Employing asremlPlus, in conjunction with asreml, to calculate and use information criteria

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This vignette illustrates the facilities in asremlPlus (Brien, 2025), in conjunction with asreml (Butler et al., 2023), for calculating and using information. Here, asremlPlus and asreml are packages for the R Statistical Computing environment (R Core Team, 2025).

It is divided into the following main sections:

- 1. Set up the maximal model for this experiment
- 2. Obtaining information criteria for separate models
- 3. Obtaining information criteria for a prescribed sequence of model changes
- 4. Using information criteria to decide model changes

1. Set up the maximal model for this experiment

```
library(knitr)
opts_chunk$set("tidy" = FALSE, comment = NA)
suppressMessages(library(asreml, quietly=TRUE))

## Offline License checked out Sun Feb 16 14:21:52 2025

packageVersion("asreml")

## [1] '4.2.0.370'
suppressMessages(library(asremlPlus))
packageVersion("asremlPlus")

## [1] '4.4.46'
options(width = 100)
```

Get data available in asremlPlus

The data are from a 1976 spring wheat experiment and are taken from Gilmour et al. (1995). An analysis is presented in the asrem1 manual by Butler et al. (2023, Section 7.6), although they suggest that it is a barley experiment.

```
data(Wheat.dat)
```

Fit the maximal model

-691.7711

In the following a model is fitted that has the terms that would be included for a balanced lattice. In addition, a term WithinColPairs has been included to allow for extraneous variation arising between pairs of adjacent lanes. Also, separable ar1 residual autocorrelation has been included. This model represents the maximal anticipated model,

```
ASReml Version 4.2 16/02/2025 14:21:52
         LogLik
                                  DF
                        Sigma2
                                          wall
       -724.1213
 1
                      23034.14
                                  124
                                        14:21:52
 2
      -717.4149
                     9206.931
                                 124
                                       14:21:52 ( 2 restrained)
 3
      -694.8752
                      26492.99
                                 124
                                       14:21:52 ( 2 restrained)
 4
      -694.1600
                      33101.80
                                 124
                                       14:21:52 ( 1 restrained)
 5
      -692.0020
                      36912.26
                                 124
                                       14:21:52 ( 1 restrained)
 6
      -691.7892
                      46701.51
                                 124
                                       14:21:52 ( 2 restrained)
 7
      -691.8336
                      46208.51
                                 124
                                       14:21:52 ( 1 restrained)
 8
       -691.7749
                      47698.26
                                  124
                                        14:21:52
```

124

Warning in asreml(yield \sim WithinColPairs + Variety, random = \sim Rep/(Row + : Some components changed by more than 1% on the last iteration

The warning from asreml is probably due to a bound term.

47041.85

Initialize a testing sequence by loading the current fit into an asrtests object

14:21:52

```
max.asrt <- as.asrtests(max.asr, NULL, NULL)
```

Check for and remove any boundary terms

```
max.asrt <- rmboundary(max.asrt)
summary(max.asrt$asreml.obj)$varcomp</pre>
```

```
std.error
                                                   z.ratio bound %ch
                         component
Rep:Row
                      4.293282e+03 3.199458e+03 1.3418779
                                                               P 0.0
Rep:Column
                      1.575689e+02 1.480357e+03 0.1064398
                                                               P 0.7
units
                                                               P 0.0
                      5.742689e+03 1.652457e+03 3.4752438
Row:Column!R
                      4.706787e+04 2.515832e+04 1.8708669
                                                               P 0.0
Row:Column!Row!cor
                     7.920301e-01 1.014691e-01 7.8056280
                                                               U 0.0
Row:Column!Column!cor 8.799559e-01 7.370402e-02 11.9390486
                                                               U 0.0
```

```
print(max.asrt, which = "testsummary")
```

Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

```
terms DF denDF p AIC BIC action
1 Rep 1 NA NA NA NA Boundary
```

Rep has been removed because it has been constrained to zero. Following the recommendation of Littel et al. (2006, p. 150), the bound on all variance components is set to unconstrained (U) using setvariances.asreml so as to avoid bias in the estimate of the residual variance. Alternatively, one could move Rep to the fixed model.

Unbind Rep, Row and Column components and reload into an asrtests object

```
ASReml Version 4.2 16/02/2025 14:21:54
```

	${ t LogLik}$	Sigma2	DF	wall		
1	-724.1213	23034.14	124	14:21:54		
2	-717.4149	9206.931	124	14:21:54	(2 restrained)
3	-694.8752	26492.99	124	14:21:54	(2 restrained)
4	-693.9744	33129.65	124	14:21:54	(1 restrained)
5	-692.8856	39662.12	124	14:21:54		
6	-691.4276	53103.83	124	14:21:54		
7	-691.2387	48092.17	124	14:21:54		
8	-691.1808	47278.94	124	14:21:54		
9	-691.1710	46850.98	124	14:21:54		
10	-691.1700	46690.46	124	14:21:54		

Warning in asreml(fixed = yield \sim WithinColPairs + Variety, random = \sim Rep/(Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:21:54] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield \sim WithinColPairs + Variety, random = \sim Rep/(Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:21:54] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield \sim WithinColPairs + Variety, random = \sim Rep/(Row + : Some components changed by more than 1% on the last iteration

```
max.asrt <- as.asrtests(max.asr, NULL, NULL)
max.asrt <- rmboundary(max.asrt)
summary(max.asrt$asreml.obj)$varcomp</pre>
```

std.error

-2462.3785858 1.191435e+03 -2.066734

z.ratio bound %ch

U 0.2

```
Rep:Row
                       5012.4021415 3.396848e+03 1.475604
                                                               U 0.1
                        920.5936391 1.704008e+03 0.540252
                                                               U 1.1
Rep:Column
units
                       5964.9099377 1.608792e+03 3.707695
                                                               P 0.1
Row:Column!R
                      46690.4620387 2.731906e+04 1.709080
                                                               P 0.0
Row:Column!Row!cor
                          0.8152180 9.988929e-02 8.161216
                                                               U 0.1
Row:Column!Column!cor
                          0.8857252 7.487875e-02 11.828793
                                                               U 0.0
print(max.asrt, which = "testsummary")
```

```
#### Sequence of model investigations for yield
```

component

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

```
[1] terms DF denDF p AIC BIC action <0 rows> (or 0-length row.names)
```

Now the Rep component estimate is negative.

The test.summary output shows that no changes have been made to the model loaded using as.asrtests. The pseudo-anova table shows that Varieties are highly significant (p < 0.001)

2. Obtaining information criteria for separate models

The method infoCriteria has two methods for calculating information criteria. One, infoCriteria.asreml, is a method for asreml objects and the other, infoCriteria.list, if for 'listobjects, the components of thelistbeingasreml' objects.

Single models

Rep

Firstly, infoCriteria is called with the default IClikelihood, which is REML. Then it is called with IClikelihood set to full (Verbyla, 2019).

```
infoCriteria(max.asr)
  fixedDF varDF NBound
                           AIC
                                    BIC loglik
                     0 1396.34 1416.082 -691.17
infoCriteria(max.asr, IClikelihood = "full")
ASReml Version 4.2 16/02/2025 14:21:56
          LogLik
                        Sigma2
                                   DF
                                          wall
       -691.1700
                      46641.98
                                         14:21:56
 1
                                  124
```

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Log-likelihood not converged

```
fixedDF varDF NBound AIC BIC loglik
1 26 7 0 1647.194 1746.545 -790.5968
```

A list of models

Now, a second model, from which the withinColPairs term has been omitted, is fitted; to be consistent, the variance components are unconstrained using setvariances.asreml. Then the asreml objects for this model and the maximal model are combined into a list and a data.frame produced that includes their information criteria.

```
ASReml Version 4.2 16/02/2025 14:21:56
```

```
LogLik
                        Sigma2
                                    DF
                                            wall
1
      -727.7742
                      22898.99
                                   125
                                          14:21:56
2
      -721.0966
                      9190.303
                                   125
                                         14:21:56
                                                       2 restrained)
3
      -698.3135
                      26671.76
                                   125
                                          14:21:56
                                                       2 restrained)
4
                      32677.28
      -697.5170
                                   125
                                                       1 restrained)
                                          14:21:56
5
      -695.4192
                                   125
                                          14:21:56
                                                       1 restrained)
                      36662.27
6
      -695.2077
                      46263.96
                                   125
                                          14:21:56
                                                       2 restrained)
7
      -695.1975
                      46156.63
                                   125
                                          14:21:56
8
                                   125
      -695.1906
                      46630.21
                                         14:21:56
```

Warning in asreml(yield \sim Variety, random = \sim Rep/(Row + Column) + units, : Some components changed by more than 1% on the last iteration

```
ASReml Version 4.2 16/02/2025 14:21:56
```

```
LogLik
                          Sigma2
                                     DF
                                             wall
       -727.7742
                       22898.99
                                    125
                                           14:21:56
 1
 2
       -721.0966
                       9190.303
                                    125
                                           14:21:56
                                                      (
                                                         2 restrained)
 3
                       26671.76
                                    125
                                                         2 restrained)
       -698.3135
                                           14:21:56
 4
       -697.3331
                       32689.33
                                    125
                                           14:21:56
                                                      ( 1 restrained)
 5
       -697.0164
                       39975.97
                                    125
                                           14:21:56
 6
       -695.0695
                       54825.30
                                    125
                                           14:21:56
 7
       -694.7571
                       47637.20
                                    125
                                           14:21:56
 8
       -694.6436
                       46775.41
                                    125
                                           14:21:56
 9
       -694.6181
                       46175.06
                                    125
                                           14:21:56
10
       -694.6152
                                    125
                       45940.69
                                           14:21:56
```

Warning in asreml(fixed = yield \sim Variety, random = \sim Rep/(Row + Column) + : Some components changed by more than 1% on the last iteration

```
WARN [2025-02-16 14:21:56] Some components changed by more than 1% on the last iteration
```

Warning in asreml(fixed = yield ~ Variety, random = ~Rep/(Row + Column) + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:21:56] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep/(Row + Column) + : Some components changed by more than 1% on the last iteration

```
mods <- list(max = max.asr, m1 = m1.asr)
ic <- infoCriteria(mods, IClikelihood = "full")
print(ic)</pre>
```

```
fixedDF varDF NBound AIC BIC loglik
max 26 7 0 1647.194 1746.545 -790.5968
m1 25 7 0 1645.326 1741.666 -790.6629
```

3. Obtaining information criteria for a prescribed sequence of model changes

The use of changeTerms.asrtests is demonstrated for a sequence of models, starting with the maximal model.

Drop the term for within Column pairs (a post hoc factor)

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Log-likelihood not converged

WARN [2025-02-16 14:21:58] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield \sim WithinColPairs + Variety, random = \sim Rep/(Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:21:58] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:21:58] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:21:58] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

```
print(current.asrt, which = "testsummary", omit.columns = "p")
```

Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

```
terms DF denDF AIC BIC action
Maximal model 26 7 1647.194 1746.545 Starting model
Drop withinColPairs 25 7 1645.326 1741.666 Changed fixed
```

So the same values of the information criteria have been obtained as when infoCriteria.list was used on a list containing the asreml objects for the two models. The differences is that here there is ultimately only one fitted model, the model stored in the asreml object in the asrtests object named current.asrt: this is the model with withinColPairs omitted.

Note this use of the omit.columns argument from print.test.summary to omit the irrelevant column p from the test.summary.

Drop nugget term

WARN [2025-02-16 14:22:00] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:22:00] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield \sim Variety, random = \sim Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:22:01] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:22:01] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components changed by more than 1% on the last iteration

Check Row autocorrelation

```
current.asrt <- changeTerms(current.asrt, newResidual = "Row:ar1(Column)",</pre>
                            label="Row autocorrelation", IClikelihood = "full")
WARN [2025-02-16 14:22:02] Some components changed by more than 1% on the last iteration
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components
changed by more than 1% on the last iteration
WARN [2025-02-16 14:22:02] Some components changed by more than 1% on the last iteration
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components
changed by more than 1% on the last iteration
WARN [2025-02-16 14:22:03] Log-likelihood not converged
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Log-likelihood
not converged
WARN [2025-02-16 14:22:03] Some components changed by more than 1% on the last iteration
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components
changed by more than 1% on the last iteration
WARN [2025-02-16 14:22:03] Log-likelihood not converged
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Log-likelihood
not converged
WARN [2025-02-16 14:22:03] Some components changed by more than 1% on the last iteration
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components
changed by more than 1% on the last iteration
Warning in newfit.asreml(asreml.obj, fixed. = fix.form, random. = ran.form, :
print(current.asrt, which = "testsummary", omit.columns = "p")
```

Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

```
terms DF denDF AIC BIC action

1 Maximal model 26 7 1647.194 1746.545 Starting model

2 Drop withinColPairs 25 7 1645.326 1741.666 Changed fixed

3 Drop units 25 6 1650.115 1743.445 Changed random

4 Row autocorrelation 25 5 1660.788 1751.107 Changed residual - old uncoverged
```

4. Using information criteria to decide model changes

This sections illustrates the use of changeModelOnIC.asrtests to decide between consecutive models in a sequence of models. The default information criterion to use for this is the AIC. However, which.IC can be used to specify the use of the BIC or both. Here we use the AIC and the full likelihood.

Check the term for within Column pairs (a post hoc factor)

As before, we start with the maximal model, in which the variance components have been unconstrained and look to decide whether of not to drop the withinColPairs term.

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ~Rep/(Row + : Log-likelihood not converged

```
current.asrt <- iterate(current.asrt)</pre>
```

WARN [2025-02-16 14:22:05] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield \sim WithinColPairs + Variety, random = \sim Rep/(Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:22:05] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield \sim WithinColPairs + Variety, random = \sim Rep/(Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:22:06] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ WithinColPairs + Variety, random = ${\rm \sim Rep/(Row \ + \ : \ Some \ components}$ changed by more than 1% on the last iteration

WARN [2025-02-16 14:22:07] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield \sim WithinColPairs + Variety, random = \sim Rep/(Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:22:07] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:22:07] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

```
print(current.asrt, which = "testsummary", omit.columns = "p")
```

Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

```
terms DF denDF AIC BIC action
1 Maximal model 26 7 1647.193601 1746.544565 Starting model
2 withinColPairs -1 0 -1.874126 -4.884762 Swapped
```

Given the warning about a lack of convergence, we use iterate.asrtests to perform additional iterations of the fitting process. It seems that it was successful.

It can be seen from the test.summary that the term has been swapped out and this has the effect of reducing the number of fixed parameters by one and makes no change to the variance parameters.

Check the nugget term

WARN [2025-02-16 14:22:09] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield \sim Variety, random = \sim Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:22:09] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:22:10] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:22:10] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + Rep:Row + Rep:Column, : Some components changed by more than 1% on the last iteration

Check Row autocorrelation

```
current.asrt <- changeModelOnIC(current.asrt, newResidual = "Row:ar1(Column)",</pre>
                                label="Row autocorrelation", IClikelihood = "full",
                                allow.unconverged = FALSE)
WARN [2025-02-16 14:22:12] Some components changed by more than 1% on the last iteration
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components
changed by more than 1% on the last iteration
WARN [2025-02-16 14:22:12] Some components changed by more than 1% on the last iteration
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components
changed by more than 1% on the last iteration
WARN [2025-02-16 14:22:13] Log-likelihood not converged
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Log-likelihood not
converged
WARN [2025-02-16 14:22:13] Some components changed by more than 1% on the last iteration
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components
changed by more than 1% on the last iteration
WARN [2025-02-16 14:22:13] Log-likelihood not converged
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Log-likelihood not
converged
WARN [2025-02-16 14:22:13] Some components changed by more than 1% on the last iteration
Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components
changed by more than 1% on the last iteration
Warning in newfit.asreml(asreml.obj, fixed. = fix.form, random. = ran.form, :
```

Check Column autocorrelation (depends on whether Row autocorrelation retained)

WARN [2025-02-16 14:22:15] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

WARN [2025-02-16 14:22:15] Some components changed by more than 1% on the last iteration

Warning in asreml(fixed = yield ~ Variety, random = ~Rep + units + Rep:Row + : Some components changed by more than 1% on the last iteration

Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik, bound.exclusions = bound.exclusions):
Row:Column!Row!cor

Warning in infoCriteria.asreml(new.asrtests.obj\$asreml.obj, IClikelihood = ic.lik, : The following bound Row:Column!Row!cor

Output the results

```
print(current.asrt, which = "test", omit.columns = "p")
```

Sequence of model investigations for yield

(If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)

```
terms DF denDF
                                        ATC
                                                    BTC
                                                                              action
1
        Maximal model 26
                             7 1647.193601 1746.544565
                                                                      Starting model
2
       withinColPairs -1
                                 -1.874126
                                             -4.884762
                                                                             Swapped
                             0
                units 0
                            -1
                                   4.789424
                                               1.778789
                                                                           Unswapped
4 Row autocorrelation 0
                                   0.000000
                                               0.000000 Unchanged - new unconverged
                             0
5 Col autocorrelation 0
                            -2
                                  19.478447
                                              13.457177
                                                                           Unswapped
```

```
summary(current.asrt$asreml.obj)$varcomp
```

```
z.ratio bound %ch
                                       std.error
                          component
Rep
                      -2392.1616314 1.199592e+03 -1.9941460
                                                                 U 0.4
Rep:Row
                       5033.2850607 3.408523e+03 1.4766764
                                                                 U 0.2
                        760.1498938 1.617038e+03 0.4700879
Rep:Column
                                                                 U 2.5
units
                       5929.0518909 1.609478e+03 3.6838361
                                                                 P 0.0
Row:Column!R
                      45940.6913910 2.634982e+04 1.7434920
                                                                 P 0.0
Row: Column! Row! cor
                          0.8101561 9.995026e-02 8.1055925
                                                                 U 0.1
Row:Column!Column!cor
                          0.8846454 7.504265e-02 11.7885681
                                                                 U 0.0
```

The test.summary shows us that the model without the autocorrelation failed to converge and so no change was made to the model. It, and the messages from checking the Column autocorrelation, also show us that the omission of the Column autocorrelation resulted in the Row autocorrelation becoming bound. That is, dropping the Column autocorrelation resulted in the dropping of two variance parameters

The function printFormulae.asreml is used to display the fitted model.

Formulae from asreml object

fixed: yield ~ Variety

random: ~ Rep + units + Rep:Row + Rep:Column

residual: ~ ar1(Row):ar1(Column)

References

Brien, C. J. (2025) asremlPlus: Augments ASReml-R in fitting mixed models and packages generally in exploring prediction differences. Version 4.4.46. https://cran.r-project.org/package=asremlPlus/ or http://chris.brien.name/rpackages/.

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