

# Package ‘NetworkRiskMeasures’

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**Type** Package

**Title** Risk Measures for (Financial) Networks

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**Description** Implements some risk measures for (financial) networks, such as DebtRank, Impact Susceptibility, Impact Diffusion and Impact Fluidity.

**License** GPL-3

**LazyData** TRUE

**Suggests** testthat, igraph, covr

**Depends** Matrix

**Imports** expm, ggplot2, dplyr

**URL** <https://github.com/carloscinelli/NetworkRiskMeasures>

**BugReports** <https://github.com/carloscinelli/NetworkRiskMeasures/issues>

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**NeedsCompilation** no

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communicability\_matrix

*Computes the communicability matrix*

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### Description

The communicability of an adjacency matrix  $M$  is defined as  $\exp(M)$  where  $M[i,j]$  can be interpreted as the weighted sums of paths from  $i$  to  $j$ . Recall that  $\exp(M)$  can be cast into a Taylor series expansion with an infinite number additive terms. The function permits the evaluation of  $\exp(M)$  using the `expm` package or using a simpler mathematical approximation. In the second case, the function truncates the infinite series by simply calculating the summation terms up to a pre-defined number of factors.

### Usage

```
communicability_matrix(x, terms = Inf, sparse = TRUE)
```

### Arguments

<code>x</code>	a square <code>matrix</code> or an <code>igraph</code> object.
<code>terms</code>	truncates the communicability matrix evaluation up to a pre-defined number of terms. If <code>terms = Inf</code> the function computes the matrix exponential using <code>expm</code> .
<code>sparse</code>	should the function use sparse matrices when computing the communicability? However, if <code>terms = Inf</code> the function will use <code>expm</code> which uses <code>dgeMatrix-class</code> .

### Value

The function returns the communicability matrix.

### References

Estrada, E. Hatano, N. (2008). Communicability in complex networks. *Physical Review E*, 77:036111.

### Examples

```
# Creating example data
## Assets Matrix (bilateral exposures)
assets_matrix <- matrix(c(0, 10, 3, 1, 0, 2, 0, 3, 0), ncol = 3)
rownames(assets_matrix) <- colnames(assets_matrix) <- letters[1:3]

## Capital Buffer
buffer <- c(a = 2, b = 5, c = 2)

# Computing vulnerability
v <- vulnerability_matrix(assets_matrix, buffer, binary = TRUE)

# Computing communicability of the vulnerability matrix
communicability_matrix(v)
```

**Description**

Given a matrix of exposures, a vector of buffers and weights (optional) the functions simulates contagion for all the shock vectors provided. You may choose from the implemented propagation contagion method or create you own propagation method. Details on how to create your own method will be provided in a future version.

**Usage**

```
contagion(exposures,
          buffer,
          shock = "all",
          weights = NULL,
          method = c("debtrank", "threshold"),
          ...,
          exposure_type = c("assets", "liabilities", "impact", "vulnerability"),
          keep.history = FALSE,
          abs.tol = .Machine$double.eps ^ 0.2,
          max.it = min(1000, nrow(v)*10),
          verbose = TRUE)
```

**Arguments**

exposures	an adjacency <a href="#">matrix</a> , (sparse) <a href="#">Matrix</a> or an <a href="#">igraph</a> object with the network of bilateral exposures between vertices. By default, the function expects the exposures in the form of an assets matrix in which A -> B means that A has an asset with B. However, you can change that with the parameter <code>exposure_type</code> . When using a matrix, preferably it should have rows and columns names.
buffer	a numeric vector with the capital buffer for each vertex. Values should be in the same row/column order as the network of bilateral exposures. The buffer is not needed if <code>exposure_type = "vulnerability"</code> .
shock	a list with the shock vectors. If "all" (the default) the function will run a simulation for the default of each vertex in the network.
weights	default is NULL. You can use a numeric vector of weights to give some economic significance to the measures, like, for instance, the total assets of the nodes.
method	the contagion propagation method. Currently, you should use either "debtrank" for the DebtRank propagation method or "threshold" for the traditional default cascades. The DebtRank version implemented is the one proposed in Bardoscia et al (2015). If you want to use the old "single-hit" DebtRank of Battiston et al (2012), simply provide the argument <code>single.hit = TRUE</code> .
...	other arguments to be passed to the contagion propagation method.

exposure_type	character vector indicating the type of the bilateral exposures. It can be an "assets" network (where A -> B means that A has an asset with B), a "liabilities" network (where A -> B means that A has a debt with B), a (binary) "impact" matrix (where A -> B indicates the relative impact of A in B's capital buffer), or a (binary) "vulnerability" matrix (where A -> B indicates the relative impact A suffers from B's default). The default is "assets".
keep.history	keep all the history of stress levels? This can use a lot of memory, so the default is FALSE.
abs.tol	the desired accuracy.
max.it	the maximum number of iterations.
verbose	gives verbose output. Default is TRUE.

### Value

The function returns an object of class "contagion" with the results of the simulation.

### References

Bardoscia M, Battiston S, Caccioli F, Caldarelli G (2015) DebtRank: A Microscopic Foundation for Shock Propagation. PLoS ONE 10(6): e0130406. doi: 10.1371/journal.pone.0130406

Battiston, S., Puliga, M., Kaushik, R., Tasca, P., and Caldarelli, G. (2012). DebtRank: Too central to fail? Financial networks, the FED and systemic risk. Scientific reports, 2:541.

### Examples

```
# Loads simulated banking data
data("sim_data")
head(sim_data)

# seed for reproducibility
set.seed(15)

# minimum density estimation
# verbose = F to prevent printing
md_mat <- matrix_estimation(sim_data$assets, sim_data$liabilities, method = "md", verbose = FALSE)

# rownames and colnames for the matrix
rownames(md_mat) <- colnames(md_mat) <- sim_data$bank

# DebtRank simulation
contdr <- contagion(exposures = md_mat, buffer = sim_data$buffer, weights = sim_data$weights,
                    shock = "all", method = "debrank", verbose = FALSE)
summary(contdr)

plot(contdr)

# Traditional default cascades simulation
contthr <- contagion(exposures = md_mat, buffer = sim_data$buffer, weights = sim_data$weights,
                    shock = "all", method = "threshold", verbose = FALSE)
summary(contthr)
```

```

# simulating shock scenarios 1% to 25% shock in all vertices
s <- seq(0.01, 0.25, by = 0.01)
shocks <- lapply(s, function(x) rep(x, nrow(md_mat)))
names(shocks) <- paste(s*100, "pct shock")

cont <- contagion(exposures = md_mat, buffer = sim_data$buffer, shock = shocks,
                 weights = sim_data$weights, method = "debtrank", verbose = FALSE)
summary(cont)
plot(cont)

```

---

criticality

*Criticality of the vertices*


---

### Description

The criticality of a vertex measures its impact on its neighbors if it defaults. It is basically the [rowSums](#) of the [impact\\_matrix](#).

### Usage

```

criticality(
  exposures,
  buffer,
  binary = FALSE,
  exposure_type = c("assets", "liabilities", "impact", "vulnerability")
)

```

### Arguments

exposures	an adjacency <a href="#">matrix</a> , (sparse) <a href="#">Matrix</a> or an <a href="#">igraph</a> object with the network of bilateral exposures between vertices. By default, the function expects the exposures in the form of an assets matrix in which A -> B means that A has an asset with B. However, you can change that with the parameter <code>exposure_type</code> . When using a matrix, preferably it should have rows and columns names.
buffer	a numeric vector with the capital buffer for each vertex. Values should be in the same row/column order as the network of bilateral exposures. The buffer is not needed if <code>exposure_type = "vulnerability"</code> .
binary	if <code>binary = TRUE</code> the function computes a 'binary' impact or vulnerability matrix. It truncates all values less than 1 to 0 and all values greater than 1 to 1.
exposure_type	character vector indicating the type of the bilateral exposures. It can be an "assets" network (where A -> B means that A has an asset with B), a "liabilities" network (where A -> B means that A has a debt with B), a (binary) "impact" matrix (where A -> B indicates the relative impact of A in B's capital buffer), or a (binary) "vulnerability" matrix (where A -> B indicates the relative impact A suffers from B's default). The default is "assets".

**Value**

The function returns a (named) vector with the criticality for each vertex.

**Examples**

```
# Creating example data
## Assets Matrix (bilateral exposures)
assets_matrix <- matrix(c(0, 10, 3, 1, 0, 2, 0, 3, 0), ncol = 3)
rownames(assets_matrix) <- colnames(assets_matrix) <- letters[1:3]

## Capital Buffer
buffer <- c(a = 2, b = 5, c = 2)

# Criticality
criticality(assets_matrix, buffer)
```

---

impact\_susceptibility *Impact Susceptibility, Fluidity and Diffusion*

---

**Description**

The `impact_susceptibility` measures the feasible contagion paths that can reach a vertex in relation to its direct contagion paths. When the impact susceptibility is greater than 1, it means that the vertex is vulnerable to other vertices beyond its direct neighbors (remotely vulnerable).

The `impact_fluidity` is simply the average of the impact susceptibility in the network.

The `impact_diffusion` tries to capture the influence exercised by a node on the propagation of impacts in the network. The impact diffusion of a vertex is measured by the change it causes on the impact susceptibility of other vertices when its power to propagate contagion is removed from the network.

All these measures are based on the communicability of the vulnerability matrix (see [vulnerability\\_matrix](#) and [communicability\\_matrix](#)).

**Usage**

```
impact_susceptibility(
  exposures,
  buffer,
  weights = NULL,
  terms = Inf,
  sparse = TRUE,
  binary = TRUE,
  exposure_type = c("assets", "liabilities", "impact", "vulnerability")
)

impact_fluidity(
```

```

    exposures,
    buffer,
    weights = NULL,
    terms = Inf,
    sparse = TRUE,
    binary = TRUE,
    exposure_type = c("assets", "liabilities", "impact", "vulnerability")
)

impact_diffusion(
  exposures,
  buffer,
  weights = NULL,
  terms = Inf,
  sparse = TRUE,
  binary = TRUE,
  exposure_type = c("assets", "liabilities", "impact", "vulnerability")
)

```

### Arguments

exposures	an adjacency <a href="#">matrix</a> , (sparse) <a href="#">Matrix</a> or an <a href="#">igraph</a> object with the network of bilateral exposures between vertices. By default, the function expects the exposures in the form of an assets matrix in which A -> B means that A has an asset with B. However, you can change that with the parameter <code>exposure_type</code> . When using a matrix, preferably it should have rows and columns names.
buffer	a numeric vector with the capital buffer for each vertex. Values should be in the same row/column order as the network of bilateral exposures. The buffer is not needed if <code>exposure_type = "vulnerability"</code> .
weights	default is NULL. You can use a numeric vector of weights to give some economic significance to the measures, like, for instance, the total assets of the nodes.
terms	truncates the communicability matrix evaluation up to a pre-defined number of terms. If <code>terms = Inf</code> the function computes the matrix exponential using <a href="#">expm</a> .
sparse	should the function use sparse matrices when computing the communicability? However, if <code>terms = Inf</code> the function will use <a href="#">expm</a> which uses <a href="#">dgeMatrix-class</a> .
binary	if <code>binary = TRUE</code> the function computes a 'binary' impact or vulnerability matrix. It truncates all values less than 1 to 0 and all values greater than 1 to 1.
exposure_type	character vector indicating the type of the bilateral exposures. It can be an "assets" network (where A -> B means that A has an asset with B), a "liabilities" network (where A -> B means that A has a debt with B), a (binary) "impact" matrix (where A -> B indicates the relative impact of A in B's capital buffer), or a (binary) "vulnerability" matrix (where A -> B indicates the relative impact A suffers from B's default). The default is "assets".

### Value

The `impact_susceptibility` function returns a vector with the (weighted) impact susceptibility

The `impact_fluidity` function returns a vector with the (weighted) impact fluidity of the network. The `impact_diffusion` function returns a `data.frame` with the vertex name and the (weighted) start, intermediate and total impact diffusion.

## References

Silva, T.C.; Souza, S.R.S.; Tabak, B.M. (2015) Monitoring vulnerability and impact diffusion in financial networks. Working Paper 392, Central Bank of Brazil.

Silva, T.C.; Souza, S.R.S.; Tabak, B.M. (2015) Network structure analysis of the Brazilian interbank market. Working Paper 391, Central Bank of Brazil.

## Examples

```
# Creating example data
## Assets Matrix (bilateral exposures)
assets_matrix <- matrix(c(0, 10, 3, 1, 0, 2, 0, 3, 0), ncol = 3)
rownames(assets_matrix) <- colnames(assets_matrix) <- letters[1:3]

## Capital Buffer
buffer <- c(a = 2, b = 5, c = 2)

# Measures
impact_susceptibility(assets_matrix, buffer)
impact_fluidity(assets_matrix, buffer)
impact_diffusion(assets_matrix, buffer)
```

---

matrix_estimation	<i>Matrix Estimation</i>
-------------------	--------------------------

---

## Description

Methods for estimating matrix entries from the marginals (row and column sums).

There are currently two methods implemented: Maximum Entropy (Upper 2004) and Minimum Density (Anand et al. 2015).

You may use the `matrix_estimation()` function, setting the desired method. Or you may use directly the `max_ent()` function for maximum entropy estimation or the `min_dens()` function for minimum density estimation.

## Usage

```
matrix_estimation(
  rowsums,
  colsums,
  method = c("me", "md"),
  ...,
  max.it = 1e+05,
```



```

    abs.tol = 0.001,
    verbose = TRUE
)

max_ent(rowsums, colsums, max.it = 1e+05, abs.tol = 0.001, verbose = TRUE)

min_dens(
  rowsums,
  colsums,
  c = 1,
  lambda = 1,
  k = 100,
  alpha = 1/sum(rowsums),
  delta = 1/sum(colsums),
  theta = 1,
  remove.prob = 0.01,
  max.it = 1e+05,
  abs.tol = 0.001,
  verbose = TRUE
)

```

### Arguments

rowsums	a numeric vector with the row sums.
colsums	a numeric vector with the column sums.
method	the matrix estimation method. Choose "me" for maximum entropy or "md" for minimum density.
...	further arguments passed to or from other methods.
max.it	the maximum number of iterations.
abs.tol	the desired accuracy.
verbose	gives verbose output. Default is TRUE.
c	the 'cost' an extra link for the minimum density estimation. See Anand et al. (2015).
lambda	you should use lamda together with k. For the first k rounds of the algorithm, the function will allocate a fraction lambda of the total, thus obtaining a "low density" solution, instead of a "minimum density" solution. See Anand et al. (2015).
k	you should use lamda together with k. For the first k rounds of the algorithm, the function will allocate a fraction lambda of the total, thus obtaining a "low density" solution, instead of a "minimum density" solution. See Anand et al. (2015).
alpha	weights for the row sums deviations. See Anand et al. (2015).
delta	weights for the column sums deviations. See Anand et al. (2015).
theta	scaling parameter. Emphasizes the weight placed on finding solutions with similar characteristics to the prior matrix. See Anand et al. (2015).

remove.prob      probability to randomly remove a link during the algorithm. See Anand et al. (2015).

### Value

The functions return the estimated matrix.

### References

Upper, C. and A. Worm (2004). Estimating bilateral exposures in the German interbank market: Is there a danger of contagion? *European Economic Review* 48, 827-849.

Anand, K., Craig, B. and G. von Peter (2015). Filling in the blanks: network structure and interbank contagion. *Quantitative Finance* 15:4, 625-636.

### Examples

```
# Example from Anand, Craig and Von Peter (2015, p.628)

# Liabilities
L <- c(a = 4, b = 5, c = 5, d = 0, e = 0, f = 2, g = 4)

# Assets
A <- c(a = 7, b = 5, c = 3, d = 1, e = 3, f = 0, g = 1)

# Maximum Entropy
ME <- matrix_estimation(A, L, method = "me")
ME <- round(ME, 2)

# Minimum Density
set.seed(192)
MD <- matrix_estimation(A, L, method = "md")
```

---

risk\_matrix

*Computes the (binary) impact or vulnerability matrices*

---

### Description

The function computes an impact or vulnerability matrix given a network of bilateral exposures and a vector of capital buffers.

### Usage

```
risk_matrix(
  exposures,
  buffer,
  binary = FALSE,
  exposure_type = c("assets", "liabilities", "impact", "vulnerability"),
  returns = c("impact", "vulnerability")
```

```

)

vulnerability_matrix(
  exposures,
  buffer,
  binary = FALSE,
  exposure_type = c("assets", "liabilities", "impact", "vulnerability")
)

impact_matrix(
  exposures,
  buffer,
  binary = FALSE,
  exposure_type = c("assets", "liabilities", "impact", "vulnerability")
)

```

### Arguments

exposures	an adjacency <code>matrix</code> , (sparse) <code>Matrix</code> or an <code>igraph</code> object with the network of bilateral exposures between vertices. By default, the function expects the exposures in the form of an assets matrix in which A -> B means that A has an asset with B. However, you can change that with the parameter <code>exposure_type</code> . When using a matrix, preferably it should have rows and columns names.
buffer	a numeric vector with the capital buffer for each vertex. Values should be in the same row/column order as the network of bilateral exposures. The buffer is not needed if <code>exposure_type = "vulnerability"</code> .
binary	if <code>binary = TRUE</code> the function computes a 'binary' impact or vulnerability matrix. It truncates all values less than 1 to 0 and all values greater than 1 to 1.
exposure_type	character vector indicating the type of the bilateral exposures. It can be an "assets" network (where A -> B means that A has an asset with B), a "liabilities" network (where A -> B means that A has a debt with B), a (binary) "impact" matrix (where A -> B indicates the relative impact of A in B's capital buffer), or a (binary) "vulnerability" matrix (where A -> B indicates the relative impact A suffers from B's default). The default is "assets".
returns	will the function return the impact or the vulnerability matrix? The default is "impact".

### Details

The impact matrix represents how much a vertex impacts the capital buffer of another vertex when it defaults.

The vulnerability matrix is just the transpose of the impact matrix. It represents how much a vertex is impacted by the default of another vertex.

### Value

The function returns a (binary) impact or vulnerability matrix.

The term  $V[i,j]$  of the impact matrix represents the impact of  $i$ 's default in  $j$ 's capital buffer.

The term  $V[i,j]$  of the vulnerability matrix represents how much  $i$ 's capital buffer is impacted by  $j$ 's default.

If `binary = TRUE` the values less than 1 are truncated to zero.

### Examples

```
# Creating example data
## Assets Matrix (bilateral exposures)
assets_matrix <- matrix(c(0, 10, 3, 1, 0, 2, 0, 3, 0), ncol = 3)
rownames(assets_matrix) <- colnames(assets_matrix) <- letters[1:3]

## Capital Buffer
buffer <- c(a = 2, b = 5, c = 2)

# Vulnerability matrices
vulnerability_matrix(assets_matrix, buffer, binary = FALSE)
vulnerability_matrix(assets_matrix, buffer, binary = TRUE)
```

---

sim\_data

*Simulated Interbank Data*

---

### Description

A simulated dataset with interbank assets, liabilities, capital buffer and weights for 125 "banks". The code to generate the data is on the examples.

### Format

A data frame with 125 rows and 5 variables

### Examples

```
# Simulated data for illustration purposes

# Setting Seed
set.seed(1100)

# Heavy tailed assets
assets <- rlnorm(125, 0, 2)
assets[assets < 4] <- runif(length(assets[assets < 4]))

# Heavy tailed liabilities
liabilities <- rlnorm(125, 0, 2)
liabilities[liabilities < 4] <- runif(length(liabilities[liabilities < 4]))

# Making sure assets = liabilities
assets <- sum(liabilities) * (assets/sum(assets))
```

```
# Buffer as a function of assets
buffer <- pmax(0.01, runif(length(liabilities))*liabilities + abs(rnorm(125, 4, 2.6)))

# Weights as a function of assets, buffer and liabilities
weights <- (assets + liabilities + buffer + 1) + rlnorm(125, 0, 1)

# creating data.frame
sim_data <- data.frame(bank = paste0("b", 1:125),
                      assets = assets,
                      liabilities = liabilities,
                      buffer = buffer,
                      weights = weights)
```

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