Package 'MixedPoisson'

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Type Package

Title Mixed Poisson Models

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Depends gaussquad, Rmpfr, MASS

Description The estimation of the parameters in mixed Poisson models.

License GPL-2

NeedsCompilation no

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Contents

`MixedPoisson-package`   *Mixed Poisson Models*

Description

The package provides functions, which support to fit parameters of different mixed Poisson models using the Expectation-Maximization (EM) algorithm of estimation, cf. (Ghitany et al., 2012, pp. 6848). In the model the assumptions are: conditional $N|\theta$ is of distribution $N|\theta \sim POIS(\lambda\theta)$, parameter θ is a random variable distributed according to the density function $f_\theta(\cdot)$, $E[\theta] = 1$ and $\lambda = \exp(\mathbf{x}'_i\boldsymbol{\beta})$ – the regression component. The E-step is carried out through the numerical integration using Laquerre quadrature. The M-step estimates the parameters β using GLM Poisson with pseudo values from E-step and mixing parameters using optimize function.

Details

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Author(s)

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References

Karlis, D. (2005). EM algorithm for mixed Poisson and other discrete distributions. Astin Bulletin, 35(01), 3-24. Ghitany, M. E., Karlis, D., Al-Mutairi, D. K., & Al-Awadhi, F. A. (2012). An EM algorithm for multivariate mixed Poisson regression models and its application. Applied Mathematical Sciences, 6(137), 6843-6856.

`est.delta`   *Estimation of delta parameter of inverse-Gaussian distribution*

Description

The function estimates the value of the parameter delta using `optimize`.

Usage

```
est.delta(t)
```

Arguments

t   the vector of values

Details

The form of the distribution is as in the function `ll.invGauss`

Value

nu   the estimates of ν

ll.delta.max   the value of loglikehood

Author(s)

Michal Trzesiok

Examples

```
est.delta(t=c(3,8))
```

| est.gamma | *Estimation of gamma parameter of Gamma distribution* |

Description

The function estimates the value of the parameter gamma using `optimize`.

Usage

```
est.gamma(t)
```

Arguments

t   the vector of values

Details

The form of the distribution is as in the function `ll.gamma`

Value

gamma   the estimates of γ

ll.gamma.max   the value of loglikehood

Author(s)

Michal Trzesiok

Examples

```
est.gamma(t=c(3,8))
```

---

est.nu   *Estimation of nu parameter of log-normal distribution*

Description

The function estimates the value of the parameter nu using `optimize`.

Usage

```
est.nu(t)
```

Arguments

t   the vector of values

Details

The form of the distribution is as in the function `ll.lognorm`

Value

nu   the estimates of ν

ll.nu.max   the value of loglikehood

Author(s)

Michal Trzesiok

Examples

```
est.nu(t=c(3,8))
```

`Gamma.density`   *Gamma density*

Description

The function returns the vector of values of density function for of Gamma distribution with one parameter γ.

Usage

```
Gamma.density(theta, gamma.par)
```

Arguments

theta   the vector of values

gamma.par   the parameter of Gamma distribution

Details

The pdf of Gamma is of the form $f_\theta(\theta) = \frac{\gamma^\gamma}{\Gamma(\gamma)} \theta^{\gamma-1} \exp(-\gamma\theta)$

Value

```
Gamma.density(theta, nu)
```
              the density – the vector of values

Author(s)

Michal Trzesiok

Examples

```
Gamma.density(c(2,3,5,4,6,7,4), 5)
```

`invGauss.density`   *inverse-Gaussian Density*

Description

The function returns the vector of values of density function for of inverse-Gaussian distribution with one parameter δ.

Usage

```
invGauss.density(theta, delta)
```

Arguments

theta	the vector of values
delta	the parameter of inverse-Gaussian distribution

Details

The pdf of inverse-Gaussian is of the form $f_\theta(\theta) = \frac{\delta}{2\pi} \exp(\delta^2) \theta^{-\frac{3}{2}} \exp(-\frac{\delta^2}{2}(\frac{1}{\theta} + \theta))$

Value

invGauss.density(theta, delta)
the density – the vector of values

Author(s)

Michal Trzesiok

Examples

```
invGauss.density(c(2,3,5,4,6,7,6), 5)
```

---

lambda_m_step	*Estimation of Lambda in M-step – Expectation-Maximization (EM) algorithm*

Description

The function fits the GLM Poisson with given offset.

Usage

```
lambda_m_step(variable, X, offset)
```

Arguments

variable	the vector of numbers
X	model matrix of the form $X = model.matrix(\ regressor)$. In the model without regressor the X sould be defined as $X = as.matrix(rep(1, length(variable)))$
offset	offset in GLM Poisson

Details

It fits the GLM Poisson, where $variable \sim 1$ and the ofset is given as the vector of the variable's length. The results are used in M-step of EM algorithm, cf. [Karlis, 2012] pp. 6850.

Value

lambda	$\hat{\lambda} = \hat{\beta}X$
beta	regressor parameters
glm	output of `glm`

Author(s)

Alicja Wolny–Dominiak, Michal Trzesiok

Examples

```
set.seed(1234)
variable=rpois(50,4)
X=as.matrix(rep(1, length(variable)))
t=pseudo_values(variable, mixing=c("invGauss"), lambda=4, delta=1, n=100)
lambda_m_step(variable, X, offset=t$pseudo_values)
```

---

lambda_start	*Estimation of starting lambda in Expectation-Maximization (EM) algorithm*

Description

The function fits the GLM Poisson without regressors.

Usage

```
lambda_start(variable, X)
```

Arguments

variable	the vector of numbers
X	model matrix of the form $X = model.matrix(\,regressor)$. In the model without regressor the X sould be defined as $X = as.matrix(rep(1, length(variable)))$

Details

It fits the GLM Poisson, where $variable \sim 1$. The results are taken as the starting value of EM algorithm.

Value

lambda	$\hat{\lambda} = \hat{\beta}X$
beta	regressor parameters
glm	output of `glm`

Author(s)

Alicja Wolny–Dominiak, Michal Trzesiok

Examples

```
set.seed(1234)
variable=rpois(50,4)
X=as.matrix(rep(1, length(variable)))
t=pseudo_values(variable, mixing=c("invGauss"), lambda=4, delta=1, n=100)
lambda_m_step(variable, X, offset=t$pseudo_values)
```

---

ll.gamma   *Gamma Log-likelihood*

Description

The function returns the value of log-likelihood function for of Gamma distribution with one parameter γ.

Usage

```
ll.gamma(gamma.par, t)
```

Arguments

gamma.par γ parameter

t   the vector of values

Details

The pdf of Gamma is of the form $f_\theta(\theta) = \frac{\gamma^\gamma}{\Gamma(\gamma)} \theta^{\gamma-1} \exp(-\gamma\theta)$

Value

ll.gamma   the value

Author(s)

Michal Trzesiok

Examples

```
ll.gamma(1, c(3,8))
```

`ll.invGauss` *Inverse-Gaussian Log-likelihood*

Description

The function returns the value of log-likelihood function for of inverse-Gaussian distribution with one parameter δ.

Usage

```
ll.invGauss(delta, t)
```

Arguments

`delta`	δ parameter
`t`	the vector of values

Details

The pdf of inverse-Gaussian is of the form $f_\theta(\theta) = \frac{\delta}{2\pi} \exp(\delta^2) \theta^{-\frac{3}{2}} \exp(-\frac{\delta^2}{2}(\frac{1}{\theta} + \theta))$

Value

`ll.invGauss`	the value

Author(s)

Michal Trzesiok

Examples

```
ll.invGauss(1, c(3,8))
```

`ll.lognorm` *Log-normal Log-likelihood*

Description

The function returns the value of log-likelihood function of log-normal distribution with one parameter ν.

Usage

```
ll.lognorm(nu, t)
```

Arguments

nu	ν parameter
t	the vector of values

Details

The pdf of log-normal is of the form $f_\theta(\theta) = \frac{1}{\sqrt{2\pi}\nu\theta} \exp[-\frac{(\log(\theta) + \frac{\nu^2}{2})^2}{2\nu^2}]$

Value

ll.lognorm	the value

Author(s)

Michal Trzesiok

Examples

```
ll.lognorm(1, c(3,8))
```

---

lognorm.density	*Log-normal Density*

Description

The function returns the vector of values of density function for of log-normal distribution with one parameter ν.

Usage

```
lognorm.density(theta, nu)
```

Arguments

theta	the vector of values
nu	the parameter of log-normal distribution

Details

The pdf of log-normal is of the form $f_\theta(\theta) = \frac{1}{\sqrt{2\pi}\nu\theta} \exp[-\frac{(\log(\theta) + \frac{\nu^2}{2})^2}{2\nu^2}]$

Value

lognorm.density(theta, nu)	the density – the vector of values

Author(s)

Michal Trzesiok

Examples

```
lognorm.density(c(2,3,5,4,6,7,6), 5)
```

---

pg.dist   *Poisson-Gamma Distribution (Negative-Binomial)*

Description

The function fits a mixed Poisson distribution, in which the random parameter follows Gamma distribution (the negative-binomial distribution). As teh method of estimation Expectation-maximization algorithm is used. In M-step the analytical formulas taken from [Karlis, 2005] are applied.

Usage

```
pg.dist(variable, alpha.start, beta.start, epsylon)
```

Arguments

variable	The count variable.
alpha.start	The starting value of the parameter alpha. Default to 1.
beta.start	The starting value of the parameter beta. Default to 0.3
epsylon	Default to epsylon = 10^(-8)

Details

This function provides estimated parameters of the model $N|\lambda \sim Poisson(\lambda)$ where λ parameter is also a random variable follows Gamma distribution with hiperparameters α, β. The pdf of Gamma is of the form $f_\lambda(\lambda) = \frac{\lambda^{\alpha-1}\exp(-\beta\lambda)\beta^\lambda}{\Gamma(\alpha)}$.

Value

alpha	the parameter of mixing Gamma distribution
beta	the parameter of mixing Gamma distribution
theta	the value 1/beta
n.iter	the number of steps in EM algorithm

References

Karlis, D. (2005). EM algorithm for mixed Poisson and other discrete distributions. Astin bulletin, 35(01), 3-24.

Examples

```
library(MASS)
pGamma1 = pg.dist(variable=quine$Days)
print(pGamma1)
```

---

pl.dist	*Poisson-Lindley Distribution*

Description

The function fits a mixed Poisson distribution, in which the random parameter follows Lindley distribution. As teh method of estimation Expectation-maximization algorithm is used.

Usage

```
pl.dist(variable, p.start, epsylon)
```

Arguments

variable	The count variable.
p.start	The starting value of p parameter. Default to 0.1.
epsylon	Default to epsylon = 10^(-8)

Details

This function provides estimated parameters of the model $N|\lambda \sim Poisson(\lambda)$ where λ parameter is also a random variable follows Lindley distribution with hiperparameter p. The pdf of Lindley is of the form $f_\lambda(\lambda) = \frac{p^2}{p+1}(\lambda + 1)\exp(-\lambda p)$.

Value

p	the parameter of mixing Lindley distribution
n.iter	the number of steps in EM algorithm

References

Karlis, D. (2005). EM algorithm for mixed Poisson and other discrete distributions. Astin bulletin, 35(01), 3-24.

Examples

```
library(MASS)
pLindley = pl.dist(variable=quine$Days)
print(pLindley)
```

---

pseudo_values   *Pseudo values – Expectation-Maximization (EM) algorithm*

Description

The function returns the pseudo values t_i defined as the conditional expectation $E[\theta_i|k_1, ..., k_n]$, where $k_1, ..., k_n$ are realizations of the count variable N.

Usage

```
pseudo_values(variable, mixing, lambda, gamma.par, nu, delta, n)
```

Arguments

variable	the vector of numbers
mixing	the name of mixing distribution – "Gamma", "lognorm", "invGauss"
lambda	λ parameter in mixed Poisson model
gamma.par	γ parameter in Gamma mixing distribution
nu	ν parameter in log-normal mixing distribution
delta	δ parameter in inverse-Gaussian mixing distribution
n	The integer value for the Laguerre quadrature. Default to 100

Details

The function calculates the vector of pseudo values $t_i = E[\theta_i|k_1, ..., k_n]$ in E-step of EM algorithm. It applies the numerical integration using $laguerre.quadrature$ in the nominator and the denominator of the formula

The proper parameter γ, ν, δ should be chosen according to the mixing distribution.

Value

pseudo_values	pseudo values $t_1, ..., t_n$
nominator	nominator in the formula
denominator	denominator in the formula

Author(s)

Alicja Wolny–Dominiak, Michal Trzesiok

Examples

```
variable=rpois(30,4)
pseudo_values(variable, mixing="Gamma", lambda=4, gamma.par=0.7, n=100)
```

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