

# Package ‘MixedIndTests’

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**Type** Package

**Title** Tests of Randomness and Tests of Independence

**Version** 1.2.0

**Description** Functions for testing randomness for a univariate time series with arbitrary distribution (discrete, continuous, mixture of both types) and for testing independence between random variables with arbitrary distributions. The test statistics are based on the multilinear empirical copula and multipliers are used to compute P-values. The test of independence between random variables appeared in Genest, Nešlehová, Rémillard & Murphy (2019) and the test of randomness appeared in Nasri (2022).

**License** GPL-3

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 AutoDep

*Dependogram for Kendall's tau and Spearman's rho*


---

## Description

This function, used in EstDepSerial, draws the P-values of Kendall's tau and Spearman's rho for a given number of lags.

## Usage

```
AutoDep(out)
```

## Arguments

out                    List of the output of EstDepSerial (P-values, subsets)

## References

B.R Nasri (2021). Tests of serial dependence for arbitrary distributions

## Examples

```
out <-EstDepSerial(SimAR1Poisson(c(5,0.4),100),10)
AutoDep(out)
```

---

 Dependogram

*Dependogram for Cramer-von Mises statistics*


---

### Description

This function, used in `EstDep`, `TestIndCopula` and `TestIndSerCopula`, draws the P-values of the Moebius Cramer-von Mises statistics from the multilinear copula and their combination for a tests of randomness for k consecives values  $X(1), \dots, X(k)$  or for a test of independence between random variables.

### Usage

```
Dependogram(out, stat = "CVM")
```

### Arguments

<code>out</code>	List of the output from <code>EstDep</code> , <code>EstDepSerial</code> , <code>TestIndCopula</code> or <code>TestIndSerCopula</code> (P-values, subsets)
<code>stat</code>	Name of statistics to be used (default is "CVM")

### References

Genest, Neslehova, Remillard & Murphy (2019). Testing for independence in arbitrary distributions

### Examples

```
x <- matrix(rnorm(250), ncol=5)
out <- TestIndCopula(x)
Dependogram(out)
```

---

 DependogramZ

*Dependogram for Moebius correlations*


---

### Description

This function, used in `EstDepMoebius` and `EstDepSerialMoebius` plot the graphs of the correlation statistics of Spearman, van der Waerden and Savage as a function of the subsets for tests of randomness or test of independence between random variables. Under the null hypothesis, the statistics should be independent  $N(0,1)$ .

### Usage

```
DependogramZ(out, n)
```

**Arguments**

out	List of the output from EstDep, EstDepSerial, TestIndCopula or TestIndSerCopula (P-values, subsets)
n	Number of observations

**References**

Nasri & Remillard (2023). Tests of independence and randomness for arbitrary data using copula-based covariances

**Examples**

```
x <- matrix(rnorm(250), ncol=5)
out <- EstDepMoebius(x)
DependogramZ(out, 50)
```

---

EstDep	<i>Kendall's tau and Spearman's rho statistics for testing independence between random variables</i>
--------	--

---

**Description**

This function computes the matrix of pairs of Kendall's tau and Spearman's rho statistics between random variables with arbitrary distributions.

**Usage**

```
EstDep(x, graph = FALSE)
```

**Arguments**

x	Data matrix
graph	Set to TRUE for a dependogram for all pairs of Kendall's taus and Spearman's rhos.

**Value**

stat	List of Kendall's tau and Spearman's rho statistics from multilinear copula, and test combinations LB
pvalue	P-values for the tests statistics

**References**

Genest, Neslehova, Remillard & Murphy (2018). Testing for independence in arbitrary distributions

**Examples**

```
x <- matrix(rnorm(500),ncol=10)
out <-EstDep(x)
```

---

EstDepMoebius	<i>Dependence measures and statistics for test of independence between random variables</i>
---------------	---

---

**Description**

This function computes copula-based dependence measures for Moebius versions of Spearman's rho, van der Waerden's coefficient, and Savage's coefficient, as well as their combination for tests of independence between random variables.

**Usage**

```
EstDepMoebius(x, trunc.level = 2, graph = FALSE)
```

**Arguments**

x	Data matrix
trunc.level	Only subsets of cardinality $\leq$ trunc.level (default=2) are considered for the Moebius statistics.
graph	Set to TRUE if one wants the dependogram of P-values for the Moebius statistics

**Value**

stat	List of statistics (spearman, vdw, savage) and test combinations Ln and Ln2 (only pairs)
pvalue	P-values for the tests
cardA	Cardinaly of the subsets for the Moebius statistics
subsets	Subsets for the Moebius statistics

**References**

B.R Nasri & B.N. Remillard (2023). Tests of independence and randomness for arbitrary data using copula-based covariances

**Examples**

```
x <- matrix(rnorm(250),ncol=5)
out <-EstDepMoebius(x,3)
```

---

EstDepSerial	<i>Kendall's tau and Spearman's rho statistics for testing randomness in a univariate time series</i>
--------------	---

---

### Description

This function computes Kendall's tau and Spearman's rho statistics for tests of randomness in a time series with arbitrary distribution for pairs  $(X[i], X[i+k])$ ,  $k=1:l$ ags

### Usage

```
EstDepSerial(x, lag, graph = FALSE)
```

### Arguments

x	Time series
lag	Number of lags
graph	Set to TRUE for a dependogram for Kendall's tau and Spearman's rho

### Value

stat	List of Kendall's tau and Spearman's rho statistics from multilinear copula, and test combinations LB
pvalue	P-values for the tests statistics

### References

B.R Nasri (2022). Tests of serial dependence for arbitrary distributions

### Examples

```
out <-EstDepSerial(SimAR1Poisson(c(5,0.4),100),10)
```

---

EstDepSerialMoebius	<i>Dependence measures and statistics for test of randomness for a univariate time series</i>
---------------------	---

---

### Description

This function computes copula-based dependence measures for Moebius versions of Spearman's rho, van der Waerden's coefficient, and Savage's coefficient, as well as their combination for tests of randomness for  $p$  consecutive values  $Y(1), \dots, Y(p)$ .

### Usage

```
EstDepSerialMoebius(y, p, trunc.level = 2, graph = FALSE)
```

**Arguments**

y	Time series
p	Number of consecutive observations
trunc.level	Only subsets of cardinality $\leq$ trunc.level (default=2) are considered for the Moebius statistics.
graph	Set to TRUE if one wants the dependogram of P-values for the Moebius statistics

**Value**

stat	List of statistics (spearman, vdw, savage) and test combinations Ln and Ln2 (only pairs)
pvalue	P-values for the tests
card	Cardinaly of the subsets for the Moebius statistics
subsets	Subsets for the Moebius statistics

**References**

B.R Nasri & B.N. Remillard (2023). Tests of independence and randomness for arbitrary data using copula-based covariances

**Examples**

```
y<- SimAR1Poisson(c(5,0.2),100)
out <- EstDepSerialMoebius(y,4,4)
```

---

Finv *Quantile function of margins*

---

**Description**

This function computes the quantile of seven cdf used in the simulatuons of Nasri (2022).

**Usage**

```
Finv(u, k)
```

**Arguments**

u	Vector of probabilities
k	Marginal distribution: [1] Bernoulli(0.8), [2] Poisson(6), [3] Negative binomial with $r = 1.5$ , $p = 0.2$ , [4] Zero-inflated Poisson (10) with $w = 0.1$ and $P(6.67)$ otherwise, [5] Zero-inflated Gaussian, [6] Discretized Gaussian, [7] Discrete Pareto(1)

**Value**

x                      Vector of quantiles

**Author(s)**

Bouchra R. Nasri January 2021

**References**

B.R Nasri (2022). Tests of serial dependence for arbitrary distributions

**Examples**

```
x = Finv(runif(100),2)
```

---

horseshoecrabs                      *Horseshoecrabs dataset*

---

**Description**

Horseshoe Crab Data from Table 3.2 of Agresti(2007). This data set consists of five variables, three of which are categorical, measured on 173 female crabs, each having a male attached in her nest.

**Usage**

```
data(horseshoecrabs)
```

**Format**

Data frame with 173 rows and 5 variables:

- X1: Color of the female (1: light medium, 2: medium, 3: dark medium, 4: dark)
- X2: Spine condition (1: both good, 2: one worn or broken, 3: both worn or broken)
- X3: Carapace width (cm)
- X4: Number of satellites, i.e., other males around the female
- X5: Weight (kg)

**References**

Agresti, A. (2007). An Introduction to Categorical data analysis, John Wiley & Sons, Wiley Series in Probability and Statistics, 2nd edition.

**Examples**

```
data(horseshoecrabs)
x =data.matrix(horseshoecrabs)
out = TestIndCopula(x, trunc.level=5, graph=TRUE)
```

---

lamb	<i>Fetal lamb dataset</i>
------	---------------------------

---

**Description**

240 body movement measurements of a fetal lamb at consecutive 5 second intervals.

**Usage**

```
data(lamb)
```

**Format**

Count data.

**References**

Leroux B, Putterman M (1992). Maximum Penalized Likelihood estimation for independent and Markov-dependent Mixture models. *Biometrics*, 48, 545–558.

**Examples**

```
data(lamb)
plot(lamb)
```

---

select_p	<i>Data-driven selection of p for the test of randomness</i>
----------	--

---

**Description**

This function uses a AIC/BIC type criterion to select p based on the data.

**Usage**

```
select_p(X, p0 = 2, d = 5, q = 2.4, lambda = 0.25)
```

**Arguments**

X	Time series
p0	Minimum value of p (default is 2)
d	Maximum value of p (default is 5)
q	Constant for selecting between AIC and BIC type penalty (default is 2.4)
lambda	Penalty term (default is 0.25); small values lead to p=d, large value lead to p=p0

**Value**

p Selected value of p

**References**

B.R Nasri (2021). Tests of serial dependence for arbitrary distributions

**Examples**

```
X <- SimAR1Poisson(c(5,0.2),100)
out <- select_p(X)
```

---

SimAR1Poisson

*Simulation of a AR(1) Poisson process*

---

**Description**

Conditionally on the past,  $X[t]$  is Poisson with  $\lambda[t] = a + bX[t-1]$

**Usage**

```
SimAR1Poisson(param, n)
```

**Arguments**

param Param[1] =  $a > 0$ , param[2] =  $b$ ,  $0 \leq b < 1$  (for stationarity)  
n Length of the series.

**Value**

X Simulated series

**Examples**

```
data <- SimAR1Poisson(c(5,0.4),500)
```

---

SimCopulaSeries	<i>Simulation of a copula-based time series</i>
-----------------	---

---

### Description

This function simulates a Markovian time series (p-Markov for the Farlie-Gumbel-Morgenstern copula) with uniform margins using a copula family for the joint distribution of  $U[t]$ ,  $U[t-1]$ .

### Usage

```
SimCopulaSeries(family, n, tau = 0, param = NULL)
```

### Arguments

family	"ind", "tent", "gaussian", "t", "clayton", "fgm", "frank", "gumbel", "joe", "plackett"
n	length of the time series
tau	Kendall's tau of the copula family
param	extra copula parameter: for "fgm", param is the dimension of the copula; for "t", param = nu

### Value

U	Simulated time series
---	-----------------------

### Author(s)

Bouchra R. Nasri January 2021

### References

B.R Nasri (2022). Tests of serial dependence for arbitrary distributions

### Examples

```
U = SimCopulaSeries("fgm",100,0.2, 3) # for the FGM, |tau|<= 2/9
```

---

TestIndCopula	<i>Statistics and P-values for a test of independence between random variables</i>
---------------	--

---

### Description

This function computes Cramer-von Mises statistics and their combination for a tests of independence between random variables with arbitrary distributions. The P-values are computed using Gaussian multipliers.

### Usage

```
TestIndCopula(
  x,
  trunc.level = 2,
  B = 1000,
  par = FALSE,
  ncores = 2,
  graph = FALSE
)
```

### Arguments

x	Data matrix
trunc.level	Only subsets of cardinality $\leq$ trunc.level (default=2) are considered for the Moebius statistics.
B	Number of multipliers samples (default = 1000)
par	Set to TRUE if one prefers parallel computing (slower)
ncores	Number of cores for parallel computing (default is 2)
graph	Set to TRUE if one wants the dependogram of P-values for the Moebius statistics

### Value

stat	List of Cramer-von Mises statistics cvm, $S_n$ from the multilinear copula, and test combinations $T_n$ and $T_{n2}$ (only pairs)
pvalue	Approximated P-values for the tests using Gaussian multipliers
card	Cardinaly of the subsets for the Moebius statistics
subsets	Subsets for the Moebius statistics

### References

Genest, Neslehova, Remillard & Murphy (2019). Testing for independence in arbitrary distributions

### Examples

```
x <- matrix(rnorm(250),ncol=5)
out <-TestIndCopula(x)
```

---

TestIndSerCopula	<i>Statistics and P-values for a test of randomness for a univariate time series</i>
------------------	--

---

### Description

This function computes Cramer-von Mises statistics from the multilinear copula and their combination for tests of randomness of  $p$  consecutive values  $X(1), \dots, X(p)$ . The p-values are computed using Gaussian multipliers.

### Usage

```
TestIndSerCopula(
  x,
  p,
  trunc.level = 2,
  B = 1000,
  par = FALSE,
  ncores = 2,
  graph = FALSE
)
```

### Arguments

<code>x</code>	Time series
<code>p</code>	Number of consecutive observations
<code>trunc.level</code>	Only subsets of cardinality $\leq$ <code>trunc.level</code> (default=2) are considered for the Moebius statistics.
<code>B</code>	Number of multipliers samples (default = 1000)
<code>par</code>	Set to TRUE if one prefers parallel computing (slower)
<code>ncores</code>	Number of cores for parallel computing (default = 2)
<code>graph</code>	Set to TRUE if one wants the dependogram of P-values for the Moebius statistics

### Value

<code>stat</code>	List of Cramer-von Mises statistics $cvm$ , $S_n$ , and test combinations $T_n$ and $T_{n2}$ (only pairs)
<code>pvalue</code>	Approximated P-values for the tests using Gaussian multipliers
<code>card</code>	Cardinaly of the subsets for the Moebius statistics
<code>subsets</code>	Subsets for the Moebius statistics

### References

B.R Nasri (2022). Tests of serial dependence for arbitrary distributions

**Examples**

```
X <- SimAR1Poisson(c(5,0.2),100)
out <- TestIndSerCopula(X,5,3)
```

---

TestIndSerCopulaMulti *Statistics and P-values for a test of randomness for a multivariate time series*

---

**Description**

This function computes Cramer-von Mises statistics from the multilinear copula and their combination for a tests of randomness for  $p$  consecutive values of random vectors  $X(1), \dots, X(p)$ . The  $p$ -values are computed using Gaussian multipliers.

**Usage**

```
TestIndSerCopulaMulti(x, p, trunc.level = 2, B = 1000, graph = FALSE)
```

**Arguments**

x	Time series matrix
p	Number of consecutive vectors
trunc.level	Only subsets of cardinality $\leq$ trunc.level (default=2) are considered for the Moebius statistics.
B	Number of multipliers samples (default = 1000)
graph	Set to TRUE if one wants the dependogram of P-values for the Moebius statistics

**Value**

stat	List of Cramer-von Mises statistics $cvm$ , tilde $S_n$ , and test combinations tilde $T_n$ and tilde $T_{n2}$ (only pairs), as defined in Nasri(2022).
pvalue	Approximated P-values for the tests using Gaussian multipliers

**References**

B.R Nasri (2022). Tests of serial dependence for arbitrary distributions

**Examples**

```
data(Y)
out <- TestIndSerCopulaMulti(Y,5,5)
```

---

X	<i>AR(1) Poisson with parameters</i>
---	--------------------------------------

---

**Description**

Simulated AR(1) Poisson sequence of length  $n=100$  with parameters  $c(5,0.4)$ .

**Usage**

```
data(X)
```

**Format**

Count data.

**Examples**

```
data(X)  
acf(X)
```

---

Xbin	<i>Bernoulli sequence</i>
------	---------------------------

---

**Description**

Simulated Bernoulli sequence.

**Usage**

```
data(Xbin)
```

**Format**

Count data.

**Examples**

```
data(Xbin)  
plot(Xbin)
```

---

Y

*VAR(1) Poisson with parameters*

---

**Description**

Simulated VAR(1) Poisson sequence of length n=100.

**Usage**

data(Y)

**Format**

Count data.

**Examples**

data(Y)  
acf(Y)

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