE-methods: coef.sgs(), fit_goscar(), fit_goscar_cv(), fit_gslope_cv(), plot.sgs(), predict.sgs(), print.sgs()
```

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run gSLOPE
model = fit_gslope(X = data$X, y = data$y, groups = groups, type="linear", path_length = 5,gFDR=0.1, standardise = "12", intercept = TRUE, verbose=FALSE)
```

fit_gslope_cv

Fit a gSLOPE model using k-fold cross-validation.

Description

Function to fit a pathwise solution of group SLOPE (gSLOPE) models using k-fold cross-validation. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
fit_gslope_cv(
   X,
   y,
   groups,
   type = "linear",
   lambda = "path",
   path_length = 20,
```

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```
min_frac = 0.05,
nfolds = 10,
gFDR = 0.1,
pen_method = 1,
backtracking = 0.7,
max_iter = 5000,
max_iter_backtracking = 100,
tol = 1e-05,
standardise = "12",
intercept = TRUE,
error_criteria = "mse",
screen = TRUE,
verbose = FALSE,
w_weights = NULL
)
```

Arguments

X Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix"

from the Matrix package).

Output vector of dimension n. For type="linear" should be continuous and

for type="logistic" should be a binary variable.

groups A grouping structure for the input data. Should take the form of a vector of

group indices.

type The type of regression to perform. Supported values are: "linear" and "logistic".

lambda The regularisation parameter. Defines the level of sparsity in the model. A higher value leads to sparser models:

"path" computes a path of regularisation parameters of length "path_length".
The path will begin just above the value at which the first predictor enters

the model and will terminate at the value determined by "min_frac".
User-specified single value or sequence. Internal scaling is applied based on the type of standardisation. The returned "lambda" value will be the

original unscaled value(s).

path_length The number of λ values to fit the model for. If "lambda" is user-specified, this

is ignored.

min_frac Smallest value of λ as a fraction of the maximum value. That is, the final λ will

be "min_frac" of the first λ value.

nfolds The number of folds to use in cross-validation.

gFDR Defines the desired group false discovery rate (FDR) level, which determines

the shape of the penalties. Must be between 0 and 1.

pen_method The type of penalty sequences to use (see Brzyski et al. (2019)):

• "1" uses the gMean gSLOPE sequence.

• "2" uses the gMax gSLOPE sequence.

backtracking The backtracking parameter, τ , as defined in Pedregosa and Gidel (2018).

max_iter Maximum number of ATOS iterations to perform.

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max_iter_backtracking

Maximum number of backtracking line search iterations to perform per global

iteration.

tol Convergence tolerance for the stopping criteria.

standardise Type of standardisation to perform on X:

• "12" standardises the input data to have ℓ_2 norms of one.

• "11" standardises the input data to have ℓ_1 norms of one.

• "sd" standardises the input data to have standard deviation of one.

• "none" no standardisation applied.

intercept Logical flag for whether to fit an intercept.

error_criteria The criteria used to discriminate between models along the path. Supported

values are: "mse" (mean squared error) and "mae" (mean absolute error).

screen Logical flag for whether to apply screening rules (see Feser and Evangelou

(2024)). Screening discards irrelevant groups before fitting, greatly improving

speed.

verbose Logical flag for whether to print fitting information.

w_weights Optional vector for the group penalty weights. Overrides the penalties from

pen_method if specified. When entering custom weights, these are multiplied

internally by λ . To void this behaviour, set $\lambda = 1$.

Details

Fits gSLOPE models under a pathwise solution using adaptive three operator splitting (ATOS), picking the 1se model as optimum. Warm starts are implemented.

Value

A list containing:

errors A table containing fitting information about the models on the path.

all_models Fitting information for all models fit on the path, which is a "gslope" object

type.

fit The 1se chosen model, which is a "gslope" object type.

best_lambda The value of λ which generated the chosen model.

best_lambda_id The path index for the chosen model.

References

Brzyski, D., Gossmann, A., Su, W., Bodgan, M. (2019). *Group SLOPE – Adaptive Selection of Groups of Predictors*, https://www.tandfonline.com/doi/full/10.1080/01621459.2017.1411269

Feser, F., Evangelou, M. (2024). *Strong screening rules for group-based SLOPE models*, https://proceedings.mlr.press/v80/pedregosa18a.html

See Also

```
fit_gslope()
Other gSLOPE-methods: coef.sgs(), fit_goscar(), fit_goscar_cv(), fit_gslope(), plot.sgs(),
predict.sgs(), print.sgs()
Other model-selection: as_sgs(), fit_goscar_cv(), fit_sgo_cv(), fit_sgs_cv(), scaled_sgs()
```

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run gSLOPE with cross-validation
cv_model = fit_gslope_cv(X = data$X, y = data$y, groups=groups, type = "linear", path_length = 5,
nfolds=5, gFDR = 0.1, min_frac = 0.05, standardise="12",intercept=TRUE,verbose=TRUE)
```

fit_sgo

Fit an SGO model.

Description

Sparse-group OSCAR (SGO) main fitting function. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
fit_sgo(
  Χ,
  у,
  groups,
  type = "linear",
  lambda = "path",
  path_length = 20,
 min_frac = 0.05,
  alpha = 0.95,
  max_iter = 5000,
  backtracking = 0.7,
  max_iter_backtracking = 100,
  tol = 1e-05,
  standardise = "12",
  intercept = TRUE,
  screen = TRUE,
  verbose = FALSE,
 w_weights = NULL,
  v_weights = NULL
)
```

Arguments

type

Χ Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package).

Output vector of dimension n. For type="linear" should be continuous and У for type="logistic" should be a binary variable.

A grouping structure for the input data. Should take the form of a vector of groups group indices.

The type of regression to perform. Supported values are: "linear" and "logistic". lambda

The regularisation parameter. Defines the level of sparsity in the model. A higher value leads to sparser models:

- "path" computes a path of regularisation parameters of length "path_length". The path will begin just above the value at which the first predictor enters the model and will terminate at the value determined by "min_frac".
- User-specified single value or sequence. Internal scaling is applied based on the type of standardisation. The returned "lambda" value will be the original unscaled value(s).

path_length The number of λ values to fit the model for. If "lambda" is user-specified, this is ignored.

Smallest value of λ as a fraction of the maximum value. That is, the final λ will be "min_frac" of the first λ value.

alpha The value of α , which defines the convex balance between OSCAR and gOSCAR. Must be between 0 and 1. Recommended value is 0.95.

max_iter Maximum number of ATOS iterations to perform.

backtracking The backtracking parameter, τ , as defined in Pedregosa and Gidel (2018). max_iter_backtracking

> Maximum number of backtracking line search iterations to perform per global iteration.

tol Convergence tolerance for the stopping criteria.

standardise Type of standardisation to perform on X:

- "12" standardises the input data to have ℓ_2 norms of one. When using this "lambda" is scaled internally by $1/\sqrt{n}$.
- "11" standardises the input data to have ℓ_1 norms of one. When using this "lambda" is scaled internally by 1/n.
- "sd" standardises the input data to have standard deviation of one.
- "noBaone" no standardisation applied.

Logical flag for whether to fit an intercept.

Logical flag for whether to apply screening rules (see Feser and Evangelou (2024)). Screening discards irrelevant groups before fitting, greatly improving

verbose Logical flag for whether to print fitting information.

> Optional vector for the group penalty weights. Overrides the OSCAR penalties when specified. When entering custom weights, these are multiplied internally by λ and $1 - \alpha$. To void this behaviour, set $\lambda = 2$ and $\alpha = 0.5$.

min_frac

intercept

screen

w_weights

v_weights

Optional vector for the variable penalty weights. Overrides the OSCAR penalties when specified. When entering custom weights, these are multiplied internally by λ and α . To void this behaviour, set $\lambda=2$ and $\alpha=0.5$.

Details

fit_sgo() fits an SGO model (Feser and Evangelou (2024)) using adaptive three operator splitting (ATOS). SGO uses the same model set-up as for SGS, but with different weights (see Bao et al. (2020) and Feser and Evangelou (2024)). The penalties are given by (for a group g and variable i, with p variables and m groups):

$$v_i = \sigma_1 + \sigma_2(p-i), \ w_g = \sigma_1 + \sigma_3(m-g),$$

where

$$\sigma_1 = d_i ||X^{\mathsf{T}}y||_{\infty}, \ \sigma_2 = \sigma_1/p, \ \sigma_3 = \sigma_1/m, \ d_i = i \times \exp{(-2)}.$$

Value

A list containing:

beta The fitted values from the regression. Taken to be the more stable fit between x and z, which is usually the former. A filter is applied to remove very small values, where ATOS has not been able to shrink exactly to zero. Check this against x and z. group_effects The group values from the regression. Taken by applying the ℓ_2 norm within each group on beta. A list containing the indicies of the active/selected variables for each "lambda" selected_var value. Index 1 corresponds to the first column in X. selected_grp A list containing the indicies of the active/selected groups for each "lambda" value. Index 1 corresponds to the first group in the groups vector. You can see the group order by running unique(groups). num_it Number of iterations performed. If convergence is not reached, this will be max_iter. success Logical flag indicating whether ATOS converged, according to tol. certificate Final value of convergence criteria. The solution to the original problem (see Pedregosa and Gidel (2018)). х z The updated values from applying the first proximal operator (see Pedregosa and Gidel (2018)). The solution to the dual problem (see Pedregosa and Gidel (2018)). screen_set_var List of variables that were kept after screening step for each "lambda" value. (corresponds to S_v in Feser and Evangelou (2024)).

screen_set_grp List of groups that were kept after screening step for each "lambda" value. (cor-

responds to S_g in Feser and Evangelou (2024)).

epsilon_set_var

List of variables that were used for fitting after screening for each "lambda" value. (corresponds to \mathcal{E}_v in Feser and Evangelou (2024)).

epsilon_set_grp

List of groups that were used for fitting after screening for each "lambda" value. (corresponds to \mathcal{E}_q in Feser and Evangelou (2024)).

kkt_violations_var

List of variables that violated the KKT conditions each "lambda" value. (corresponds to K_v in Feser and Evangelou (2024)).

kkt_violations_grp

List of groups that violated the KKT conditions each "lambda" value. (corresponds to K_q in Feser and Evangelou (2024)).

pen_slope Vector of the variable penalty sequence.

pen_gslope Vector of the group penalty sequence.

screen Logical flag indicating whether screening was performed.

type Indicates which type of regression was performed.intercept Logical flag indicating whether an intercept was fit.

lambda Value(s) of λ used to fit the model.

References

Bao, R., Gu B., Huang, H. (2020). Fast OSCAR and OWL Regression via Safe Screening Rules, https://proceedings.mlr.press/v119/bao20b

Feser, F., Evangelou, M. (2023). *Sparse-group SLOPE: adaptive bi-level selection with FDR-control*, https://arxiv.org/abs/2305.09467

Feser, F., Evangelou, M. (2024). *Strong screening rules for group-based SLOPE models*, https://arxiv.org/abs/2405.15357

Pedregosa, F., Gidel, G. (2018). *Adaptive Three Operator Splitting*, https://proceedings.mlr.press/v80/pedregosa18a.html

See Also

```
Other SGS-methods: as_sgs(), coef.sgs(), fit_sgo_cv(), fit_sgs(), fit_sgs_cv(), plot.sgs(), predict.sgs(), print.sgs(), scaled_sgs()
```

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run SGO
model = fit_sgo(X = data$X, y = data$y, groups = groups, type="linear", path_length = 5, alpha=0.95, standardise = "l2", intercept = TRUE, verbose=FALSE)
```

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fit_sgo_cv

Fit an SGO model using k-fold cross-validation.

Description

Function to fit a pathwise solution of sparse-group SLOPE (SGO) models using k-fold cross-validation. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
fit_sgo_cv(
 Χ,
 у,
  groups,
  type = "linear",
  lambda = "path",
  path_length = 20,
 min_frac = 0.05,
  alpha = 0.95,
  nfolds = 10,
  backtracking = 0.7,
 max_iter = 5000,
 max_iter_backtracking = 100,
  tol = 1e-05,
  standardise = "12",
  intercept = TRUE,
  error_criteria = "mse",
  screen = TRUE,
  verbose = FALSE,
  v_weights = NULL,
  w_wights = NULL
)
```

Arguments

X	Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package).
у	Output vector of dimension n . For type="linear" should be continuous and for type="logistic" should be a binary variable.
groups	A grouping structure for the input data. Should take the form of a vector of group indices.
type	The type of regression to perform. Supported values are: "linear" and "logistic".
lambda	The regularisation parameter. Defines the level of sparsity in the model. A higher value leads to sparser models:

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• "path" computes a path of regularisation parameters of length "path_length". The path will begin just above the value at which the first predictor enters the model and will terminate at the value determined by "min_frac".

• User-specified single value or sequence. Internal scaling is applied based on the type of standardisation. The returned "lambda" value will be the original unscaled value(s).

path_length The number of λ values to fit the model for. If "lambda" is user-specified, this

is ignored.

min_frac Smallest value of λ as a fraction of the maximum value. That is, the final λ will

be "min_frac" of the first λ value.

alpha The value of α , which defines the convex balance between OSCAR and gOSCAR.

Must be between 0 and 1. Recommended value is 0.95.

nfolds The number of folds to use in cross-validation.

backtracking The backtracking parameter, τ , as defined in Pedregosa and Gidel (2018).

max_iter Maximum number of ATOS iterations to perform.

max_iter_backtracking

Maximum number of backtracking line search iterations to perform per global

iteration.

tol Convergence tolerance for the stopping criteria.

standardise Type of standardisation to perform on X:

• "12" standardises the input data to have ℓ_2 norms of one.

• "11" standardises the input data to have ℓ_1 norms of one.

• "sd" standardises the input data to have standard deviation of one.

• "none" no standardisation applied.

intercept Logical flag for whether to fit an intercept.

error_criteria The criteria used to discriminate between models along the path. Supported

values are: "mse" (mean squared error) and "mae" (mean absolute error).

screen Logical flag for whether to apply screening rules (see Feser and Evangelou

(2024)). Screening discards irrelevant groups before fitting, greatly improving

speed.

verbose Logical flag for whether to print fitting information.

v_weights Optional vector for the variable penalty weights. Overrides the OSCAR penal-

ties when specified. When entering custom weights, these are multiplied inter-

nally by λ and α . To void this behaviour, set $\lambda = 2$ and $\alpha = 0.5$.

w_weights Optional vector for the group penalty weights. Overrides the OSCAR penalties

when specified. When entering custom weights, these are multiplied internally

by λ and $1 - \alpha$. To void this behaviour, set $\lambda = 2$ and $\alpha = 0.5$.

Details

Fits SGO models under a pathwise solution using adaptive three operator splitting (ATOS), picking the 1se model as optimum. Warm starts are implemented.

Value

A list containing:

all_models A list of all the models fitted along the path.

fit The 1se chosen model, which is a "sgs" object type. best_lambda The value of λ which generated the chosen model.

best_lambda_id The path index for the chosen model.

errors A table containing fitting information about the models on the path.

type Indicates which type of regression was performed.

References

Bao, R., Gu B., Huang, H. (2020). Fast OSCAR and OWL Regression via Safe Screening Rules, https://proceedings.mlr.press/v119/bao20b

Feser, F., Evangelou, M. (2023). *Sparse-group SLOPE: adaptive bi-level selection with FDR-control*, https://arxiv.org/abs/2305.09467

Feser, F., Evangelou, M. (2024). *Strong screening rules for group-based SLOPE models*, https://arxiv.org/abs/2405.15357

See Also

```
fit_sgo()
```

```
Other model-selection: as_sgs(), fit_goscar_cv(), fit_gslope_cv(), fit_sgs_cv(), scaled_sgs()
Other SGS-methods: as_sgs(), coef.sgs(), fit_sgo(), fit_sgs(), fit_sgs_cv(), plot.sgs(),
predict.sgs(), print.sgs(), scaled_sgs()
```

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run SGO with cross-validation
cv_model = fit_sgo_cv(X = data$X, y = data$y, groups=groups, type = "linear",
path_length = 5, nfolds=5, alpha = 0.95, min_frac = 0.05,
standardise="12",intercept=TRUE,verbose=TRUE)
```

fit_sgs

Fit an SGS model.

Description

Sparse-group SLOPE (SGS) main fitting function. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
fit_sgs(
 Χ,
 у,
  groups,
  type = "linear",
  lambda = "path",
  path_length = 20,
 min_frac = 0.05,
  alpha = 0.95,
  vFDR = 0.1,
  gFDR = 0.1,
  pen_method = 1,
 max_iter = 5000,
  backtracking = 0.7,
 max_iter_backtracking = 100,
  tol = 1e-05,
  standardise = "12",
  intercept = TRUE,
  screen = TRUE,
  verbose = FALSE,
 w_weights = NULL,
  v_weights = NULL
)
```

Arguments

type

Χ Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix"

from the Matrix package).

Output vector of dimension n. For type="linear" should be continuous and У

for type="logistic" should be a binary variable.

groups A grouping structure for the input data. Should take the form of a vector of group indices.

The type of regression to perform. Supported values are: "linear" and "logistic".

lambda The regularisation parameter. Defines the level of sparsity in the model. A higher value leads to sparser models:

- "path" computes a path of regularisation parameters of length "path_length". The path will begin just above the value at which the first predictor enters the model and will terminate at the value determined by "min_frac".
- User-specified single value or sequence. Internal scaling is applied based on the type of standardisation. The returned "lambda" value will be the original unscaled value(s).

path_length The number of λ values to fit the model for. If "lambda" is user-specified, this is ignored.

min_frac Smallest value of λ as a fraction of the maximum value. That is, the final λ will be "min_frac" of the first λ value.

alpha The value of α , which defines the convex balance between SLOPE and gSLOPE.

Must be between 0 and 1. Recommended value is 0.95.

vFDR Defines the desired variable false discovery rate (FDR) level, which determines

the shape of the variable penalties. Must be between 0 and 1.

gFDR Defines the desired group false discovery rate (FDR) level, which determines

the shape of the group penalties. Must be between 0 and 1.

pen_method The type of penalty sequences to use (see Feser and Evangelou (2023)):

• "1" uses the vMean SGS and gMean gSLOPE sequences.

• "2" uses the vMax SGS and gMean gSLOPE sequences.

• "3" uses the BH SLOPE and gMean gSLOPE sequences, also known as SGS Original.

max_iter Maximum number of ATOS iterations to perform.

backtracking The backtracking parameter, τ , as defined in Pedregosa and Gidel (2018).

max_iter_backtracking

Maximum number of backtracking line search iterations to perform per global iteration

tol Convergence tolerance for the stopping criteria.

standardise Type of standardisation to perform on X:

• "12" standardises the input data to have ℓ_2 norms of one. When using this "lambda" is scaled internally by $1/\sqrt{n}$.

• "11" standardises the input data to have ℓ_1 norms of one. When using this "lambda" is scaled internally by 1/n.

• "sd" standardises the input data to have standard deviation of one.

• "none" no standardisation applied.

intercept Logical flag for whether to fit an intercept.

screen Logical flag for whether to apply screening rules (see Feser and Evangelou

(2024)). Screening discards irrelevant groups before fitting, greatly improving

speed.

verbose Logical flag for whether to print fitting information.

w_weights Optional vector for the group penalty weights. Overrides the penalties from

pen_method if specified. When entering custom weights, these are multiplied

internally by λ and $1 - \alpha$. To void this behaviour, set $\lambda = 2$ and $\alpha = 0.5$.

v_weights Optional vector for the variable penalty weights. Overrides the penalties from

pen_method if specified. When entering custom weights, these are multiplied

internally by λ and α . To void this behaviour, set $\lambda = 2$ and $\alpha = 0.5$.

Details

fit_sgs() fits an SGS model (Feser and Evangelou (2023)) using adaptive three operator splitting (ATOS). SGS is a sparse-group method, so that it selects both variables and groups. Unlike group selection approaches, not every variable within a group is set as active. It solves the convex optimisation problem given by

$$\frac{1}{2n}f(b;y,\mathbf{X}) + \lambda\alpha \sum_{i=1}^{p} v_i|b|_{(i)} + \lambda(1-\alpha) \sum_{g=1}^{m} w_g \sqrt{p_g} ||b^{(g)}||_2,$$

where $f(\cdot)$ is the loss function and p_g are the group sizes. The penalty parameters in SGS are sorted so that the largest coefficients are matched with the largest penalties, to reduce the FDR. For the variables: $|\beta|_{(1)} \ge \ldots \ge |\beta|_{(p)}$ and $v_1 \ge \ldots \ge v_p \ge 0$. For the groups: $\sqrt{p_1} \|\beta^{(1)}\|_2 \ge \ldots \ge \sqrt{p_m} \|\beta^{(m)}\|_2$ and $w_1 \ge \ldots \ge w_g \ge 0$. In the case of the linear model, the loss function is given by the mean-squared error loss:

$$f(b; y, \mathbf{X}) = \|y - \mathbf{X}b\|_2^2.$$

In the logistic model, the loss function is given by

$$f(b; y, \mathbf{X}) = -1/n \log(\mathcal{L}(b; y, \mathbf{X})).$$

where the log-likelihood is given by

$$\mathcal{L}(b; y, \mathbf{X}) = \sum_{i=1}^{n} \left\{ y_i b^{\mathsf{T}} x_i - \log(1 + \exp(b^{\mathsf{T}} x_i)) \right\}.$$

SGS can be seen to be a convex combination of SLOPE and gSLOPE, balanced through alpha, such that it reduces to SLOPE for alpha = 1 and to gSLOPE for alpha = 0. The penalty parameters in SGS are sorted so that the largest coefficients are matched with the largest penalties, to reduce the FDR. For the group penalties, see fit_gslope(). For the variable penalties, the vMean SGS sequence (pen_method=1) (Feser and Evangelou (2023)) is given by

$$v_i^{\text{mean}} = \overline{F}_{\mathcal{N}}^{-1} \left(1 - \frac{q_v i}{2p} \right), \text{ where } \overline{F}_{\mathcal{N}}(x) := \frac{1}{m} \sum_{j=1}^m F_{\mathcal{N}} \left(\alpha x + \frac{1}{3} (1 - \alpha) a_j w_j \right), i = 1, \dots, p,$$

where F_N is the cumulative distribution functions of a standard Gaussian distribution. The vMax SGS sequence (pen_method=2) (Feser and Evangelou (2023)) is given by

$$v_i^{\max} = \max_{j=1,\dots,m} \left\{ \frac{1}{\alpha} F_{\mathcal{N}}^{-1} \left(1 - \frac{q_v i}{2p} \right) - \frac{1}{3\alpha} (1 - \alpha) a_j w_j \right\},$$

The BH SLOPE sequence (pen_method=3) (Bogdan et al. (2015)) is given by

$$v_i = z(1 - iq_v/2p),$$

where z is the quantile function of a standard normal distribution.

Value

A list containing:

The fitted values from the regression. Taken to be the more stable fit between x and z, which is usually the former. A filter is applied to remove very small values, where ATOS has not been able to shrink exactly to zero. Check this against x and z.

group_effects The group values from the regression. Taken by applying the ℓ_2 norm within each group on beta.

selected_var A list containing the indicies of the active/selected variables for each "lambda" value. Index 1 corresponds to the first column in X.

selected_grp A list containing the indicies of the active/selected groups for each "lambda" value. Index 1 corresponds to the first group in the groups vector. You can see the group order by running unique (groups). Number of iterations performed. If convergence is not reached, this will be num_it max_iter. success Logical flag indicating whether ATOS converged, according to tol. certificate Final value of convergence criteria. The solution to the original problem (see Pedregosa and Gidel (2018)). Х z The updated values from applying the first proximal operator (see Pedregosa and Gidel (2018)). The solution to the dual problem (see Pedregosa and Gidel (2018)). screen_set_var List of variables that were kept after screening step for each "lambda" value. (corresponds to S_v in Feser and Evangelou (2024)). screen_set_grp List of groups that were kept after screening step for each "lambda" value. (corresponds to S_q in Feser and Evangelou (2024)). epsilon_set_var List of variables that were used for fitting after screening for each "lambda" value. (corresponds to \mathcal{E}_v in Feser and Evangelou (2024)). epsilon_set_grp List of groups that were used for fitting after screening for each "lambda" value. (corresponds to \mathcal{E}_q in Feser and Evangelou (2024)). kkt_violations_var List of variables that violated the KKT conditions each "lambda" value. (corresponds to K_v in Feser and Evangelou (2024)). kkt_violations_grp List of groups that violated the KKT conditions each "lambda" value. (corre-

sponds to \mathcal{K}_g in Feser and Evangelou (2024)).

pen_slope Vector of the variable penalty sequence. pen_gslope Vector of the group penalty sequence.

Logical flag indicating whether screening was performed. screen

Indicates which type of regression was performed. type Logical flag indicating whether an intercept was fit. intercept

lambda Value(s) of λ used to fit the model.

References

Bogdan, M., van den Berg, E., Sabatti, C., Candes, E. (2015). SLOPE - Adaptive variable selection via convex optimization, https://projecteuclid.org/journals/annals-of-applied-statistics/ volume-9/issue-3/SLOPEAdaptive-variable-selection-via-convex-optimization/10.1214/ 15-AOAS842.full

Feser, F., Evangelou, M. (2023). Sparse-group SLOPE: adaptive bi-level selection with FDRcontrol, https://arxiv.org/abs/2305.09467

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```
Feser, F., Evangelou, M. (2024). Strong screening rules for group-based SLOPE models, https://arxiv.org/abs/2405.15357
```

Pedregosa, F., Gidel, G. (2018). *Adaptive Three Operator Splitting*, https://proceedings.mlr.press/v80/pedregosa18a.html

See Also

```
Other SGS-methods: as_sgs(), coef.sgs(), fit_sgo(), fit_sgo_cv(), fit_sgs_cv(), plot.sgs(), predict.sgs(), print.sgs(), scaled_sgs()
```

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run SGS
model = fit_sgs(X = data$X, y = data$y, groups = groups, type="linear", path_length = 5,
alpha=0.95, vFDR=0.1, gFDR=0.1, standardise = "l2", intercept = TRUE, verbose=FALSE)
```

fit_sgs_cv

Fit an SGS model using k-fold cross-validation.

Description

Function to fit a pathwise solution of sparse-group SLOPE (SGS) models using k-fold cross-validation. Supports both linear and logistic regression, both with dense and sparse matrix implementations.

Usage

```
fit_sgs_cv(
 Χ,
 у,
  groups,
  type = "linear",
  lambda = "path",
  path_length = 20,
 min_frac = 0.05,
  alpha = 0.95,
  vFDR = 0.1,
  gFDR = 0.1,
  pen_method = 1,
 nfolds = 10,
 backtracking = 0.7,
 max_iter = 5000,
 max_iter_backtracking = 100,
  tol = 1e-05,
```

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```
standardise = "12",
  intercept = TRUE,
  error_criteria = "mse",
  screen = TRUE,
  verbose = FALSE,
  v_weights = NULL,
 w_weights = NULL
)
```

Arguments

Χ Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix" from the Matrix package).

У Output vector of dimension n. For type="linear" should be continuous and for type="logistic" should be a binary variable.

A grouping structure for the input data. Should take the form of a vector of groups

group indices.

The type of regression to perform. Supported values are: "linear" and "logistic". type

The regularisation parameter. Defines the level of sparsity in the model. A lambda higher value leads to sparser models:

> • "path" computes a path of regularisation parameters of length "path_length". The path will begin just above the value at which the first predictor enters the model and will terminate at the value determined by "min_frac".

• User-specified single value or sequence. Internal scaling is applied based on the type of standardisation. The returned "lambda" value will be the original unscaled value(s).

The number of λ values to fit the model for. If "lambda" is user-specified, this path_length

is ignored.

min frac Smallest value of λ as a fraction of the maximum value. That is, the final λ will

be "min_frac" of the first λ value.

The value of α , which defines the convex balance between SLOPE and gSLOPE. alpha

Must be between 0 and 1. Recommended value is 0.95.

Defines the desired variable false discovery rate (FDR) level, which determines

the shape of the variable penalties. Must be between 0 and 1.

Defines the desired group false discovery rate (FDR) level, which determines gFDR

the shape of the group penalties. Must be between 0 and 1.

pen_method The type of penalty sequences to use (see Feser and Evangelou (2023)):

• "1" uses the vMean SGS and gMean gSLOPE sequences.

• "2" uses the vMax SGS and gMean gSLOPE sequences.

• "3" uses the BH SLOPE and gMean gSLOPE sequences, also known as SGS Original.

nfolds The number of folds to use in cross-validation.

backtracking The backtracking parameter, τ , as defined in Pedregosa and Gidel (2018).

max_iter Maximum number of ATOS iterations to perform.

vFDR

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max_iter_backtracking

Maximum number of backtracking line search iterations to perform per global

iteration.

tol Convergence tolerance for the stopping criteria.

standardise Type of standardisation to perform on X:

• "12" standardises the input data to have ℓ_2 norms of one.

• "11" standardises the input data to have ℓ_1 norms of one.

• "sd" standardises the input data to have standard deviation of one.

• "none" no standardisation applied.

intercept Logical flag for whether to fit an intercept.

error_criteria The criteria used to discriminate between models along the path. Supported

values are: "mse" (mean squared error) and "mae" (mean absolute error).

screen Logical flag for whether to apply screening rules (see Feser and Evangelou

(2024)). Screening discards irrelevant groups before fitting, greatly improving

speed.

verbose Logical flag for whether to print fitting information.

v_weights Optional vector for the variable penalty weights. Overrides the penalties from

pen_method if specified. When entering custom weights, these are multiplied

internally by λ and α . To void this behaviour, set $\lambda = 2$ and $\alpha = 0.5$

w_weights Optional vector for the group penalty weights. Overrides the penalties from

pen_method if specified. When entering custom weights, these are multiplied internally by λ and $1-\alpha$. To void this behaviour, set $\lambda=2$ and $\alpha=0.5$

Details

Fits SGS models under a pathwise solution using adaptive three operator splitting (ATOS), picking the 1se model as optimum. Warm starts are implemented.

Value

A list containing:

all_models A list of all the models fitted along the path.

fit The 1se chosen model, which is a "sgs" object type. best_lambda The value of λ which generated the chosen model.

best_lambda_id The path index for the chosen model.

errors A table containing fitting information about the models on the path.

type Indicates which type of regression was performed.

References

Feser, F., Evangelou, M. (2023). *Sparse-group SLOPE: adaptive bi-level selection with FDR-control*, https://arxiv.org/abs/2305.09467

Feser, F., Evangelou, M. (2024). *Strong screening rules for group-based SLOPE models*, https://arxiv.org/abs/2405.15357

gen_pens

See Also

```
fit_sgs()
Other model-selection: as_sgs(), fit_goscar_cv(), fit_gslope_cv(), fit_sgo_cv(), scaled_sgs()
Other SGS-methods: as_sgs(), coef.sgs(), fit_sgo(), fit_sgo_cv(), fit_sgs(), plot.sgs(),
predict.sgs(), print.sgs(), scaled_sgs()
```

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run SGS with cross-validation
cv_model = fit_sgs_cv(X = data$X, y = data$y, groups=groups, type = "linear",
path_length = 5, nfolds=5, alpha = 0.95, vFDR = 0.1, gFDR = 0.1, min_frac = 0.05,
standardise="12",intercept=TRUE,verbose=TRUE)
```

gen_pens

Generate penalty sequences for SGS.

Description

Generates variable and group penalties for SGS.

Usage

```
gen_pens(gFDR, vFDR, pen_method, groups, alpha)
```

Arguments

gFDR	Defines the desired group false discovery rate (FDR) level, which determines the shape of the group penalties.	
vFDR	Defines the desired variable false discovery rate (FDR) level, which determines the shape of the variable penalties.	
pen_method	The type of penalty sequences to use (see Feser and Evangelou (2023)):	
 "1" uses the vMean SGS and gMean gSLOPE sequences. "2" uses the vMax SGS and gMean gSLOPE sequences. "3" uses the BH SLOPE and gMean gSLOPE sequences, also known SGS Original. 		
	• "4" uses the gMax gSLOPE sequence. For a gSLOPE model only.	
groups	A grouping structure for the input data. Should take the form of a vector of group indices.	
alpha	The value of α , defines the convex balance between SLOPE and gSLOPE.	

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Details

The vMean and vMax SGS sequences are variable sequences derived specifically to give variable false discovery rate (FDR) control for SGS under orthogonal designs (see Feser and Evangelou (2023)). The BH SLOPE sequence is derived in Bodgan et al. (2015) and has links to the Benjamini-Hochberg critical values. The sequence provides variable FDR-control for SLOPE under orthogonal designs. The gMean gSLOPE sequence is derived in Brzyski et al. (2015) and provides group FDR-control for gSLOPE under orthogonal designs.

Value

```
A list containing:
```

```
pen_slope_org A vector of the variable penalty sequence.
pen_gslope_org A vector of the group penalty sequence.
```

References

```
Bogdan, M., Van den Berg, E., Sabatti, C., Su, W., Candes, E. (2015). SLOPE — Adaptive variable selection via convex optimization, https://projecteuclid.org/journals/annals-of-applied-statistics/volume-9/issue-3/SLOPEAdaptive-variable-selection-via-convex-optimization/10.1214/15-AOAS842.full
```

Brzyski, D., Gossmann, A., Su, W., Bodgan, M. (2019). *Group SLOPE – Adaptive Selection of Groups of Predictors*, https://www.tandfonline.com/doi/full/10.1080/01621459.2017.1411269

Feser, F., Evangelou, M. (2023). *Sparse-group SLOPE: adaptive bi-level selection with FDR-control*, https://arxiv.org/abs/2305.09467

Examples

gen_toy_data

Generate toy data.

Description

Generates different types of datasets, which can then be fitted using sparse-group SLOPE.

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Usage

```
gen_toy_data(
 p,
  n,
  rho = 0,
  seed_id = 2,
  grouped = TRUE,
  groups,
 noise_level = 1,
  group_sparsity = 0.1,
  var\_sparsity = 0.5,
 orthogonal = FALSE,
  data_mean = 0,
  data_sd = 1,
  signal_mean = 0,
  signal_sd = sqrt(10)
)
```

Arguments

p	The number of input variables.
n	The number of observations.
rho	Correlation coefficient. Must be in range $[0,1]$.
seed_id	Seed to be used to generate the data matrix X .
grouped	A logical flag indicating whether grouped data is required.
groups	If grouped=TRUE, the grouping structure is required. Each input variable should have a group id.
noise_level	Defines the level of noise (σ) to be used in generating the response vector y .
group_sparsity	Defines the level of group sparsity. Must be in the range $[0,1]$.
var_sparsity	Defines the level of variable sparsity. Must be in the range $[0,1]$. If grouped=TRUE, this defines the level of sparsity within each group, not globally.
orthogonal	Logical flag as to whether the input matrix should be orthogonal.
data_mean	Defines the mean of input predictors.
data_sd	Defines the standard deviation of the signal (β) .
signal_mean	Defines the mean of the signal (β) .
signal_sd	Defines the standard deviation of the signal (β) .

Details

The data is generated under a Gaussian linear model. The generated data can be grouped and sparsity can be provided at both a group and/or variable level.

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Value

A list containing:

y The response vector.
X The input matrix.

true_beta The true values of β used to generate the response. true_grp_id Indices of which groups are non-zero in true_beta.

Examples

Description

```
Plots the pathwise solution of a cross-validation fit, from a call to one of the following: fit_sgs(), fit_sgs_cv(), fit_gslope(), fit_gslope_cv(), fit_sgo(), fit_sgo_cv(), fit_goscar(), fit_goscar_cv().
```

Usage

```
## S3 method for class 'sgs'
plot(x, how_many = 10, ...)
```

Arguments

A Object of one of the following classes: "sgs", "sgs_cv", "gslope", "gslope_cv".
 how_many
 Defines how many predictors to plot. Plots the predictors in decreasing order of largest absolute value.
 ... further arguments passed to base function.

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Value

A list containing:

response The predicted response. In the logistic case, this represents the predicted class

probabilities.

class The predicted class assignments. Only returned if type = "logistic" in the model

object.

See Also

```
fit_sgs(), fit_sgs_cv(), fit_gslope(), fit_gslope_cv(), fit_sgo(), fit_sgo_cv(), fit_goscar(),
fit_goscar_cv()
Other SGS-methods: as_sgs(), coef.sgs(), fit_sgo(), fit_sgo_cv(), fit_sgs(), fit_sgs_cv(),
predict.sgs(), print.sgs(), scaled_sgs()
Other gSLOPE-methods: coef.sgs(), fit_goscar(), fit_goscar_cv(), fit_gslope(), fit_gslope_cv(),
predict.sgs(), print.sgs()
```

Examples

```
# specify a grouping structure
groups = c(1,1,2,2,3)
# generate data
data = gen_toy_data(p=5, n=4, groups = groups, seed_id=3,signal_mean=20,group_sparsity=1)
# run SGS
model = fit_sgs(X = data$X, y = data$y, groups=groups, type = "linear",
path_length = 20, alpha = 0.95, vFDR = 0.1, gFDR = 0.1,
min_frac = 0.05, standardise="l2",intercept=TRUE,verbose=FALSE)
plot(model, how_many = 10)
```

Description

```
Performs prediction from one of the following fits: fit_sgs(), fit_sgs_cv(), fit_gslope(), fit_gslope_cv(), fit_sgo(), fit_sgo_cv(), fit_goscar(), fit_goscar_cv(). The predictions are calculated for each "lambda" value in the path.
```

Usage

```
## S3 method for class 'sgs'
predict(object, x, ...)
```

print.sgs 39

Arguments

object	Object of one of the following classes: "sgs", "sgs_cv", "gslope", "gslope_cv".
Х	Input data to use for prediction.
	further arguments passed to stats function.

Value

A list containing:

response The predicted response. In the logistic case, this represents the predicted class

probabilities.

class The predicted class assignments. Only returned if type = "logistic" in the model

object.

See Also

```
fit_sgs(), fit_sgs_cv(), fit_gslope(), fit_gslope_cv(), fit_sgo(), fit_sgo_cv(), fit_goscar(),
fit_goscar_cv()
Other SGS-methods: as_sgs(), coef.sgs(), fit_sgo(), fit_sgo_cv(), fit_sgs(), fit_sgs_cv(),
plot.sgs(), print.sgs(), scaled_sgs()
Other gSLOPE-methods: coef.sgs(), fit_goscar(), fit_goscar_cv(), fit_gslope(), fit_gslope_cv(),
plot.sgs(), print.sgs()
```

Examples

```
# specify a grouping structure
groups = c(1,1,1,2,2,3,3,3,4,4)
# generate data
data = gen_toy_data(p=10, n=5, groups = groups, seed_id=3,group_sparsity=1)
# run SGS
model = fit_sgs(X = data$X, y = data$y, groups = groups, type="linear", lambda = 1, alpha=0.95,
vFDR=0.1, gFDR=0.1, standardise = "l2", intercept = TRUE, verbose=FALSE)
# use predict function
model_predictions = predict(model, x = data$X)
```

Description

Prints out useful metric from a model fit.

Usage

```
## S3 method for class 'sgs'
print(x, ...)
```

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Arguments

```
x Object of one of the following classes: "sgs", "sgs_cv", "gslope", "gslope_cv".
... further arguments passed to base function.
```

Value

A summary of the model fit(s).

See Also

```
fit_sgs(), fit_sgs_cv(), fit_gslope(), fit_gslope_cv(), fit_sgo(), fit_sgo_cv(), fit_goscar(),
fit_goscar_cv()
Other SGS-methods: as_sgs(), coef.sgs(), fit_sgo(), fit_sgo_cv(), fit_sgs(), fit_sgs_cv(),
plot.sgs(), predict.sgs(), scaled_sgs()
Other gSLOPE-methods: coef.sgs(), fit_goscar(), fit_goscar_cv(), fit_gslope(), fit_gslope_cv(),
plot.sgs(), predict.sgs()
```

Examples

scaled_sgs

Fits a scaled SGS model.

Description

Fits an SGS model using the noise estimation procedure (Algorithm 5 from Bogdan et al. (2015)). This estimates λ and then fits the model using the estimated value. It is an alternative approach to cross-validation (fit_sgs_cv()).

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Usage

```
scaled_sgs(
   X,
   y,
   groups,
   type = "linear",
   pen_method = 1,
   alpha = 0.95,
   vFDR = 0.1,
   gFDR = 0.1,
   standardise = "12",
   intercept = TRUE,
   verbose = FALSE
)
```

Arguments

X Input matrix of dimensions $n \times p$. Can be a sparse matrix (using class "sparseMatrix"

from the Matrix package).

y Output vector of dimension n. For type="linear" should be continuous and

for type="logistic" should be a binary variable.

groups A grouping structure for the input data. Should take the form of a vector of

group indices.

type The type of regression to perform. Supported values are: "linear" and "logistic".

pen_method The type of penalty sequences to use.

• "1" uses the vMean SGS and gMean gSLOPE sequences.

• "2" uses the vMax SGS and gMean gSLOPE sequences.

 $\bullet\,$ "1" uses the BH SLOPE and gMean gSLOPE sequences, also known as

SGS Original.

alpha The value of α , which defines the convex balance between SLOPE and gSLOPE.

Must be between 0 and 1.

vFDR Defines the desired variable false discovery rate (FDR) level, which determines

the shape of the variable penalties. Must be between 0 and 1.

gFDR Defines the desired group false discovery rate (FDR) level, which determines

the shape of the group penalties. Must be between 0 and 1.

standardise Type of standardisation to perform on X:

• "12" standardises the input data to have ℓ_2 norms of one.

• "11" standardises the input data to have ℓ_1 norms of one.

• "sd" standardises the input data to have standard deviation of one.

• "none" no standardisation applied.

intercept Logical flag for whether to fit an intercept.

verbose Logical flag for whether to print fitting information.

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Value

An object of type "sgs" containing model fit information (see fit_sgs()).

References

```
Bogdan, M., Van den Berg, E., Sabatti, C., Su, W., Candes, E. (2015). SLOPE — Adaptive variable selection via convex optimization, https://projecteuclid.org/journals/annals-of-applied-statistics/volume-9/issue-3/SLOPEAdaptive-variable-selection-via-convex-optimization/10.1214/15-AOAS842.full
```

See Also

```
as_sgs()
Other model-selection: as_sgs(), fit_goscar_cv(), fit_gslope_cv(), fit_sgo_cv(), fit_sgs_cv()
Other SGS-methods: as_sgs(), coef.sgs(), fit_sgo(), fit_sgo_cv(), fit_sgs(), fit_sgs_cv(),
plot.sgs(), predict.sgs(), print.sgs()
```

Examples

```
# specify a grouping structure
groups = c(1,1,2,2,3)
# generate data
data = gen_toy_data(p=5, n=4, groups = groups, seed_id=3,
signal_mean=20,group_sparsity=1,var_sparsity=1)
# run noise estimation
model = scaled_sgs(X=data$X, y=data$y, groups=groups, pen_method=1)
```

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