Package ‘SLSEdesign’

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Title Optimal Regression Design under the Second-Order Least Squares Estimator

Version 0.0.3

Description With given inputs that include number of points, discrete design space, a measure of skewness, models and parameter value, this package calculates the objective value, optimal designs and plot the equivalence theory under A- and D-optimal criteria under the second-order Least squares estimator. This package is based on the paper ``Properties of optimal regression designs under the second-order least squares estimator'' by Chi-Kuang Yeh and Julie Zhou (2021) <doi:10.1007/s00362-018-01076-6>.

URL https://github.com/chikuang/SLSEdesign

BugReports https://github.com/chikuang/SLSEdesign/issues

License GPL-3

Encoding UTF-8

RoxygenNote 7.3.1

Imports CVXR

Suggests knitr, rmarkdown

VignetteBuilder knitr

NeedsCompilation no

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`Aopt` *Calculate the A-optimal design under the second-order Least squares estimator*

Description

Calculate the A-optimal design under the second-order Least squares estimator

Usage

```
Aopt(N, u, tt, FUN, theta, num_iter = 1000)
```

Arguments

`N`	The number of sample points in the design space.
`u`	The discretized design space.
`tt`	The level of skewness between 0 to 1 (inclusive). When tt=0, it is equivalent to compute the A-optimal design under the ordinary least squares estimator.
`FUN`	The function to calculate the derivative of the given model.
`theta`	The parameter value of the model.
`num_iter`	Maximum number of iteration.

Details

This function calculates the A-optimal design and the loss function under the A-optimality. The loss function under A-optimality is defined as the trace of the inverse of the Fisher information matrix

Value

A list that contains 1. Value of the objective function at solution. 2. Status. 3. Optimal design

Examples

```
poly3 <- function(xi, theta){
  matrix(c(1, xi, xi^2, xi^3), ncol = 1)
}
Npt <- 101
my_design <- Aopt(N = Npt, u = seq(-1, +1, length.out = Npt),
   tt = 0, FUN = poly3, theta = rep(0,4), num_iter = 2000)
round(my_design$design, 3)
my_design$val
```

calc_phiA — *Calculate the loss function of the A-optimal design*

Description

Calculate the loss function of the A-optimal design

Usage

```
calc_phiA(design, theta, FUN, tt, A)
```

Arguments

design	The resulted design that contains the design points and the associated weights
theta	The parameter value of the model
FUN	The function to calculate the derivative of the given model.
tt	The level of skewness
A	The calculated covariance matrix

Details

This function calculates the loss function of the design problem under the A-optimality. The loss function under A-optimality is defined as the trace of the inverse of the Fisher information matrix

Value

The loss of the model at each design points

Examples

```
my_design <- data.frame(location = c(0, 180), weight = c(1/2, 1/2))
theta <- c(0.05, 0.5)
peleg <- function(xi, theta){
   deno <- (theta[1] + xi * theta[2])^2
   rbind(-xi/deno, -xi^2/deno)
}
A <- matrix(c(1, 0, 0, 0, 0.2116, 1.3116, 0, 1.3116, 15.462521), byrow = TRUE, ncol = 3)
res <- calc_phiA(my_design, theta, peleg, 0, A)
res
```

`calc_phiD` *Calculate the loss function of the D-optimal design*

Description

Calculate the loss function of the D-optimal design

Usage

```
calc_phiD(design, theta, FUN, tt, A)
```

Arguments

design	The resulted design that contains the design points and the associated weights
theta	The parameter value of the model
FUN	The function to calculate the derivative of the given model.
tt	The level of skewness
A	The calculated covariance matrix

Details

This function calculates the loss function of the design problem under the D-optimality. The loss function under D-optimality is defined as the log determinant of the inverse of the Fisher information matrix

Value

The loss of the model at each design points

Examples

```
my_design <- data.frame(location = c(0, 180), weight = c(1/2, 1/2))
theta <- c(0.05, 0.5)
peleg <- function(xi, theta){
   deno <- (theta[1] + xi * theta[2])^2
   rbind(-xi/deno, -xi^2/deno)
}
A <- matrix(c(1, 0, 0, 0, 0.2116, 1.3116, 0, 1.3116, 15.462521), byrow = TRUE, ncol = 3)
res <- calc_phiA(my_design, theta, peleg, 0, A)
res
```

`Dopt` *Calculate the D-optimal design under the SLSE*

Description

Calculate the D-optimal design under the SLSE

Usage

```
Dopt(N, u, tt, FUN, theta, num_iter = 1000)
```

Arguments

`N`	The number of sample points in the design space.
`u`	The discretized design space.
`tt`	The level of skewness. When tt=0, it is equivalent to compute the D-optimal design under the ordinary least squares estimator.
`FUN`	The function to calculate the derivative of the given model.
`theta`	The parameter value of the model.
`num_iter`	Maximum number of iteration.

Details

This function calculates the D-optimal design and the loss function under the D-optimality. The loss function under D-optimality is defined as the log determinant of the inverse of the Fisher information matrix.

Value

A list that contains 1. Value of the objective function at solution. 2. Status. 3. Optimal design

Examples

```
poly3 <- function(xi, theta){
  matrix(c(1, xi, xi^2, xi^3), ncol = 1)
}
Npt <- 101
my_design <- Dopt(N = Npt, u = seq(-1, +1, length.out = Npt),
   tt = 0, FUN = poly3, theta = rep(0,4), num_iter = 2000)
round(my_design$design, 3)
my_design$val
```

`plot_direction_Aopt` *Verify the optimality condition for the A-optimal design*

Description

Verify the optimality condition for the A-optimal design

Usage

```
plot_direction_Aopt(u, design, tt, FUN, theta)
```

Arguments

`u`	The discretized design points.
`design`	The A-optimal design that contains the design points and the associated weights
`tt`	The level of skewness.
`FUN`	The function to calculate the derivative of the given model.
`theta`	The parameter value of the model.

Details

This function produces the figure for the directional derivative of the given A-optimal design of the compact supports. According to the general equivalence theorem, for an optimal design, all the negative value of the directional derivative should be below zero line.

Value

The plot of the negative value of the directional derivative of an A-optimal design

Examples

```
poly3 <- function(xi, theta){
  matrix(c(1, xi, xi^2, xi^3), ncol = 1)
}
design = data.frame(location = c(-1, -0.464, 0.464, 1),
                    weight = c(0.151, 0.349, 0.349, 0.151))
u = seq(-1, 1, length.out = 201)
plot_direction_Aopt(u=u, design=design, tt=0, FUN = poly3, theta = rep(0,4))
```

plot_direction_Dopt — *Verify the optimality condition for the D-optimal design*

Description

Verify the optimality condition for the D-optimal design

Usage

```
plot_direction_Dopt(u, design, tt, FUN, theta)
```

Arguments

`u`	The discretized design points.
`design`	The D-optimal design that contains the design points and the associated weights.
`tt`	The level of skewness.
`FUN`	The function to calculate the derivative of the given model.
`theta`	The parameter value of the model.

Details

This function produces the figure for the negative value of the directional derivative of the given D-optimal design of the compact supports. According to the general equivalence theorem, for an optimal design, all the directional derivative should be below zero line.

Value

The plot of the negative value of the directional derivative of a D-optimal design

Examples

```
poly3 <- function(xi, theta){
    matrix(c(1, xi, xi^2, xi^3), ncol = 1)
}
design = data.frame(location = c(-1, -0.447, 0.447, 1),
 weight = rep(0.25, 4))
u = seq(-1, 1, length.out = 201)
plot_direction_Dopt(u, design, tt=0, FUN = poly3,
  theta = rep(0, 4))
```

plot_weight *Plot the weight distribution of the optimal design for univaraite regression model*

Description

Plot the weight distribution of the optimal design for univaraite regression model

Usage

```
plot_weight(design)
```

Arguments

design	The resulted design that contains the design points and the associated weights

Details

This functions produce a figure that contains the location and their associated weights of the resulted optimal design measures.

Value

The plot that shows the given optimal design

Examples

```
Des = list(location = c(-1, +1), weight = c(0.5, 0.5))
plot_weight(Des)
```

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