Package ‘PosRatioDist’

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Type Package

Title Quotient of Random Variables Conditioned to the Positive Quadrant

Version 1.2.1

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Description Computes the exact probability density function of X/Y conditioned on positive quadrant for series of bivariate distributions,for more details see Nadarajah,Song and Si (2019) <DOI:10.1080/03610926.2019.1576893>.

Depends R (>= 3.5.0)

License GPL (>= 2)

Encoding UTF-8

Imports mvtnorm, stats

NeedsCompilation no

RoxygenNote 7.1.1

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R topics documented:

`dBibs_expPR` *Bibs_expPR*

Description

probability density function of quotient of Balakrishna and Shiji's bivariate exponential random variables conditioned to the positive quadrant.For more detailed information please read the first reference paper.

Usage

```
dBibs_expPR(x, a, r)
```

Arguments

`x`	vector of positive quantiles.
`a`	parameter for Balakrishna and Shiji's bivariate exponential distribution
`r`	parameter for Balakrishna and Shiji's bivariate exponential distribution

Details

Probability density function

$$f_R(r \mid X > 0, Y > 0) = \frac{a}{2\sqrt{r}}\left(r + \frac{a^2}{4r}\right)^{-3/2}$$

For $r > 0$,$a > 0$

Value

`dBibs_expPR` gives the probability density function for quotient of Balakrishna and Shiji's bivariate exponential random variables conditioned to the positive quadrant.

Invalid arguments will return an error message.

References

Yuancheng Si and Saralees Nadarajah and Xiaodong Song, (2020). On the distribution of quotient of random variables conditioned to the positive quadrant. *Communications in Statistics - Theory and Methods*, **49**, pp2514-2528.

Balakrishnan, N. and Lai, C. -D. (2009).*Continuous Bivariate Distributions*.Springer Verlag, New York.

Balakrishna, N. and Shiji, K. (2014).On a class of bivariate exponential distributions.*Statistics and Probability Letters*, **85**, pp153-160.

Examples

```
x <- seq(0.1,5,0.1)
y <- dBibs_expPR(x, 2, 2)
plot(x,y,type = 'l')
```

`dBicauchyPR` *BicauchyPR*

Description

probability density function of quotient of Bivariate cauchy random variables conditioned to the positive quadrant.For more detailed information please read the first reference paper.

Usage

```
dBicauchyPR(x, a, b)
```

Arguments

`x`	single real positive scalar
`a`	parameter for bivaraite cauchy distribution
`b`	parameter for bivaraite cauchy distribution

Details

Probability density function

$$f_R(r \mid X > 0, Y > 0) = \frac{1}{2\pi \Pr(X > 0, Y > 0)} J_1\left(r^2 + 1, Ar + B, C, \frac{3}{2}\right)$$

For $-\infty < x < \infty$,$-\infty < y < \infty, r > 0, -\infty < a < \infty, -\infty < b < \infty$,where $A = -2a, B = -2b, C = 1 + a^2 + b^2$ and J_1 is given by first reference paper section (2.5).

Value

`dBicauchyPR` gives the probability density function for quotient of Bivariate cauchy random variables conditioned to the positive quadrant.

Invalid arguments will return an error message.

Author(s)

Saralees Nadarajah & Yuancheng Si <siyuanchengman@gmail.com>

References

Yuancheng Si and Saralees Nadarajah and Xiaodong Song, (2020). On the distribution of quotient of random variables conditioned to the positive quadrant. *Communications in Statistics - Theory and Methods*, **49**, pp2514-2528.

Balakrishnan, N. and Lai, C. -D. (2009).*Continuous Bivariate Distributions*.Springer Verlag, New York.

Caginalp, C. and Caginalp, G. (2018).The quotient of normal random variables and application to asset price fat tails.*Physica A—Statistical Mechanics and Its Applications*, **499**, pp457-471.

Louzada, F., Ara, A. and Fernandes, G. (2017).The bivariate alpha-skew-normal distribution.*Communications in Statistics - Theory and Methods*, **46**, pp7147-7156.

Nadarajah, S. (2009).A bivariate Pareto model for drought.*Stochastic Environmental Research and Risk Assessment*, **23**, pp811-822.

Nadarajah, S. and Kotz, S. (2006).Reliability models based on bivariate exponential distributions.*Probabilistic Engineering Mechanics*, **21**, pp338-351.

Nadarajah, S. and Kotz, S. (2007).Financial Pareto ratios.*Quantitative Finance*, **7**, pp257-260.

Examples

```
x <- seq(0.1,5,0.1)
y <- c()
for (i in x){y=c(y,dBicauchyPR(i,1,2))}
plot(x,y,type = 'l')
```

`dBiexpweightedPR` *BiexpweightedPR*

Description

probability density function of quotient of Bivariate exponential random variables resulting from weighted linear combinations conditioned to the positive quadrant.For more detailed information please read the first reference paper.

Usage

```
dBiexpweightedPR(x, a, b, c)
```

Arguments

x	vector of positive quantiles.
a	parameter for Bivariate exponential random variables resulting from weighted linear combinations
b	parameter for Bivariate exponential random variables resulting from weighted linear combinations
c	parameter for Bivariate exponential random variables resulting from weighted linear combinations

Details

Probability density function

$$f_R(r \mid X > 0, Y > 0) = \frac{(1-2c)\exp\left[(1-2c)a+b\right]}{\Pr(X > 0, Y > 0)\left[1+(1-2c)r\right]^2}$$

For $x > a > -\infty$,$y > b > -\infty, r > 0, 0 < c < 1$,These correlated exponential random variables can be used to model the stress and strength components of a system, hence the quotient distribution can be used to estimate the probability of failure of the system

Value

`dBiexpweightedPR` gives the probability density function for quotient of Bivariate exponential random variables resulting from weighted linear combinations conditioned to the positive quadrant.

Invalid arguments will return an error message.

Author(s)

Saralees Nadarajah & Yuancheng Si <siyuanchengman@gmail.com>

References

Yuancheng Si and Saralees Nadarajah and Xiaodong Song, (2020). On the distribution of quotient of random variables conditioned to the positive quadrant. *Communications in Statistics - Theory and Methods*, **49**, pp2514-2528.

Balakrishnan, N. and Lai, C. -D. (2009).*Continuous Bivariate Distributions*.Springer Verlag, New York.

Caginalp, C. and Caginalp, G. (2018).The quotient of normal random variables and application to asset price fat tails.*Physica A—Statistical Mechanics and Its Applications*, **499**, pp457-471.

Louzada, F., Ara, A. and Fernandes, G. (2017).The bivariate alpha-skew-normal distribution.*Communications in Statistics - Theory and Methods*, **46**, pp7147-7156.

Nadarajah, S. (2009).A bivariate Pareto model for drought.*Stochastic Environmental Research and Risk Assessment*, **23**, pp811-822.

Nadarajah, S. and Kotz, S. (2006).Reliability models based on bivariate exponential distributions.*Probabilistic Engineering Mechanics*, **21**, pp338-351.

Nadarajah, S. and Kotz, S. (2007).Financial Pareto ratios.*Quantitative Finance*, **7**, pp257-260.

Examples

```
x <- seq(0.1,5,0.1)
y <- dBiexpweightedPR(x, 4, 2, 0.2)
plot(x,y,type = 'l')
```

`dBilomaxPR` *BilomaxPR*

Description

probability density function of quotient of Bivariate Lomax random variables conditioned to the positive quadrant.For more detailed information please read the first reference paper.

Usage

```
dBilomaxPR(x, a, b, c, alpha, beta, theta)
```

Arguments

`x`	single positive scalar for quotient
`a`	parameter for Bivariate lomax distribution
`b`	parameter for Bivariate lomax distribution
`c`	parameter for Bivariate lomax distribution
`alpha`	parameter for Bivariate lomax distribution
`beta`	parameter for Bivariate lomax distribution
`theta`	parameter for Bivariate lomax distribution

Details

Probability density function

$$f_R(r \mid X>0, Y>0) = \frac{c^2\theta^2 r}{\Pr(X>0,Y>0)} J_3\left(\theta r, \beta-\theta a+(\alpha-\theta b)\,r, 1-\alpha a-\beta b+\theta ab, c+2\right) + \frac{c^2\theta\left[(\alpha-\theta b)r+\beta\right.}{\Pr(X>0,Y>}$$

For $r>0$,$\alpha>0$, $\beta>0$, $\theta>0$, $0\le\theta\le(c+1)\alpha\beta$ where J_1, J_2, J_3 are given by first reference paper section (2.5)

Value

`dBilomaxPR` gives the probability density function for bivariate lomax random variables conditioned to the positive quadrant.

Invalid arguments will return an error message.

Author(s)

Saralees Nadarajah & Yuancheng Si <siyuanchengman@gmail.com>

References

Yuancheng Si and Saralees Nadarajah and Xiaodong Song, (2020). On the distribution of quotient of random variables conditioned to the positive quadrant. *Communications in Statistics - Theory and Methods*, **49**, pp2514-2528.

Balakrishnan, N. and Lai, C. -D. (2009).*Continuous Bivariate Distributions*.Springer Verlag, New York.

`dBiMG_expPR` *BiMG_expPR*

Description

probability density function of quotient of Morgenstern type bivariate exponential random variables conditioned to the positive quadrant.For more detailed information please read the first reference paper.

Usage

```
dBiMG_expPR(x, a, b, alpha)
```

Arguments

`x`	vector of positive quantiles.
`a`	parameter for Morgenstern type bivariate exponential distribution
`b`	parameter for Morgenstern type bivariate exponential distribution
`alpha`	parameter for Morgenstern type bivariate exponential distribution

Details

Probability density function

$$f_R(r \mid X > 0, Y > 0) = \frac{(1+\alpha)\exp(a+b)}{\Pr(X>0,Y>0)(1+r)^2} - \frac{2\alpha\exp(a+2b)}{\Pr(X>0,Y>0)(2+r)^2} - \frac{2\alpha\exp(2a+b)}{\Pr(X>0,Y>0)(1+2r)^2} + \frac{}{\Pr(X}$$

For $r > 0$,$-1 \le \alpha \le 1, a > -\infty, b > -\infty$ These correlated exponential random variables can also be used to model the stress and strength components of a system, hence the quotient distribution can be used to estimate the probability of failure of the system

Value

`dBiMG_expPR` gives the probability density function for quotient of Morgenstern type bivariate exponential random variables conditioned to the positive quadrant

Invalid arguments will return an error message.

Author(s)

Saralees Nadarajah & Yuancheng Si <siyuanchengman@gmail.com>

References

Yuancheng Si and Saralees Nadarajah and Xiaodong Song, (2020). On the distribution of quotient of random variables conditioned to the positive quadrant. *Communications in Statistics - Theory and Methods*, **49**, pp2514-2528.

Balakrishnan, N. and Lai, C. -D. (2009).*Continuous Bivariate Distributions*.Springer Verlag, New York.

Balakrishna, N. and Shiji, K. (2014).On a class of bivariate exponential distributions.*Statistics and Probability Letters*, **85**, pp153-160.

Examples

```
x <- seq(0.1,5,0.1)
y <- dBiMG_expPR(x, 3, 2, 0.5)
plot(x,y,type = 'l')
```

`dBinormalPR` *BinormalPR*

Description

probability density function of quotient of Bivariate normal random variables conditioned to the positive quadrant.For more detailed information please read the first reference paper.

Usage

```
dBinormalPR(x, a, b, rho)
```

Arguments

x	vector of positive quantiles.
a	parameter
b	parameter
rho	correlation coefficient,$-1 < \rho < 1$

Details

Probability density function

$$f_R(r \mid X > 0, Y > 0) = \frac{1}{2\pi\sqrt{1-\rho^2}\,\Pr(X > 0, Y > 0)} \exp\left[-\frac{a^2+b^2-2\rho ab}{2\left(1-\rho^2\right)}\right] I_1\left(\frac{1+Cr+r^2}{2\left(1-\rho^2\right)}, \frac{Ar+B}{2\left(1-\rho^2\right)}\right)$$

For $-\infty < x < \infty$,$-\infty < y < \infty, r > 0, -\infty < a < \infty, -\infty < b < \infty, -1 < \rho < 1$,where $A = -2a + 2\rho b, B = -2b + 2\rho a, C = -2\rho$

Value

dBinormalPR gives the probability density function for quotient of Bivariate normal random variables conditioned to the positive quadrant.

Invalid arguments will return an error message.

Author(s)

Saralees Nadarajah & Yuancheng Si <siyuanchengman@gmail.com>

References

Yuancheng Si and Saralees Nadarajah and Xiaodong Song, (2020). On the distribution of quotient of random variables conditioned to the positive quadrant. *Communications in Statistics - Theory and Methods*, **49**, pp2514-2528.

Balakrishna, N. and Shiji, K. (2014). On a class of bivariate exponential distributions.*Statistics and Probability Letters*, **85**, pp153-160.

Arnold, B. C. and Strauss, D. (1988).Pseudolikelihood estimation.*Sankhya B* , **53**, pp233-243.

Caginalp, C. and Caginalp, G. (2018).The quotient of normal random variables and application to asset price fat tails.*Physica A—Statistical Mechanics and Its Applications*, **499**, pp457-471.

Louzada, F., Ara, A. and Fernandes, G. (2017).The bivariate alpha-skew-normal distribution.*Communications in Statistics - Theory and Methods*, **46**, pp7147-7156.

Nadarajah, S. (2009).A bivariate Pareto model for drought.*Stochastic Environmental Research and Risk Assessment*, **23**, pp811-822.

Nadarajah, S. and Kotz, S. (2006).Reliability models based on bivariate exponential distributions.*Probabilistic Engineering Mechanics*, **21**, pp338-351.

Nadarajah, S. and Kotz, S. (2007).Financial Pareto ratios.*Quantitative Finance*, **7**, pp257-260.

Examples

```
x <- seq(0.1,5,0.1)
y <- dBinormalPR(x, 2, 1, 0.5)
plot(x,y,type = 'l')
```

---

dBiparetoPR *BiparetoPR*

Description

probability density function of quotient of Bivariate Pareto random variables conditioned to the positive quadrant.For more detailed information please read the first reference paper.

Usage

```
dBiparetoPR(x)
```

Arguments

`x`	vector of positive quantiles.

Details

Probability density function

$$f_R(r \mid X > 0, Y > 0) = (r+1)^{-2}$$

For $r > 0$,Nadarajah (2009) used this distribution to model the proportion of droughts defined as a quotient of drought durations and non-drought durations.

Value

`dBiparetoPR` gives the probability density function for quotient of Bivariate Pareto random variables conditioned to the positive quadrant.

Invalid arguments will return an error message.

Author(s)

Saralees Nadarajah & Yuancheng Si `<siyuanchengman@gmail.com>`

References

Yuancheng Si and Saralees Nadarajah and Xiaodong Song, (2020). On the distribution of quotient of random variables conditioned to the positive quadrant. *Communications in Statistics - Theory and Methods*, **49**, pp2514-2528.

Mardia, K. V. (1962).Multivariate Pareto distributions.*Annals of Mathematical Statistics*, **33**, 1008-1015.

Nadarajah, S. (2009) A bivariate Pareto model for drought.*Stochastic Environmental Research and Risk Assessment*, **23**, pp811-822.

Examples

```
x <- seq(0.1,5,0.1)
y <- dBiparetoPR(x)
plot(x,y,type = 'l')
```

dBitPR *BitPR*

Description

probability density function of quotient of Bivariate t random variables conditioned to the positive quadrant.For more detailed information please read the first reference paper.

Usage

```
dBitPR(x, a, b, rho, v)
```

Arguments

x	single positive scalar,for quotient of Bivariate t random variables conditioned to the positive quadrant
a	parameter for Bivariate t distribution
b	parameter for Bivariate t distribution
rho	correlation coefficient, $-1 < \rho < 1$
v	parameter, degree of freedom of Bivariate t distribution

Details

Probability density function

$$f_R(r \mid X>0, Y>0) = \frac{\Gamma\left(\frac{\nu+2}{2}\right)\nu^{\frac{\nu}{2}}\left(1-\rho^2\right)^{\frac{\nu+1}{2}}}{\Gamma\left(\frac{\nu}{2}\right)\pi \Pr(X>0, Y>0)} J_1\left(r^2-2\rho r+1, Ar+B, C+\nu\left(1-\rho^2\right), \frac{\nu}{2}+1\right)$$

For $-\infty < x < \infty$,$-\infty < y < \infty, r > 0, -\infty < a < \infty, -\infty < b < \infty, -1 < \rho < 1$,where $A = -2a + 2\rho b, B = -2b + 2\rho a, C = a^2 + b^2 - 2\rho ab$ and J_1 is given by first reference paper section (2.5).

Value

`dBitPR` gives the probability density function for quotient of Bivariate t random variables conditioned to the positive quadrant.

Invalid arguments will return an error message.

Author(s)

Saralees Nadarajah & Yuancheng Si <siyuanchengman@gmail.com>

References

Yuancheng Si and Saralees Nadarajah and Xiaodong Song, (2020). On the distribution of quotient of random variables conditioned to the positive quadrant. *Communications in Statistics - Theory and Methods*, **49**, pp2514-2528.

Balakrishnan, N. and Lai, C. -D. (2009).*Continuous Bivariate Distributions*.Springer Verlag, New York.

Arnold, B. C. and Strauss, D. (1988).Pseudolikelihood estimation.*Sankhya B* , **53**, pp233-243.

Caginalp, C. and Caginalp, G. (2018).The quotient of normal random variables and application to asset price fat tails.*Physica A—Statistical Mechanics and Its Applications*, **499**, pp457-471.

Louzada, F., Ara, A. and Fernandes, G. (2017).The bivariate alpha-skew-normal distribution.*Communications in Statistics - Theory and Methods*, **46**, pp7147-7156.

Nadarajah, S. (2009).A bivariate Pareto model for drought.*Stochastic Environmental Research and Risk Assessment*, **23**, pp811-822.

Nadarajah, S. and Kotz, S. (2006).Reliability models based on bivariate exponential distributions.*Probabilistic Engineering Mechanics*, **21**, pp338-351.

Nadarajah, S. and Kotz, S. (2007).Financial Pareto ratios.*Quantitative Finance*, **7**, pp257-260.

Examples

```
x <- seq(0.1,5,0.1)
y <- c()
for (i in x){y=c(y,dBitPR(i,1,2,0.5,2))}
plot(x,y,type = 'l')
```

---

f21hyper *f21hyper*

Description

Computes the value of a Gaussian hypergeometric function $F(a, b, c, z)$ for $-1 \le z \le 1$ and $a, b, c \ge 0$

Usage

```
f21hyper(a, b, c, z)
```

Arguments

a	The parameter a of the Gaussian hypergeometric function, must be a positive scalar here
b	The parameter b of the Gaussian hypergeometric function, must be a positive scalar here

`c` The parameter `c` of the Gaussian hypergeometric function, must be a positive scalar here

`z` The parameter `z` of the Gaussian hypergeometric function, must be between -1 and 1 here

Details

The function `f21hyper` complements the analysis of the 'hyper-g prior' introduced by Liang et al. (2008).
For parameter values, compare cf. `https://en.wikipedia.org/wiki/Hypergeometric_function#The_series_2F1`.

Value

Invalid arguments will return an error message.

Author(s)

Martin Feldkircher and Stefan Zeugner

References

Liang F., Paulo R., Molina G., Clyde M., Berger J.(2008): Mixtures of g-priors for Bayesian variable selection. J. Am. Statist. Assoc. 103, p. 410-423

Yuancheng Si and Saralees Nadarajah and Xiaodong Song, (2020). On the distribution of quotient of random variables conditioned to the positive quadrant. *Communications in Statistics - Theory and Methods*, **49**, pp2514-2528.

Saralees Nadarajah and Y.Si (2020) A note on the "L-logistic regression models: Prior sensitivity analysis, robustness to outliers and applications". *Brazilian Journal of Probability and Statistics*,**34**,p. 183-187.

Examples

```
f21hyper(30,1,20,.8) #returns about 165.8197
f21hyper(30,10,20,0) #returns one
f21hyper(10,15,20,-0.1) # returns about 0.4872972
```

I_1 *Lemma*

Description

Technical Lemmas for calculating quotient of random variables conditioned to the positive quadrant.For more detailed information please read the first reference paper section 2.2.

Usage

```
I_1(a, b)

I_2(a, b)

I_3(a, b)

J_1(a, b, c, alpha)

J_2(a, b, c, alpha)

J_3(a, b, c, alpha)
```

Arguments

a	parameter
b	parameter
c	parameter
alpha	parameter

Details

I_n Type I Integration

$$I_n(a,b) = \int_0^\infty y^n \exp\left(-ay^2 - by\right) dy$$

For $-\infty < a < \infty, -\infty < b < \infty$,where n is positive integer.

In particular,for $a > 0$,we have expressions below

$$I_1(a,b) = -\frac{\sqrt{\pi} b}{4a^{3/2}} \exp\left(\frac{b^2}{4a}\right) \text{ erfc}\left(\frac{b}{2\sqrt{a}}\right) + \frac{1}{2a}$$

$$I_2(a,b) = \frac{\sqrt{\pi}}{4a^{3/2}} \exp\left(\frac{b^2}{4a}\right) \text{ erfc}\left(\frac{b}{2\sqrt{a}}\right) + \frac{\sqrt{\pi} b^2}{8a^{5/2}} \exp\left(\frac{b^2}{4a}\right) \text{ erfc}\left(\frac{b}{2\sqrt{a}}\right) - \frac{b}{4a^2}$$

$$I_3(a,b) = -\frac{3\sqrt{\pi} b}{8a^{5/2}} \exp\left(\frac{b^2}{4a}\right) \text{ erfc}\left(\frac{b}{2\sqrt{a}}\right) - \frac{\sqrt{\pi} b^3}{16a^{7/2}} \exp\left(\frac{b^2}{4a}\right) \text{ erfc}\left(\frac{b}{2\sqrt{a}}\right) + \frac{1}{2a^2} + \frac{b^2}{8a^3}$$

J_n Type J Integration

$$J_n(a,b,c,\alpha) = \int_0^\infty y^n \left(ay^2 + by + c\right)^{-\alpha} dy$$

In particular,for $a > 0, b^2 < 4ac, -1 < n < 2\alpha - 1$,we have expressions below

$$J_1(a,b,c,\alpha) = a^{-1} c^{1-\alpha} B\left(2, 2\alpha - 2\right) \ {}_2F_1\left(1, \alpha - 1; \alpha + \frac{1}{2}; 1 - \frac{b^2}{4ac}\right)$$

$$J_2(a,b,c,\alpha) = a^{-\frac{3}{2}} c^{\frac{3}{2}-\alpha} B\left(3, 2\alpha - 3\right) \ {}_2F_1\left(\frac{3}{2}, \alpha - \frac{3}{2}; \alpha + \frac{1}{2}; 1 - \frac{b^2}{4ac}\right)$$

$$J_3(a,b,c,\alpha) = a^{-2} c^{2-\alpha} B\left(4, 2\alpha - 4\right) \ {}_2F_1\left(2, \alpha - 2; \alpha + \frac{1}{2}; 1 - \frac{b^2}{4ac}\right)$$

Value

`I_1` gives value of Type I integration with $n = 1$

`I_2` gives value of Type I integration with $n = 2$

`I_3` gives value of Type I integration with $n = 3$

`J_1` gives value of Type J integration with $n = 1$

`J_2` gives value of Type J integration with $n = 2$

`J_3` gives value of Type J integration with $n = 3$

Invalid arguments will return an error message.

Author(s)

Saralees Nadarajah & Yuancheng Si <siyuanchengman@gmail.com>

References

Yuancheng Si and Saralees Nadarajah and Xiaodong Song, (2020). On the distribution of quotient of random variables conditioned to the positive quadrant. *Communications in Statistics - Theory and Methods*, **49**, pp2514-2528.

Balakrishna, N. and Shiji, K. (2014). On a class of bivariate exponential distributions.*Statistics and Probability Letters*, **85**, pp153-160.

Arnold, B. C. and Strauss, D. (1988).Pseudolikelihood estimation.*Sankhya B* , **53**, pp233-243.

Caginalp, C. and Caginalp, G. (2018).The quotient of normal random variables and application to asset price fat tails.*Physica A—Statistical Mechanics and Its Applications*, **499**, pp457-471.

Louzada, F., Ara, A. and Fernandes, G. (2017).The bivariate alpha-skew-normal distribution.*Communications in Statistics - Theory and Methods*, **46**, pp7147-7156.

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Nadarajah, S. and Kotz, S. (2006).Reliability models based on bivariate exponential distributions.*Probabilistic Engineering Mechanics*, **21**, pp338-351.

Nadarajah, S. and Kotz, S. (2007).Financial Pareto ratios.*Quantitative Finance*, **7**, pp257-260.

Examples

```
I_1(1,2)
I_2(1,2)
I_3(1,2)
J_1(1,2,3,3)
J_2(1,2,3,3)
J_3(1,2,3,3)
```

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