Package ‘orthopolynom’

October 14, 2022

Version 1.0-6.1

Date 2022-05-06

Title Collection of Functions for Orthogonal and Orthonormal Polynomials

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Depends R (>= 2.0.1)

Imports polynom, stats

Description A collection of functions to construct sets of orthogonal polynomials and their recurrence relations. Additional functions are provided to calculate the derivative, integral, value and roots of lists of polynomial objects.

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NeedsCompilation no

Repository CRAN

Date/Publication 2022-10-03 10:11:45 UTC

R topics documented:

`chebyshev.c.inner.products` *Inner products of Chebyshev polynomials*

Description

This function returns a vector with $n + 1$ elements containing the inner product of an order k Chebyshev polynomial of the first kind, $C_k(x)$, with itself (i.e. the norm squared) for orders $k = 0, 1, \ldots, n$.

Usage

```
chebyshev.c.inner.products(n)
```

Arguments

`n`	integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$$h_n = \langle C_n | C_n \rangle = \begin{cases} 4\,\pi & n \neq 0 \\ 8\,\pi & n = 0 \end{cases}$$

Value

A vector with $n + 1$ elements

1	inner product of order 0 orthogonal polynomial
2	inner product of order 1 orthogonal polynomial
...	
n+1	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

gegenbauer.inner.products

Examples

```
###
### generate the inner products vector for the
### C Chebyshev polynomials of orders 0 to 10
###
h <- chebyshev.c.inner.products( 10 )
print( h )
```

`chebyshev.c.polynomials` *Create list of Chebyshev polynomials*

Description

This function returns a list with $n + 1$ elements containing the order k Chebyshev polynomials of the first kind, $C_k\left(x\right)$, for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
chebyshev.c.polynomials(n, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`normalized`	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `chebyshev.c.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

`1`	order 0 Chebyshev polynomial
`2`	order 1 Chebyshev polynomial
...	
`n+1`	order n Chebyshev polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

chebyshev.c.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized C Chebyshev polynomials of orders 0 to 10
###
normalized.p.list <- chebyshev.c.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized C Chebyshev polynomials of orders 0 to 10
###
unnormalized.p.list <- chebyshev.c.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )
```

---

chebyshev.c.recurrences

Recurrence relations for Chebyshev polynomials

Description

This function returns a data frame with $n+1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k Chebyshev polynomial of the first kind, $C_k(x)$, and for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
chebyshev.c.recurrences(n, normalized=FALSE)
```

Arguments

n	integer value for the highest polynomial order
normalized	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

chebyshev.c.inner.products

Examples

```
###
### generate the recurrences data frame for
### the normalized Chebyshev C polynomials
### of orders 0 to 10.
###
normalized.r <- chebyshev.c.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the normalized Chebyshev C polynomials
### of orders 0 to 10.
###
unnormalized.r <- chebyshev.c.recurrences( 10, normalized=FALSE )
print( unnormalized.r )
```

---

chebyshev.c.weight — *Weight function for the Chebyshev polynomial*

Description

This function returns the value of the weight function for the order k Chebyshev polynomial of the first kind, $C_k\left(x\right)$.

Usage

```
chebyshev.c.weight(x)
```

Arguments

x	the function argument which can be a vector

Details

The function takes on non-zero values in the interval $(-2, 2)$. The formula used to compute the weight function is as follows.

$w\left(x\right) = \frac{1}{\sqrt{1 - \frac{x^2}{4}}}$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the C Chebyshev weight function for arguments between -3 and 3
###
x <- seq( -3, 3, .01 )
y <- chebyshev.c.weight( x )
plot( x, y )
```

---

chebyshev.s.inner.products *Inner products of Chebyshev polynomials*

Description

This function returns a vector with $n + 1$ elements containing the inner product of an order k Chebyshev polynomial of the second kind, $S_k\left(x\right)$, with itself (i.e. the norm squared) for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
chebyshev.s.inner.products(n)
```

Arguments

n	integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$h_n = \left\langle S_n | S_n \right\rangle = \pi$.

Value

A vector with $n + 1$ elements

1	inner product of order 0 orthogonal polynomial
2	inner product of order 1 orthogonal polynomial
...	
n+1	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### S Chebyshev polynomials of orders 0 to 10
###
h <- chebyshev.s.inner.products( 10 )
print( h )
```

---

chebyshev.s.polynomials

Create list of Chebyshev polynomials

Description

This function returns a list with $n + 1$ elements containing the order k Chebyshev polynomials of the second kind, $S_k(x)$, for orders $k = 0, 1, \ldots, n$.

Usage

```
chebyshev.s.polynomials(n, normalized=FALSE)
```

Arguments

n	integer value for the highest polynomial order
normalized	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `chebyshev.s.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

1	order 0 Chebyshev polynomial
2	order 1 Chebyshev polynomial
...	
n+1	order n Chebyshev polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`chebyshev.s.recurrences`, `orthogonal.polynomials`, `orthonormal.polynomials`

Examples

```
###
### gemerate a list of normalized S Chebyshev polynomials of orders 0 to 10
###
normalized.p.list <- chebyshev.s.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized S Chebyshev polynomials of orders 0 to 10
###
unnormalized.p.list <- chebyshev.s.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )
```

chebyshev.s.recurrences *Recurrence relations for Chebyshev polynomials*

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k Chebyshev polynomial of the second kind, $S_k(x)$, and for orders $k = 0, 1, \ldots, n$.

Usage

```
chebyshev.s.recurrences(n, normalized=FALSE)
```

Arguments

n	integer value for the highest polynomial order
normalized	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

chebyshev.s.inner.products

Examples

```
###
### generate the recurrences data frame for
### the normalized Chebyshev S polynomials
### of orders 0 to 10.
###
```

```
normalized.r <- chebyshev.s.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the normalized Chebyshev S polynomials
### of orders 0 to 10.
###
unnormalized.r <- chebyshev.s.recurrences( 10, normalized=FALSE )
print( unnormalized.r )
```

---

chebyshev.s.weight *Weight function for the Chebyshev polynomial*

Description

This function returns the value of the weight function for the order k Chebyshev polynomial of the second kind, $S_k\left(x\right)$.

Usage

```
chebyshev.s.weight(x)
```

Arguments

x the function argument which can be a vector

Details

The function takes on non-zero values in the interval $(-2, 2)$. The formula used to compute the weight function is as follows.

$w\left(x\right) = \sqrt{1 - \frac{x^2}{4}}$

Value

The value of the weight function.

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the S Chebyshev weight function for arguments between -2 and 2
###
x <- seq( -2, 2, .01 )
y <- chebyshev.s.weight( x )
plot( x, y )
```

`chebyshev.t.inner.products`

Inner products of Chebyshev polynomials

Description

This function returns a vector with $n+1$ elements containing the inner product of an order k Chebyshev polynomial of the first kind, $T_k(x)$, with itself (i.e. the norm squared) for orders $k = 0, 1, \ldots, n$.

Usage

```
chebyshev.t.inner.products(n)
```

Arguments

`n`	integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$$h_n = \langle T_n | T_n \rangle = \begin{cases} \frac{\pi}{2} & n \neq 0 \\ \pi & n = 0 \end{cases}$$

Value

A vector with $n+1$ elements

`1`	inner product of order 0 orthogonal polynomial
`2`	inner product of order 1 orthogonal polynomial
...	
`n+1`	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### T Chybyshev polynomials of orders 0 to 10
###
h <- chebyshev.t.inner.products( 10 )
print( h )
```

---

chebyshev.t.polynomials

Create list of Chebyshev polynomials

Description

This function returns a list with $n+1$ elements containing the order k Chebyshev polynomials of the first kind, $T_k(x)$, for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
chebyshev.t.polynomials(n, normalized=FALSE)
```

Arguments

n	integer value for the highest polynomial order
normalized	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `chebyshev.t.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

1	order 0 Chebyshev polynomial
2	order 1 Chebyshev polynomial
...	
n+1	order n Chebyshev polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

chebyshev.u.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized T Chebyshev polynomials of orders 0 to 10
###
normalized.p.list <- chebyshev.t.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized T Chebyshev polynomials of orders 0 to 10
###
unnormalized.p.list <- chebyshev.t.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )
```

---

chebyshev.t.recurrences

Recurrence relations for Chebyshev polynomials

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k Chebyshev polynomial of the first kind, $T_k(x)$, for orders $k = 0, 1, \ldots, n$.

Usage

```
chebyshev.t.recurrences(n, normalized=FALSE)
```

Arguments

n	integer value for the highest polynomial order
normalized	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

chebyshev.t.inner.products

Examples

```
###
### generate the recurrence relations for
### the normalized T Chebyshev polynomials
### of orders 0 to 10
###
normalized.r <- chebyshev.t.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for
### the normalized T Chebyshev polynomials
### of orders 0 to 10
###
unnormalized.r <- chebyshev.t.recurrences( 10, normalized=FALSE )
print( unnormalized.r )
```

chebyshev.t.weight *Weight function for the Chebyshev polynomial*

Description

This function returns the value of the weight function for the order k Chebyshev polynomial of the first kind, $T_k(x)$.

Usage

```
chebyshev.t.weight(x)
```

Arguments

x	the function argument which can be a vector

Details

The function takes on non-zero values in the interval $(-1, 1)$. The formula used to compute the weight function is as follows.

$w(x) = \frac{1}{\sqrt{1-x^2}}$

Value

The value of the weight function.

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the T Chebyshev function for argument values between -2 and 2
x <- seq( -1, 1, .01 )
y <- chebyshev.t.weight( x )
plot( x, y )
```

chebyshev.u.inner.products

Inner products of Chebyshev polynomials

Description

This function returns a vector with $n + 1$ elements containing the inner product of an order k Chebyshev polynomial of the second kind, $U_k(x)$, with itself (i.e. the norm squared) for orders $k = 0, 1, \ldots, n$.

Usage

```
chebyshev.u.inner.products(n)
```

Arguments

n	integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$h_n = \langle U_n | U_n \rangle = \frac{\pi}{2}$

Value

A vector with $n + 1$ elements

1	inner product of order 0 orthogonal polynomial
2	inner product of order 1 orthogonal polynomial
...	
n+1	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### U Chebyshev polynomials of orders 0 to 10
###
h <- chebyshev.u.inner.products( 10 )
print( h )
```

---

chebyshev.u.polynomials   *Create list of Chebyshev polynomials*

Description

This function returns a list with $n+1$ elements containing the order k Chebyshev polynomials of the second kind, $U_k(x)$, for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
chebyshev.u.polynomials(n, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`normalized`	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `chebyshev.u.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n+1$ polynomial objects

1	order 0 Chebyshev polynomial
2	order 1 Chebyshev polynomial
...	
n+1	order n Chebyshev polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`chebyshev.u.recurrences`, `orthogonal.polynomials`, `orthonormal.polynomials`

Examples

```
###
### gemerate a list of normalized U Chebyshev polynomials of orders 0 to 10
###
normalized.p.list <- chebyshev.u.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized T Chebyshev polynomials of orders 0 to 10
###
unnormalized.p.list <- chebyshev.u.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )
```

`chebyshev.u.recurrences` *Recurrence relations for Chebyshev polynomials*

Description

This function returns a data frame with $n+1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k Chebyshev polynomial of the second kind, $U_k(x)$, for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
chebyshev.u.recurrences(n, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`normalized`	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`chebyshev.u.inner.products`

Examples

```
###
### generate the recurrence relations for
### the normalized U Chebyshev polynomials
### of orders 0 to 10
###
normalized.r <- chebyshev.u.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for
### the unnormalized U Chebyshev polynomials
### of orders 0 to 10
###
unnormalized.r <- chebyshev.u.recurrences( 10, normalized=FALSE )
print( unnormalized.r )
```

`chebyshev.u.weight` *Weight function for the Chebyshev polynomial*

Description

This function returns the value of the weight function for the order k Chebyshev polynomial of the second kind, $U_k(x)$.

Usage

```
chebyshev.u.weight(x)
```

Arguments

`x`	the function argument which can be a vector

Details

The function takes on non-zero values in the interval $(-1, 1)$. The formula used to compute the weight function is as follows.

$w(x) = \sqrt{1 - x^2}$

Value

The value of the weight function.

Author(s)

Frederick Novomestky < `fnovomes@poly.edu` >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the U Chebyshev function for argument values between -2 and 2
###
x <- seq( -1, 1, .01 )
y <- chebyshev.u.weight( x )
plot( x, y )
```

`gegenbauer.inner.products` *Inner products of Gegenbauer polynomials*

Description

This function returns a vector with $n+1$ elements containing the inner product of an order k Gegenbauer polynomial, $C_k^{(\alpha)}(x)$, with itself (i.e. the norm squared) for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
gegenbauer.inner.products(n,alpha)
```

Arguments

`n`	integer value for the highest polynomial order
`alpha`	numeric value for the polynomial parameter

Details

The formula used to compute the inner products is as follows.

$$h_n = \left\langle C_n^{(\alpha)} | C_n^{(\alpha)} \right\rangle = \begin{cases} \frac{\pi\, 2^{1-2\,\alpha}\, \Gamma(n+2\,\alpha)}{n!\, (n+\alpha)\, [\Gamma(\alpha)]^2} & \alpha \neq 0 \\ \frac{2\,\pi}{n^2} & \alpha = 0 \end{cases} .$$

Value

A vector with $n+1$ elements

`1`	inner product of order 0 orthogonal polynomial
`2`	inner product of order 1 orthogonal polynomial
...	
`n+1`	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`ultraspherical.inner.products`

Examples

```
###
### generate the inner products vector for the
### Gegenbauer polynomials of orders 0 to 10
### the polynomial parameter is 1.0
###
h <- gegenbauer.inner.products( 10, 1 )
print( h )
```

gegenbauer.polynomials *Create list of Gegenbauer polynomials*

Description

This function returns a list with $n + 1$ elements containing the order k Gegenbauer polynomials, $C_k^{(\alpha)}(x)$, for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
gegenbauer.polynomials(n, alpha, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`alpha`	polynomial parameter
`normalized`	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `gegenbauer.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

`1`	order 0 Gegenbauer polynomial
`2`	order 1 Gegenbauer polynomial
...	
`n+1`	order n Chebyshev polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

gegenbauer.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized Gegenbauer polynomials of orders 0 to 10
### polynomial parameter is 1.0
###
normalized.p.list <- gegenbauer.polynomials( 10, 1, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized Gegenbauer polynomials of orders 0 to 10
### polynomial parameter is 1.0
###
unnormalized.p.list <- gegenbauer.polynomials( 10, 1, normalized=FALSE )
print( unnormalized.p.list )
```

---

gegenbauer.recurrences

Recurrence relations for Gegenbauer polynomials

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k Gegenbauer polynomial, $C_k^{(\alpha)}(x)$, and for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
gegenbauer.recurrences(n, alpha, normalized=FALSE)
```

Arguments

n	integer value for the highest polynomial order
alpha	numeric value for polynomial parameter
normalized	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`gegenbauer.inner.products`

Examples

```
###
### generate the recurrences data frame for
### the normalized Gegenbauer polynomials
### of orders 0 to 10.
### polynomial parameter value is 1.0
###
normalized.r <- gegenbauer.recurrences( 10, 1, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the normalized Gegenbauer polynomials
### of orders 0 to 10.
### polynomial parameter value is 1.0
###
unnormalized.r <- gegenbauer.recurrences( 10, 1, normalized=FALSE )
print( unnormalized.r )
```

`gegenbauer.weight` *Weight function for the Gegenbauer polynomial*

Description

This function returns the value of the weight function for the order k Gegenbauer polynomial, $C_k^{(\alpha)}(x)$.

Usage

```
gegenbauer.weight(x,alpha)
```

Arguments

`x`	the function argument which can be a vector
`alpha`	polynomial parameter

Details

The function takes on non-zero values in the interval $(-1, 1)$. The formula used to compute the weight function is as follows.

$w(x) = \left(1 - x^2\right)^{\alpha - 0.5}$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the Gegenbauer weight function for argument values between -1 and 1
###
x <- seq( -1, 1, .01 )
y <- gegenbauer.weight( x, 1 )
plot( x, y )
```

---

ghermite.h.inner.products

Inner products of generalized Hermite polynomials

Description

This function returns a vector with $n+1$ elements containing the inner product of an order k generalized Hermite polynomial, $H_k^{(\mu)}(x)$, with itself (i.e. the norm squared) for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
ghermite.h.inner.products(n, mu)
```

Arguments

`n`	`n` integer value for the highest polynomial order
`mu`	`mu` polynomial parameter

Details

The parameter μ must be greater than -0.5. The formula used to compute the inner products is as follows.

$h_n(\mu) = \left\langle H_m^{(\mu)} | H_n^{(\mu)} \right\rangle = 2^{2\,n}\ \left[\frac{n}{2}\right]!\ \Gamma\left(\left[\frac{n+1}{2}\right] + \mu + \frac{1}{2}\right)$

Value

A vector with $n+1$ elements

`1`	inner product of order 0 orthogonal polynomial
`2`	inner product of order 1 orthogonal polynomial

...

`n+1`	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### generalized Hermite polynomials of orders 0 to 10
### polynomial parameter is 1
###
h <- ghermite.h.inner.products( 10, 1 )
print( h )
```

`ghermite.h.polynomials` *Create list of generalized Hermite polynomials*

Description

This function returns a list with $n+1$ elements containing the order k generalized Hermite polynomials, $H_k^{(\mu)}(x)$, for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
ghermite.h.polynomials(n, mu, normalized = FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`mu`	numeric value for the polynomial parameter
`normalized`	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Details

The parameter μ must be greater than -0.5. The function `ghermite.h.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n+1$ polynomial objects

`1`	order 0 generalized Hermite polynomial
`2`	order 1 generalized Hermite polynomial
...	
`n+1`	order n generalized Hermite polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Alvarez-Nordase, R., M. K. Atakishiyeva and N. M. Atakishiyeva, 2004. A q-extension of the generalized Hermite polynomials with continuous orthogonality property on R, *International Journal of Pure and Applied Mathematics*, 10(3), 335-347.

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`ghermite.h.recurrences`, `orthogonal.polynomials`, `orthonormal.polynomials`

Examples

```
###
### gemerate a list of normalized generalized Hermite polynomials of orders 0 to 10
### polynomial parameter is 1.0
###
normalized.p.list <- ghermite.h.polynomials( 10, 1, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized generalized Hermite polynomials of orders 0 to 10
### polynomial parameter is 1.0
###
unnormalized.p.list <- ghermite.h.polynomials( 10, 1, normalized=FALSE )
print( unnormalized.p.list )
```

`ghermite.h.recurrences` *Recurrence relations for generalized Hermite polynomials*

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k generalized Hermite polynomial, $H_k^{(\mu)}(x)$, and for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
ghermite.h.recurrences(n, mu, normalized = FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`mu`	numeric value for the polynomial parameter
`normalized`	`normalized` boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Details

The parameter μ must be greater than -0.5.

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Alvarez-Nordase, R., M. K. Atakishiyeva and N. M. Atakishiyeva, 2004. A q-extension of the generalized Hermite polynomials with continuous orthogonality property on R, *International Journal of Pure and Applied Mathematics*, 10(3), 335-347.

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`ghermite.h.inner.products`

Examples

```
###
### generate the recurrences data frame for
### the normalized generalized Hermite polynomials
### of orders 0 to 10.
### polynomial parameter value is 1.0
###
normalized.r <- ghermite.h.recurrences( 10, 1, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the unnormalized generalized Hermite polynomials
### of orders 0 to 10.
### polynomial parameter value is 1.0
###
```

```
unnormalized.r <- ghermite.h.recurrences( 10, 1, normalized=FALSE )
print( unnormalized.r )
```

`ghermite.h.weight` *Weight function for the generalized Hermite polynomial*

Description

This function returns the value of the weight function for the order k generalized Hermite polynomial, $H_k^{(\mu)}(x)$.

Usage

```
ghermite.h.weight(x, mu)
```

Arguments

x	a numeric vector function argument
mu	polynomial parameter

Details

The function takes on non-zero values in the interval $(-\infty, \infty)$. The parameter μ must be greater than -0.5. The formula used to compute the generalized Hermite weight function is as follows.
$w(x, \mu) = |x|^{2\,\mu}\ \exp\left(-x^2\right)$

Value

The value of the weight function

Author(s)

Frederick Novomestky < `fnovomes@poly.edu` >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the generalized Hermite weight function for argument values
### between -3 and 3
###
x <- seq( -3, 3, .01 )
y <- ghermite.h.weight( x, 1 )
```

`glaguerre.inner.products` *Inner products of generalized Laguerre polynomials*

Description

This function returns a vector with $n + 1$ elements containing the inner product of an order k generalized Laguerre polynomial, $L_n^{(\alpha)}(x)$, with itself (i.e. the norm squared) for orders $k = 0, 1, \ldots, n$.

Usage

```
glaguerre.inner.products(n,alpha)
```

Arguments

`n`	integer highest polynomial order
`alpha`	polynomial parameter

Details

The formula used to compute the inner products is as follows.

$h_n = \left\langle L_n^{(\alpha)} | L_n^{(\alpha)} \right\rangle = \frac{\Gamma(\alpha+n+1)}{n!}$.

Value

A vector with $n + 1$ elements

`1`	inner product of order 0 orthogonal polynomial
`2`	inner product of order 1 orthogonal polynomial
...	
`n+1`	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### generalized Laguerre polynomial inner products of orders 0 to 10
### polynomial parameter is 1.
###
h <- glaguerre.inner.products( 10, 1 )
print( h )
```

`glaguerre.polynomials` *Create list of generalized Laguerre polynomials*

Description

This function returns a list with $n+1$ elements containing the order n generalized Laguerre polynomials, $L_n^{(\alpha)}(x)$, for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
glaguerre.polynomials(n, alpha, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`alpha`	numeric value for the polynomial parameter
`normalized`	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `glaguerre.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

1	order 0 generalized Laguerre polynomial
2	order 1 generalized Laguerre polynomial
...	
n+1	order n generalized Laguerre polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

glaguerre.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized generalized Laguerre polynomials of orders 0 to 10
### polynomial parameter is 1.0
###
normalized.p.list <- glaguerre.polynomials( 10, 1, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized generalized Laguerre polynomials of orders 0 to 10
### polynomial parameter is 1.0
###
unnormalized.p.list <- glaguerre.polynomials( 10, 1, normalized=FALSE )
```

`glaguerre.recurrences` *Recurrence relations for generalized Laguerre polynomials*

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k generalized Laguerre polynomial, $L_n^{(\alpha)}(x)$ and for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
glaguerre.recurrences(n, alpha, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`alpha`	numeric value for the polynomial parameter
`normalized`	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`glaguerre.inner.products`

Examples

```
###
### generate the recurrences data frame for
### the normalized generalized Laguerre polynomials
### of orders 0 to 10.  the polynomial parameter value is 1.0.
###
normalized.r <- glaguerre.recurrences( 10, 1, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the unnormalized generalized Laguerre polynomials
### of orders 0 to 10.  the polynomial parameter value is 1.0.
###
unnormalized.r <- glaguerre.recurrences( 10, 1, normalized=FALSE )
print( unnormalized.r )
```

`glaguerre.weight` *Weight function for the generalized Laguerre polynomial*

Description

This function returns the value of the weight function for the order k generalized Laguerre polynomial, $L_n^{(\alpha)}(x)$.

Usage

```
glaguerre.weight(x,alpha)
```

Arguments

`x`	the function argument which can be a vector
`alpha`	polynomial parameter

Details

The function takes on non-zero values in the interval $(0, \infty)$. The formula used to compute the weight function is as follows.

$w(x) = e^{-x} x^{\alpha}$

Value

The value of the weight function

Author(s)

Frederick Novomestky < `fnovomes@poly.edu` >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the generalized Laguerre weight function for argument values
### between -3 and 3
### polynomial parameter value is 1.0
###
x <- seq( -3, 3, .01 )
y <- glaguerre.weight( x, 1 )
```

---

hermite.h.inner.products

Inner products of Hermite polynomials

Description

This function returns a vector with $n+1$ elements containing the inner product of an order k Hermite polynomial, $H_k(x)$, with itself (i.e. the norm squared) for orders $k = 0, 1, \ldots, n$.

Usage

```
hermite.h.inner.products(n)
```

Arguments

`n`	integer value for highest polynomial order

Details

The formula used to compute the innner product is as follows.

$h_n = \langle H_n | H_n \rangle = \sqrt{\pi}\, 2^n\, n!$.

Value

A vector with $n + 1$ elements

1	inner product of order 0 orthogonal polynomial
2	inner product of order 1 orthogonal polynomial
...	
n+1	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### Hermite polynomials of orders 0 to 10
###
h <- hermite.h.inner.products( 10 )
print( h )
```

---

hermite.h.polynomials *Create list of Hermite polynomials*

Description

This function returns a list with $n+1$ elements containing the order k Hermite polynomials, $H_k\left(x\right)$, for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
hermite.h.polynomials(n, normalized=FALSE)
```

Arguments

n	integer value for the highest polynomial order
normalized	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `hermite.h.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

1	order 0 Hermite polynomial
2	order 1 Hermite polynomial
...	
n+1	order n Hermite polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`hermite.h.recurrences`, `orthogonal.polynomials`, `orthonormal.polynomials`

Examples

```
###
### gemerate a list of normalized Hermite polynomials of orders 0 to 10
###
normalized.p.list <- hermite.h.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized Hermite polynomials of orders 0 to 10
###
unnormalized.p.list <- hermite.h.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )
```

hermite.h.recurrences *Recurrence relations for Hermite polynomials*

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k Hermite polynomial, $H_k\left(x\right)$, and for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
hermite.h.recurrences(n, normalized=FALSE)
```

Arguments

n	integer value for the highest polynomial order
normalized	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

hermite.h.inner.products,

Examples

```
###
### generate the recurrences data frame for
### the normalized Hermite H polynomials
### of orders 0 to 10.
###
normalized.r <- hermite.h.recurrences( 10, normalized=TRUE )
print( normalized.r )
```

```
###
### generate the recurrences data frame for
### the unnormalized Hermite H polynomials
### of orders 0 to 10.
###
unnormalized.r <- hermite.h.recurrences( 10, normalized=FALSE )
print( unnormalized.r )
```

---

hermite.h.weight *Weight function for the Hermite polynomial*

Description

This function returns the value of the weight function for the order k Hermite polynomial, $H_k\left(x\right)$.

Usage

```
hermite.h.weight(x)
```

Arguments

x	the function argument which can be a vector

Details

The function takes on non-zero values in the interval $\left(-\infty,\infty\right)$. The formula used to compute the weight function.

$w\left(x\right)=\exp\left(-x^2\right)$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the Hermite weight function for argument values
### between -3 and 3
x <- seq( -3, 3, .01 )
y <- hermite.h.weight( x )
plot( x, y )
```

`hermite.he.inner.products` *Inner products of Hermite polynomials*

Description

This function returns a vector with $n+1$ elements containing the inner product of an order k Hermite polynomial, $He_k\left(x\right)$, with itself (i.e. the norm squared) for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
hermite.he.inner.products(n)
```

Arguments

`n`	integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$h_n = \langle He_n | He_n \rangle = \sqrt{2\,\pi}\ n!$.

Value

A vector with $n+1$ elements

`1`	inner product of order 0 orthogonal polynomial
`2`	inner product of order 1 orthogonal polynomial
...	
`n+1`	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### scaled Hermite polynomials of orders 0 to 10
###
h <- hermite.he.inner.products( 10 )
print( h )
```

---

hermite.he.polynomials

Create list of Hermite polynomials

Description

This function returns a list with $n + 1$ elements containing the order k Hermite polynomials, $He_k(x)$, for orders $k = 0, 1, \ldots, n$.

Usage

```
hermite.he.polynomials(n, normalized=FALSE)
```

Arguments

`n`	integer value for thehighest polynomial order
`normalized`	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `hermite.he.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

1	order 0 Hermite polynomial
2	order 1 Hermite polynomial
...	
n+1	order n Hermite polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

hermite.he.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized Hermite polynomials of orders 0 to 10
###
normalized.p.list <- hermite.he.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized Hermite polynomials of orders 0 to 10
###
unnormalized.p.list <- hermite.he.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )
```

---

hermite.he.recurrences   *Recurrence relations for Hermite polynomials*

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k Hermite polynomial, $He_k(x)$, and for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
hermite.he.recurrences(n, normalized=FALSE)
```

Arguments

n	integer value for the highest polynomial order
normalized	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

hermite.he.inner.products

Examples

```
###
### generate the recurrences data frame for
### the normalized Hermite H polynomials
### of orders 0 to 10.
###
normalized.r <- hermite.he.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the unnormalized Hermite H polynomials
### of orders 0 to 10.
###
unnormalized.r <- hermite.he.recurrences( 10, normalized=FALSE )
print( unnormalized.r )
```

`hermite.he.weight` *Weight function for the Hermite polynomial*

Description

This function returns the value of the weight function for the order k Hermite polynomial, $He_k\left(x\right)$.

Usage

```
hermite.he.weight(x)
```

Arguments

x	the function argument which can be a vector

Details

The function takes on non-zero values in the interval $(-\infty, \infty)$. The formula used to compute the weight function is as follows.

$w\left(x\right) = \exp\left(-\frac{x^2}{2}\right)$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the scaled Hermite weight function for argument values
### between -3 and 3
###
x <- seq( -3, 3, .01 )
y <- hermite.he.weight( x )
```

`jacobi.g.inner.products`

Inner products of Jacobi polynomials

Description

This function returns a vector with $n + 1$ elements containing the inner product of an order k Jacobi polynomial, $G_k\,(p, q, x)$, with itself (i.e. the norm squared) for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
jacobi.g.inner.products(n,p,q)
```

Arguments

`n`	integer value for the highest polynomial order
`p`	numeric value for the first polynomial parameter
`q`	numeric value for the first polynomial parameter

Details

The formula used to compute the inner products is as follows.

$h_n = \langle G_n | G_n \rangle = \frac{n!\,\Gamma(n+q)\,\Gamma(n+p)\,\Gamma(n+p-q+1)}{(2\,n+p)\,[\Gamma(2\,n+p)]^2}$.

Value

A vector with $n + 1$ elements

`1`	inner product of order 0 orthogonal polynomial
`2`	inner product of order 1 orthogonal polynomial
...	
`n+1`	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### G Jacobi polynomials of orders 0 to 10
### parameter p is 3 and parameter q is 2
###
h <- jacobi.g.inner.products( 10, 3, 2 )
print( h )
```

`jacobi.g.polynomials` *Create list of Jacobi polynomials*

Description

This function returns a list with $n{+}1$ elements containing the order k Jacobi polynomials, $G_k\left(p, q, x\right)$, for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
jacobi.g.polynomials(n, p, q, normalized=FALSE)
```

Arguments

n	integer value for the highest polynomial order
p	numeic value for the first polynomial parameter
q	numeric value for the second polynomial parameter
normalized	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `jacobi.g.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

1	order 0 Jacobi polynomial
2	order 1 Jacobi polynomial
...	
n+1	order n Jacobi polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

jacobi.g.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized Jacobi G polynomials of orders 0 to 10
### first parameter value p is 3 and second parameter value q is 2
###
normalized.p.list <- jacobi.g.polynomials( 10, 3, 2, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of normalized Jacobi G polynomials of orders 0 to 10
### first parameter value p is 3 and second parameter value q is 2
###
unnormalized.p.list <- jacobi.g.polynomials( 10, 3, 2, normalized=FALSE )
print( unnormalized.p.list )
```

---

jacobi.g.recurrences *Recurrence relations for Jacobi polynomials*

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k Jacobi polynomial, $G_k\left(p, q, x\right)$, and for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
jacobi.g.recurrences(n, p, q, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`p`	numeric value for the first polynomial parameter
`q`	numeric value for the second polynomial parameter
`normalized`	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`jacobi.g.inner.products`, `pochhammer`

Examples

```
###
### generate the recurrences data frame for
### the normalized Jacobi G polynomials
### of orders 0 to 10.
### parameter p is 3 and parameter q is 2
###
normalized.r <- jacobi.g.recurrences( 10, 3, 2, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the normalized Jacobi G polynomials
### of orders 0 to 10.
### parameter p is 3 and parameter q is 2
###
unnormalized.r <- jacobi.g.recurrences( 10, 3, 2, normalized=FALSE )
print( unnormalized.r )
```

`jacobi.g.weight` *Weight function for the Jacobi polynomial*

Description

This function returns the value of the weight function for the order k Jacobi polynomial, $G_k(p, q, x)$.

Usage

```
jacobi.g.weight(x,p,q)
```

Arguments

x	the function argument which can be a vector
p	the first polynomial parameter
q	the second polynomial parameter

Details

The function takes on non-zero values in the interval $(0, 1)$. The formula used to compute the weight function is as follows.

$w(x) = (1 - x)^{p-q}\ x^{q-1}$

Value

The value of the weight function

Author(s)

Frederick Novomestky < `fnovomes@poly.edu` >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the Jacobi G weight function for argument values
### between 0 and 1
### parameter p is 3 and q is 2
###
x <- seq( 0, 1, .01 )
y <- jacobi.g.weight( x, 3, 2 )
```

---

jacobi.matrices — *Create list of Jacobi matrices from monic recurrence parameters*

Description

Return a list of n real symmetric, tri-diagonal matrices which are the principal minors of the $n \times n$ Jacobi matrix derived from the monic recurrence parameters, a and b, for orthogonal polynomials.

Usage

```
jacobi.matrices(r)
```

Arguments

`r`	a data frame containing the parameters a and b

Value

A list of symmetric, tri-diagnonal matrices

`1`	a 1×1 matrix
`2`	a 2×2 matrix

...

`n`	an $n \times n$ matrix

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Examples

```
r <- chebyshev.t.recurrences( 5 )
m.r <- monic.polynomial.recurrences( r )
j.m <- jacobi.matrices( m.r )
```

jacobi.p.inner.products *Inner products of Jacobi polynomials*

Description

This function returns a vector with $n+1$ elements containing the inner product of an order k Jacobi polynomial, $P_k^{(\alpha,\beta)}(x)$, with itself (i.e. the norm squared) for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
jacobi.p.inner.products(n,alpha,beta)
```

Arguments

n	integer value for the highest polynomial order
alpha	numeric value for the first polynomial parameter
beta	numeric value for the first polynomial parameter

Details

The formula used to compute the innser products is as follows.
$h_n = \left\langle P_n^{(\alpha,\beta)} | P_n^{(\alpha,\beta)} \right\rangle = \frac{2^{\alpha+\beta+1}}{2\,n+\alpha+\beta+1} \frac{\Gamma(n+\alpha+1)\,\Gamma(n+\beta+1)}{n!\;\Gamma(n+\alpha+\beta+1)}$.

Value

A vector with $n+1$ elements

1	inner product of order 0 orthogonal polynomial
2	inner product of order 1 orthogonal polynomial
...	
n+1	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner product vector for the P Jacobi polynomials of orders 0 to 10
###
h <- jacobi.p.inner.products( 10, 2, 2 )
print( h )
```

`jacobi.p.polynomials` *Create list of Jacobi polynomials*

Description

This function returns a list with $n+1$ elements containing the order k Jacobi polynomials, $P_k^{(\alpha,\beta)}(x)$, for orders $k = 0, 1, \ldots, n$.

Usage

```
jacobi.p.polynomials(n, alpha, beta, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`alpha`	numeric value for the first polynomial parameter
`beta`	numeric value for the second polynomial parameter
`normalized`	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `jacobi.p.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

`1`	order 0 Jacobi polynomial
`2`	order 1 Jacobi polynomial
...	
`n+1`	order n Chebyshev polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

jacobi.p.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized Jacobi P polynomials of orders 0 to 10
### first parameter value a is 2 and second parameter value b is 2
###
normalized.p.list <- jacobi.p.polynomials( 10, 2, 2, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized Jacobi P polynomials of orders 0 to 10
### first parameter value a is 2 and second parameter value b is 2
###
unnormalized.p.list <- jacobi.p.polynomials( 10, 2, 2, normalized=FALSE )
print( unnormalized.p.list )
```

---

jacobi.p.recurrences *Recurrence relations for Jacobi polynomials*

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k Jacobi polynomial, $P_k^{(\alpha,\beta)}(x)$, and for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
jacobi.p.recurrences(n, alpha, beta, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`alpha`	numeric value for the first polynomial parameter
`beta`	numeric value for the second polynomial parameter
`normalized`	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

jacobi.p.inner.products, pochhammer

Examples

```
###
### generate the recurrences data frame for
### the normalized Jacobi P polynomials
### of orders 0 to 10.
### parameter a is 2 and parameter b is 2
###
normalized.r <- jacobi.p.recurrences( 10, 2, 2, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the unnormalized Jacobi P polynomials
### of orders 0 to 10.
### parameter a is 2 and parameter b is 2
###
unnormalized.r <- jacobi.p.recurrences( 10, 2, 2, normalized=FALSE )
print( unnormalized.r )
```

---

jacobi.p.weight *Weight function for the Jacobi polynomial*

Description

This function returns the value of the weight function for the order k Jacobi polynomial, $P_k^{(\alpha,\beta)}(x)$.

Usage

```
jacobi.p.weight(x,alpha,beta)
```

Arguments

`x`	the function argument which can be a vector
`alpha`	the first polynomial parameter
`beta`	the second polynomial parameter

Details

The function takes on non-zero values in the interval $(-1, 1)$. The formula used to compute the weight function is as follows.

$w(x) = (1-x)^{\alpha}\ (1+x)^{\beta}$

Value

The value of the weight function

Author(s)

Frederick Novomestky < `fnovomes@poly.edu` >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the Jacobi P weight function for argument values
### between -1 and 1
###
x <- seq( -1, 1, .01 )
y <- jacobi.p.weight( x, 2, 2 )
```

`laguerre.inner.products` *Inner products of Laguerre polynomials*

Description

This function returns a vector with $n + 1$ elements containing the inner product of an order k Laguerre polynomial, $L_n(x)$, with itself (i.e. the norm squared) for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
laguerre.inner.products(n)
```

Arguments

`n`	integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$h_n = \langle L_n | L_n \rangle = 1$.

Value

A vector with $n + 1$ elements

`1`	inner product of order 0 orthogonal polynomial
`2`	inner product of order 1 orthogonal polynomial
...	
`n+1`	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### Laguerre polynomial inner products of orders 0 to 10
###
h <- laguerre.inner.products( 10 )
print( h )
```

`laguerre.polynomials` *Create list of Laguerre polynomials*

Description

This function returns a list with $n+1$ elements containing the order k Laguerre polynomials, $L_n(x)$, for orders $k = 0, 1, \ldots, n$.

Usage

```
laguerre.polynomials(n, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`normalized`	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `laguerre.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

`1`	order 0 Laguerre polynomial
`2`	order 1 Laguerre polynomial
...	
`n+1`	order n Laguerre polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

laguerre.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized Laguerre polynomials of orders 0 to 10
###
normalized.p.list <- laguerre.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized Laguerre polynomials of orders 0 to 10
###
unnormalized.p.list <- laguerre.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )
```

`laguerre.recurrences` *Recurrence relations for Laguerre polynomials*

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k Laguerre polynomial, $L_n(x)$, and for orders $k = 0, 1, \ldots, n$.

Usage

```
laguerre.recurrences(n, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`normalized`	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

glaguerre.recurrences

Examples

```
###
### generate the recurrences data frame for
### the normalized Laguerre polynomials
### of orders 0 to 10.
###
normalized.r <- laguerre.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the normalized Laguerre polynomials
### of orders 0 to 10.
###
unnormalized.r <- laguerre.recurrences( 10, normalized=FALSE )
print( unnormalized.r )
```

`laguerre.weight` *Weight function for the Laguerre polynomial*

Description

This function returns the value of the weight function for the order k Laguerre polynomial, $L_n(x)$.

Usage

```
laguerre.weight(x)
```

Arguments

x	the function argument which can be a vector

Details

The function takes on non-zero values in the interval $(0, \infty)$. The formula used to compute the weight function is as follows. $w(x) = e^{-x}$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the Laguerre weight function for argument values
### between 0 and 3
x <- seq( -0, 3, .01 )
y <- laguerre.weight( x )
plot( x, y )
```

`legendre.inner.products` *Inner products of Legendre polynomials*

Description

This function returns a vector with $n + 1$ elements containing the inner product of an order k Legendre polynomial, $P_k(x)$, with itself (i.e. the norm squared) for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
legendre.inner.products(n)
```

Arguments

n	integer value for the highest polynomial order

Details

The formula used compute the inner products is as follows.

$h_n = \langle P_n | P_n \rangle = \frac{2}{2\,n+1}$.

Value

A vector with $n+1$ elements

1	inner product of order 0 orthogonal polynomial
2	inner product of order 1 orthogonal polynomial
...	
n+1	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

spherical.inner.products

Examples

```
###
### compute the inner product for the
###  Legendre polynomials of orders 0 to 1
###
h <- legendre.inner.products( 10 )
print( h )
```

`legendre.polynomials` *Create list of Legendre polynomials*

Description

This function returns a list with $n+1$ elements containing the order k Legendre polynomials, $P_k\left(x\right)$, for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
legendre.polynomials(n, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`normalized`	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `legendre.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

`1`	order 0 Legendre polynomial
`2`	order 1 Legendre polynomial
...	
`n+1`	order n Legendre polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

legendre.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized Laguerre polynomials of orders 0 to 10
###
normalized.p.list <- legendre.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized Laguerre polynomials of orders 0 to 10
###
unnormalized.p.list <- legendre.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )
```

---

legendre.recurrences *Recurrence relations for Legendre polynomials*

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k Legendre polynomial, $P_k(x)$, and for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
legendre.recurrences(n, normalized=FALSE)
```

Arguments

n	integer value for the highest polynomial order
normalized	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

legendre.inner.products

Examples

```
###
### generate the recurrences data frame for
### the normalized Legendre polynomials
### of orders 0 to 10.
###
normalized.r <- legendre.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrences data frame for
### the normalized Legendre polynomials
### of orders 0 to 10.
###
unnormalized.r <- legendre.recurrences( 10, normalized=FALSE )
print( unnormalized.r )
```

---

legendre.weight *Weight function for the Legendre polynomial*

Description

This function returns the value of the weight function for the order k Legendre polynomial, $P_k(x)$.

Usage

```
legendre.weight(x)
```

Arguments

x	the function argument which can be a vector

Details

The function takes on non-zero values in the interval $(-1, 1)$. The formula used to compute the weight function is as follows.

$w(x) = 1$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the Legendre weight function for argument values
### between -1 and 1
###
x <- seq( -1, 1, .01 )
y <- legendre.weight( x )
plot( x, y )
```

`lpochhammer` *Calculate the logarithm of Pochhammer's symbol*

Description

`lpochhammer` returns the value of the natural logarithm of Pochhammer's symbol calculated as

$$\ln\left[(z)_n\right] = \ln\Gamma\left(z+n\right) - \ln\Gamma\left(z\right)$$

where $\Gamma\left(z\right)$ is the Gamma function

Usage

```
lpochhammer(z, n)
```

Arguments

`z`	argument of the symbol
`n`	integer number of terms in the symbol

Value

The value of the logarithm of the symbol

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

See Also

pochhammer

Examples

```
lpochhammer( pi, 5 )
```

---

monic.polynomial.recurrences

Create data frame of monic recurrences

Description

This function returns a data frame with parameters required to construct monic orthogonal polynomials based on the standard recurrence relation for the non-monic polynomials. The recurrence relation for monic orthogonal polynomials is as follows.

$$q_{k+1}(x) = (x - a_k)\ q_k(x) - b_k\ q_{k-1}(x)$$

We require that $q_{-1}(x) = 0$ and $q_0(x) = 1$. The recurrence for non-monic orthogonal polynomials is given by

$$c_k\ p_{k+1}(x) = (d_k + e_k\ x)\ p_k(x) - f_k\ p_{k-1}(x)$$

We require that $p_{-1}(x) = 0$ and $p_0(x) = 1$. The monic polynomial recurrence parameters, **a** and **b**, are related to the non-monic polynomial parameter vectors **c**, **d**, **e** and **f** in the following manner.

$$a_k = -\frac{d_k}{e_k}$$

$$b_k = \frac{c_{k-1}\ f_k}{e_{k-1}\ e_k}$$

with $b_0 = 0$.

Usage

```
monic.polynomial.recurrences(recurrences)
```

Arguments

`recurrences`	the data frame of recurrence parameter vectors **c**, **d**, **e** and **f**

Value

A data frame with $n + 1$ rows and two named columns, **a** and **b**.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`orthogonal.polynomials`,

Examples

```
###
### construct a list of the recurrences for the T Chebyshev polynomials of
### orders 0 to 10
###
r <- chebyshev.t.recurrences( 10, normalized=TRUE )
###
### construct the monic polynomial recurrences from the above list
###
m.r <- monic.polynomial.recurrences( r )
```

`monic.polynomials` *Create list of monic orthogonal polynomials*

Description

This function returns a list with $n+1$ elements containing the order k monic polynomials for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
monic.polynomials(monic.recurrences)
```

Arguments

`monic.recurrences`
: a data frame containing the parameters a and b

Value

A list with $n + 1$ polynomial objects

`1` order 0 monic orthogonal polynomial

`2` order 1 monic orthogonal polynomial

...

`n+1` order n monic orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

See Also

`monic.polynomial.recurrences`

Examples

```
###
### generate the recurrences for the T Chebyshev polynomials
### of orders 0 to 10
###
r <- chebyshev.t.recurrences( 10, normalized=TRUE )
###
### get the corresponding monic polynomial recurrences
###
m.r <- monic.polynomial.recurrences( r )
###
### obtain the list of monic polynomials
###
p.list <- monic.polynomials( m.r )
```

orthogonal.polynomials

Create orthogonal polynomials

Description

Create list of orthogonal polynomials from the following recurrence relations for $k = 0,\ 1,\ \ldots,\ n$.

$$c_k p_{k+1}(x) = (d_k + e_k x)\, p_k(x) - f_k p_{k-1}(x)$$

We require that $p_{-1}(x) = 0$ and $p_0(x) = 1$. The coefficients are the column vectors $\mathbf{c}$, $\mathbf{d}$, $\mathbf{e}$ and $\mathbf{f}$.

Usage

```
orthogonal.polynomials(recurrences)
```

Arguments

`recurrences`	a data frame containing the parameters of the orthogonal polynomial recurrence relations

Details

The argument is a data frame with $n + 1$ rows and four named columns. The column names are `c`, `d`, `e` and `f`. These columns correspond to the column vectors described above.

Value

A list of $n + 1$ polynomial objects

`1`	Order 0 orthogonal polynomial
`2`	Order 1 orthogonal polynomial
...	
`n+1`	Order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the recurrence relations for T Chebyshev polynomials of orders 0 to 10
###
r <- chebyshev.t.recurrences( 10, normalized=FALSE )
print( r )
###
### generate the list of orthogonal polynomials
###
p.list <- orthogonal.polynomials( r )
print( p.list )
```

`orthonormal.polynomials` *Create orthonormal polynomials*

Description

Create list of orthonormal polynomials from the following recurrence relations for $k = 0,\ 1,\ \ldots,\ n$.

$$c_k p_{k+1}(x) = (d_k + e_k x)\, p_k(x) - f_k p_{k-1}(x)$$

We require that $p_{-1}(x) = 0$ and $p_0(x) = 1$. The coefficients are the column vectors $\mathbf{c}$, $\mathbf{d}$, $\mathbf{e}$ and $\mathbf{f}$.

Usage

```
orthonormal.polynomials(recurrences, p.0)
```

Arguments

`recurrences`	a data frame containing the parameters of the orthonormal polynomial recurrence relations
`p.0`	a polynomial object for the order 0 orthonormal polynomial

Details

The argument is a data frame with $n + 1$ rows and four named columns. The column names are `c`, `d`, `e` and `f`. These columns correspond to the column vectors described above.

Value

A list of $n + 1$polynomial objects

`1`	Order 0 orthonormal polynomial
`2`	Order 1 orthonormal polynomial
...	
`n+1`	Order n orthonormal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate a data frame with the recurrences parameters for normalized T Chebyshev
### polynomials of orders 0 to 10
###
r <- chebyshev.t.recurrences( 10, normalized=TRUE )
print( r )
norm <- sqrt( pi )
###
### create the order 0 orthonormal polynomial
###
library("polynom")
p.0 <- polynomial( c( 1 / norm ) )
###
### generate a list of orthonormal polynomial objects
###
p.list <- orthonormal.polynomials( r, p.0 )
print( p.list )
```

`pochhammer` *Calculate the value of Pochhammer's symbol*

Description

`pochhammer` returns the value of Pochhammer's symbol calculated as

$$(z)_n = z\ (z+1)\ \ldots\ (z+n-1) = \frac{\Gamma\left(z+n\right)}{\Gamma\left(z\right)}$$

where $\Gamma\left(z\right)$ is the Gamma function

Usage

```
pochhammer(z, n)
```

Arguments

z	numeric value for the argument of the symbol
n	integer value for the number of terms in the symbol

Value

The value of Pochhammer's symbol

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### compute the Pochhamer's symbol fo z equal to 1 and
### n equal to 5
###
pochhammer( 1, 5 )
```

`polynomial.coefficients`

Create list of polynomial coefficient vectors

Description

This function returns a list with $n+1$ elements containing the vector of coefficients of the order k polynomials for orders $k = 0,\ 1,\ \ldots,\ n$. Each element in the list is a vector.

Usage

```
polynomial.coefficients(polynomials)
```

Arguments

polynomials	list of polynomial objects

Value

A list of $n+1$ polynomial objects where each element is a vector of coefficients.

1	Coefficient(s) of the order 0 polynomial
2	Coefficient(s) of the order 1 polynomial
...	
n+1	Coefficient(s) of the order n polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### generate a list of normalized T Chebyshev polynomials
### of orders 0 to 10
###
p.list <- chebyshev.t.polynomials( 10, normalized=TRUE )
###
### obtain the list of coefficients for these polynomials
###
p.coef <- polynomial.coefficients( p.list )
```

`polynomial.derivatives` *Create list of polynomial derivatives*

Description

This function returns a list with $n+1$ elements containing polynomial objects which are the derivatives of the order k polynomials for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
polynomial.derivatives(polynomials)
```

Arguments

`polynomials`	list of polynomial objects

Details

The polynomial objects in the argument `polynomials` are as follows

- 1order 0 polynomial
- 2order 1 polynomial ...
- n+1order n polynomial

Value

List of $n + 1$ polynomial objects

1	derivative of `polynomials[[1]]`
2	derivative of `polynomials[[2]]`
...	
n+1	derivative of `polynomials[[n+1]]`

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### generate a list of normalized T Chebyshev polynomials of
### orders 0 to 10
###
p.list <- chebyshev.t.polynomials( 10, normalized=TRUE )
###
### generate the corresponding list of polynomial derivatives
###
p.deriv <- polynomial.derivatives( p.list )
```

`polynomial.functions` *Coerce polynomials to functions*

Description

This function returns a list with $n{+}1$ elements containing the functions of the order k polynomials for orders $k = 0,\ 1,\ \ldots,\ n$ and for the given argument x.

Usage

```
polynomial.functions(polynomials, ...)
```

Arguments

`polynomials`	a list of polynomial objects
`...`	further arguments to be passed to or from methods

Details

The function uses the method as.function.polynomial to coerce each polynomial object to a function object.

Value

A list of $n + 1$ polynomial objects where each element is the function for the polynomial.

1	Function for the order 0 polynomial
2	Function for the order 1 polynomial

...

n+1	Function for the order n polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### generate a list of T Chebyshev polynomials of
### orders 0 to 10
###
p.list <- chebyshev.t.polynomials( 10, normalized=FALSE )
###
### create the list of functions for each polynomial
###
f.list <- polynomial.functions( p.list )
```

`polynomial.integrals` *Create list of polynomial integrals*

Description

This function returns a list with $n + 1$ elements containing polynomial objects which are the indefinite integrals of the order k polynomials for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
polynomial.integrals(polynomials)
```

Arguments

`polynomials`	list of polynomial objects

Details

The polynomial objects in the argument `polynomials` are as follows

- 1order 0 polynomial
- 2order 1 polynomial ...
- n+1order n polynomial

Value

List of $n + 1$ polynomial objects

`1`	integral of `polynomials[[1]]`
`2`	integral of `polynomials[[2]]`
...	
`n+1`	integral of `polynomials[[n+1]]`

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### generate a list of normalized T Chebyshev polynomials
### of orders 0 to 10
###
p.list <- chebyshev.t.polynomials( 10, normalized=TRUE )
###
### generate the corresponding list of polynomial integrals
###
p.int <- polynomial.integrals( p.list )
```

`polynomial.orders` *Create vector of polynomial orders*

Description

This function returns a vector with n elements containing the orders of the polynomials

Usage

```
polynomial.orders(polynomials)
```

Arguments

`polynomials`	list of n polynomial objects

Value

A vector of n values

1	Order of polynomials[[1]]
2	Order of polynomials[[2]]

...

n	Order of polynomials[[n]]

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### generate a list of normalized T Chebyshev polynomials
### of orders 0 to 10
###
p.list <- chebyshev.t.polynomials( 10, normalized=TRUE )
###
### get the vector of polynomial orders
###
p.order <- polynomial.orders( p.list )
```

`polynomial.powers` *Create a list of polynomial linear combinations*

Description

This function returns a list with $n+1$ elements containing the vector of linear combinations of the order k polynomials for orders $k = 0,\ 1,\ \ldots,\ n$. Each element in the list is a vector.

Usage

```
polynomial.powers(polynomials)
```

Arguments

`polynomials`	A list of polynomials

Details

The j-th component in the list is a vector of linear combinations of the order k polynomials for orders $k = 0,\ 1,\ \ldots,\ j-1$ equal to the monomial x raised to the power $j-1$.

Value

A list of $n+1$ elements where each element is a vector of linear combinations.

`1`	Linear combination(s) of polynomials up to order 0
`2`	Linear combination(s) of polynomials up to order 1
...	
`n+1`	Linear combination(s) of polynomials up to order n

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### generate Legendre polynomials of orders 0 to 10
###
polynomials <- legendre.polynomials( 10 )
###
### generate list of linear combinations of these polynomials
###
alphas <- polynomial.powers( polynomials )
print( alphas )
```

`polynomial.roots` *Create a list of polynomial roots*

Description

This function returns a list with n elements containing the roots of the order k monic orthogonal polynomials for orders $k = 0,\ 1,\ \ldots,\ n$ using a data frame with the monic polynomial recurrence parameter vectors $\mathbf{a}$ and $\mathbf{b}$

Usage

```
polynomial.roots(m.r)
```

Arguments

`m.r`	monic recurrence data frame with parameters a and b

Details

The parameter `m.r` is a data frame with n+1 rows and two names columns. The columns which are names `a` and `b` correspond to the above referenced vectors. Function `jacobi.matrices` is used to create a list of symmetric, tridiagonal Jacobi matrices from these named columns. The eigenvalues of the $k \times k$ Jacobi matrix are the roots or zeros of the order k monic orthogonal polynomial.

Value

A list with n elements each of which is a vector of polynomial roots

1	roots of the order 1 monic polynomial
2	roots of the order 2 monic polynomial
...	
n	roots of the order n monic polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

monic.polynomial.recurrences, jacobi.matrices

Examples

```
###
### generate the recurrences data frame for
### the normalized Chebyshev polynomials of
### orders 0 to 10
###
r <- chebyshev.t.recurrences( 10, normalized=TRUE )
###
### construct the corresponding monic polynomial
### recurrences
###
m.r <- monic.polynomial.recurrences( r )
###
### obtain the polynomial roots from the monic polynomial
### recurrences
p.roots <- polynomial.roots( m.r )
```

---

polynomial.values *Create vector of polynomial values*

Description

This function returns a list with $n+1$ elements containing the values of the order k polynomials for orders $k = 0,\ 1,\ \ldots,\ n$ and for the given argument x.

Usage

```
polynomial.values( polynomials, x )
```

Arguments

polynomials	list of polynomial objects
x	the argument which can be any numeric object

Value

A list of $n+1$ polynomial objects where each element is the value of the polynomial.

1	Value(s) for the order 0 polynomial
2	Value(s) for the order 1 polynomial
...	
n+1	Value(s) for the order n polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

Examples

```
###
### generate a list of T Chebyshev polynomials of
### orders 0 to 10
###
p.list <- chebyshev.t.polynomials( 10, normalized=FALSE )
x <- seq( -2, 2, .01 )
###
### compute the value of the polynomials for the given range of values in x
###
y <- polynomial.values( p.list, x )
print( y )
```

---

scaleX *Scale values from [a,b] to [u.v]*

Description

This function returns a vector of values that have been mapped from the interval [a,b] to the interval [u.v].

Usage

```
scaleX(x, a = min(x, na.rm = TRUE), b = max(x, na.rm = TRUE), u, v)
```

Arguments

x	A numerical vector of values to be mapped into a target interval
a	A numerical lower bound for the domain interval with min(x) as the default value
b	A numerical upper bound for the domain interval with max(x) as the default value
u	A numerical lower bound for the target interval
v	A numerical upper bound for the target interval

Details

Target lower and/or upper bounds can be $-\infty$ and ∞, respectively. This accomodates finite target intervals, semi-infinite target intervals and infinite target intervals.

Value

A vector of transformed values with four attributes. The first attribute is called "a" and it is the domain interval lower bound. The second attribute is called "b" and it is the domain interval upper bound. The third attribute is called "u" and it is the target interval lower bound. The fourth attribute is called "v" and it is the target interval upper bound.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>, Gregor Gorjanc <gregor.gorjanc@bfro-uni-lj.si>

References

Seber, G. A. F. (1997) Linear Regression Analysis, New York.

Examples

```
x <- rnorm( 1000, 0, 10 )
y0 <- scaleX( x, u=0 , v=1 )
y1 <- scaleX( x, u=-1, v=1 )
y2 <- scaleX( x, u=-Inf, v=0 )
y3 <- scaleX( x, u=0, v=Inf )
y4 <- scaleX( x, u=-Inf, v=Inf )
```

---

schebyshev.t.inner.products

Inner products of shifted Chebyshev polynomials

Description

This function returns a vector with $n + 1$ elements containing the inner product of an order k shifted Chebyshev polynomial of the first kind, $T_k^*(x)$, with itself (i.e. the norm squared) for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
schebyshev.t.inner.products(n)
```

Arguments

`n`	integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$$h_n = \langle T_n^* | T_n^* \rangle = \begin{cases} \frac{\pi}{2} & n \neq 0 \\ \pi & n = 0 \end{cases}$$

Value

A vector with $n + 1$ elements

1	inner product of order 0 orthogonal polynomial
2	inner product of order 1 orthogonal polynomial
...	
n+1	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., NY.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### generate the inner products vector for the
### shifted T Chebyshev polynomials of orders 0 to 10
###
h <- schebyshev.t.inner.products( 10 )
print( h )
```

`schebyshev.t.polynomials` *Create list of shifted Chebyshev polynomials*

Description

This function returns a list with $n + 1$ elements containing the order k shifted Chebyshev polynomials of the first kind, $T_k^*(x)$, for orders $k = 0, 1, \ldots, n$.

Usage

```
schebyshev.t.polynomials(n, normalized)
```

Arguments

`n`	integer value for the highest polynomial order
`normalized`	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `schebyshev.t.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

`1`	order 0 shifted Chebyshev polynomial
`2`	order 1 shifted Chebyshev polynomial

...

`n+1`	order n shifted Chebyshev polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`schebyshev.u.recurrences`, `orthogonal.polynomials`, `orthonormal.polynomials`

Examples

```
###
### gemerate a list of normalized shifted T Chebyshev polynomials of orders 0 to 10
###
normalized.p.list <- schebyshev.t.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized shifted T Chebyshev polynomials of orders 0 to 10
###
unnormalized.p.list <- schebyshev.t.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )
```

---

schebyshev.t.recurrences

Recurrence relations for shifted Chebyshev polynomials

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k shifted Chebyshev polynomial of the first kind, $T_k^*(x)$, and for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
schebyshev.t.recurrences(n, normalized)
```

Arguments

`n`	integer value for the highest polynomial order
`normalized`	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

schebyshev.t.inner.products

Examples

```
###
### generate the recurrence relations for
### the normalized shifted T Chebyshev polynomials
### of orders 0 to 10
###
normalized.r <- schebyshev.t.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for
### the unnormalized shifted T Chebyshev polynomials
### of orders 0 to 10
###
unnormalized.r <- schebyshev.t.recurrences( 10, normalized=FALSE )
print( unnormalized.r )
```

---

schebyshev.t.weight *Weight function for the shifted Chebyshev polynomial*

Description

This function returns the value of the weight function for the order k shifted Chebyshev polynomial of the first kind, $T_k^*\left(x\right)$.

Usage

```
schebyshev.t.weight(x)
```

Arguments

x	the function argument which can be a vector

Details

The function takes on non-zero values in the interval $\left(0,1\right)$. The formula used to compute the weight function is as follows.

$w\left(x\right) = \frac{1}{\sqrt{x - x^2}}$

Value

The value of the weight function

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the shifted T Chebyshev weight function for argument values
### between 0 and 1
x <- seq( 0, 1, .01 )
y <- schebyshev.t.weight( x )
plot( x, y )
```

`schebyshev.u.inner.products`

Inner products of shifted Chebyshev polynomials

Description

This function returns a vector with $n+1$ elements containing the inner product of an order k shifted Chebyshev polynomial of the second kind, $U_k^*(x)$, with itself (i.e. the norm squared) for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
schebyshev.u.inner.products(n)
```

Arguments

n	integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$h_n = \langle U_n^* | U_n^* \rangle = \frac{\pi}{8}$.

Value

A vector with $n+1$ elements

`1`	inner product of order 0 orthogonal polynomial
`2`	inner product of order 1 orthogonal polynomial
...	
`n+1`	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
h <- schebyshev.u.inner.products( 10 )
```

---

schebyshev.u.polynomials

Create list of shifted Chebyshev polynomials

Description

This function returns a list with $n+1$ elements containing the order k shifted Chebyshev polynomials of the second kind, $U_k^*(x)$, for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
schebyshev.u.polynomials(n, normalized)
```

Arguments

`n`	integer value for highest polynomial order
`normalized`	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `schebyshev.u.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

1	order 0 shifted Chebyshev polynomial
2	order 1 shifted Chebyshev polynomial
...	
n+1	order n shifted Chebyshev polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`schebyshev.u.recurrences`, `orthogonal.polynomials`, `orthonormal.polynomials`

Examples

```
###
### gemerate a list of normalized shifted U Chebyshev polynomials of orders 0 to 10
###
normalized.p.list <- schebyshev.u.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized shifted U Chebyshev polynomials of orders 0 to 10
###
unnormalized.p.list <- schebyshev.u.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )
```

schebyshev.u.recurrences *Recurrence relations for shifted Chebyshev polynomials*

Description

This function returns a data frame with $n+1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k shifted Chebyshev polynomial of the second kind, $U_k^*(x)$, and for orders $k = 0, 1, \ldots, n$.

Usage

```
schebyshev.u.recurrences(n, normalized)
```

Arguments

`n`	integer value for the highest polynomial order
`normalized`	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <`fnovomes@poly.edu`>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`schebyshev.u.inner.products`

Examples

```
###
### generate the recurrence relations for
### the normalized shifted U Chebyshev polynomials
### of orders 0 to 10
###
normalized.r <- schebyshev.u.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for
### the unnormalized shifted T Chebyshev polynomials
### of orders 0 to 10
unnormalized.r <- schebyshev.u.recurrences( 10, normalized=FALSE )
print( unnormalized.r )
```

---

schebyshev.u.weight *Weight function for the shifted Chebyshev polynomial*

Description

This function returns the value of the weight function for the order k shifted Chebyshev polynomial of the second kind, $U_k^*(x)$.

Usage

```
schebyshev.u.weight(x)
```

Arguments

x	the function argument which can be a vector

Details

The function takes on non-zero values in the interval $(0, 1)$. The formula used to compute the weight function is as follows.

$w(x) = \sqrt{x - x^2}$

Value

The value of the weight function.

Author(s)

Frederick Novomestky < fnovomes@poly.edu >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the shifted U Chebyshev weight function for argument values
### between 0 and 1
###
x <- seq( 0, 1, .01 )
y <- schebyshev.u.weight( x )
plot( x, y )
```

---

slegendre.inner.products

Inner products of shifted Legendre polynomials

Description

This function returns a vector with $n+1$ elements containing the inner product of an order k shifted Legendre polynomial, $P_k^*\left(x\right)$, with itself (i.e. the norm squared) for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
slegendre.inner.products(n)
```

Arguments

`n`	integer value for the highest polynomial order

Details

The formula used to compute the inner products is as follows.

$h_n = \langle P_n^* | P_n^* \rangle = \frac{1}{2\,n+1}$.

Value

A vector with n+1 elements

1	inner product of order 0 orthogonal polynomial
2	inner product of order 1 orthogonal polynomial
...	
n+1	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the inner products vector for the
### shifted Legendre polynomials of orders 0 to 10
###
h <- slegendre.inner.products( 10 )
print( h )
```

---

slegendre.polynomials *Create list of shifted Legendre polynomials*

Description

This function returns a list with $n+1$ elements containing the order k shifted Legendre polynomials, $P_k^*(x)$, for orders $k = 0, 1, \ldots, n$.

Usage

```
slegendre.polynomials(n, normalized=FALSE)
```

Arguments

n	integer value for the highest polynomial order
normalized	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `slegendre.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

1	order 0 shifted Legendre polynomial
2	order 1 shifted Legendre polynomial
...	
n+1	order n shifted Legendre polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`slegendre.recurrences`, `orthogonal.polynomials`, `orthonormal.polynomials`

Examples

```
###
### gemerate a list of normalized shifted Legendre polynomials of orders 0 to 10
###
normalized.p.list <- slegendre.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized shifted Legendre polynomials of orders 0 to 10
###
unnormalized.p.list <- slegendre.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )
```

slegendre.recurrences *Recurrence relations for shifted Legendre polynomials*

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k shifted Legendre polynomial, $P_k^*(x)$, and for orders $k = 0, 1, \ldots, n$.

Usage

```
slegendre.recurrences(n, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`normalized`	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky `<fnovomes@poly.edu>`

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`slegendre.inner.products`,

Examples

```
###
### generate the recurrence relations for normalized shifted Legendre polynomials
### of orders 0 to 10
###
normalized.r <- slegendre.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for normalized shifted Legendre polynomials
### of orders 0 to 10
###
unnormalized.r <- slegendre.recurrences( 10, normalized=FALSE )
print( unnormalized.r )
```

`slegendre.weight` *Weight function for the shifted Legendre polynomial*

Description

This function returns the value of the weight function for the order k shifted Legendre polynomial, $P_k^*(x)$.

Usage

```
slegendre.weight(x)
```

Arguments

`x`	the function argument which can be a vector

Details

The function takes on non-zero values in the interval $(0, 1)$. The formula used to compute the weight function is as follows.

$w(x) = 1$

Value

The value of the weight function

Author(s)

Frederick Novomestky < `fnovomes@poly.edu` >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the shifted Legendre weight function for argument values
### between 0 and 1
###
x <- seq( 0, 1, .01 )
y <- slegendre.weight( x )
```

---

spherical.inner.products   *Inner products of spherical polynomials*

Description

This function returns a vector with $n + 1$ elements containing the inner product of an order k spherical polynomial, $P_k(x)$, with itself (i.e. the norm squared) for orders $k = 0, 1, \ldots, n$.

Usage

```
spherical.inner.products(n)
```

Arguments

`n`	integer value for the highest polynomial order

Details

The formula used to compute the inner products of the spherical orthogonal polynomials is the same as that used for the Legendre orthogonal polynomials.

Value

A vector with $n + 1$ elements

`1`	inner product of order 0 orthogonal polynomial
`2`	inner product of order 1 orthogonal polynomial
...	
`n+1`	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

legendre.inner.products

Examples

```
###
### generate the inner products vector for the spherical polynomals
### of orders 0 to 10.
###
h <- spherical.inner.products( 10 )
print( h )
```

---

spherical.polynomials *Create list of spherical polynomials*

Description

This function returns a list with $n+1$ elements containing the order k spherical polynomials, $P_k(x)$, for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
spherical.polynomials(n, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`normalized`	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `spherical.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

1	order 0 spherical polynomial
2	order 1 spherical polynomial
...	
n+1	order n Chebyshev polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`spherical.recurrences`, `orthogonal.polynomials`, `orthonormal.polynomials`

Examples

```
###
### generate a list of spherical orthonormal polynomials of orders 0 to 10
###
normalized.p.list <- spherical.polynomials( 10, normalized=TRUE )
print( normalized.p.list )
###
### generate a list of spherical orthogonal polynomials of orders 0 to 10
###
unnormalized.p.list <- spherical.polynomials( 10, normalized=FALSE )
print( unnormalized.p.list )
```

`spherical.recurrences` *Recurrence relations for spherical polynomials*

Description

This function returns a data frame with $n + 1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k spherical polynomial, $P_k(x)$, and for orders $k = 0, 1, \ldots, n$.

Usage

```
spherical.recurrences(n, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`normalized`	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

spherical.inner.products

Examples

```
###
### generate the recurrence relations for
### the normalized spherical polynomials
### of orders 0 to 10
###
normalized.r <- spherical.recurrences( 10, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for
### the unnormalized spherical polynomials
### of orders 0 to 10
###
unnormalized.r <- spherical.recurrences( 10, normalized=FALSE )
print( unnormalized.r )
```

`spherical.weight` *Weight function for the spherical polynomial*

Description

This function returns the value of the weight function for the order k spherical polynomial, $P_k(x)$.

Usage

```
spherical.weight(x)
```

Arguments

`x`	the function argument which can be a vector or matrix

Details

The function takes on non-zero values in the interval $(-1, 1)$. The formula used to compute the weight function is as follows.

$w(x) = 1$

Value

The value of the weight function

Author(s)

Frederick Novomestky < `fnovomes@poly.edu` >

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the spherical weight function for a sequence of values between -2 and 2
###
x <- seq( -2, 2, .01 )
y <- spherical.weight( x )
plot( x, y )
```

ultraspherical.inner.products *Inner products of ultraspherical polynomials*

Description

This function returns a vector with $n+1$ elements containing the inner product of an order k ultraspherical polynomial, $C_k^{(\alpha)}(x)$, with itself (i.e. the norm squared) for orders $k = 0, 1, \ldots, n$.

Usage

```
ultraspherical.inner.products(n,alpha)
```

Arguments

`n`	integer value for the highest polynomial order
`alpha`	numeric value for the polynomial parameter

Details

This function uses the same formula as the function `gegenbauer.inner.products`.

Value

A vector with $n+1$ elements

`1`	inner product of order 0 orthogonal polynomial
`2`	inner product of order 1 orthogonal polynomial
...	
`n+1`	inner product of order n orthogonal polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., NY.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

`gegenbauer.inner.products`

Examples

```
###
### generate the inner products vector for the
### ultraspherical polynomials of orders 0 to 10.
### the polynomial parameter is 1.0
###
h <- ultraspherical.inner.products( 10, 1 )
print( h )
```

`ultraspherical.polynomials`

Create list of ultraspherical polynomials

Description

This function returns a list with $n + 1$ elements containing the order k ultraspherical polynomials, $C_k^{(\alpha)}(x)$, for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
ultraspherical.polynomials(n, alpha, normalized=FALSE)
```

Arguments

`n`	integer value for the highest polynomial order
`alpha`	polynomial parameter
`normalized`	a boolean value which, if TRUE, returns a list of normalized orthogonal polynomials

Details

The function `ultraspherical.recurrences` produces a data frame with the recurrence relation parameters for the polynomials. If the `normalized` argument is FALSE, the function `orthogonal.polynomials` is used to construct the list of orthogonal polynomial objects. Otherwise, the function `orthonormal.polynomials` is used to construct the list of orthonormal polynomial objects.

Value

A list of $n + 1$ polynomial objects

`1`	order 0 ultraspherical polynomial
`2`	order 1 ultraspherical polynomial
...	
`n+1`	order n ultraspherical polynomial

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

gegenbauer.recurrences, orthogonal.polynomials, orthonormal.polynomials

Examples

```
###
### gemerate a list of normalized ultra spherical polynomials
### of orders 0 to 10
###
normalized.p.list <- ultraspherical.polynomials( 10, 1, normalized=TRUE )
print( normalized.p.list )
###
### gemerate a list of unnormalized ultra spherical polynomials
### of orders 0 to 10
###
unnormalized.p.list <- ultraspherical.polynomials( 10, 1, normalized=FALSE )
print( unnormalized.p.list )
```

---

ultraspherical.recurrences

Recurrence relations for ultraspherical polynomials

Description

This function returns a data frame with $n+1$ rows and four named columns containing the coefficient vectors **c**, **d**, **e** and **f** of the recurrence relations for the order k ultraspherical polynomial, $C_k^{(\alpha)}(x)$, and for orders $k = 0,\ 1,\ \ldots,\ n$.

Usage

```
ultraspherical.recurrences(n, alpha, normalized=FALSE)
```

Arguments

n	integer value for the highest polynomial order
alpha	numeric value for the polynomial parameter
normalized	boolean value which, if TRUE, returns recurrence relations for normalized polynomials

Value

A data frame with the recurrence relation parameters.

Author(s)

Frederick Novomestky <fnovomes@poly.edu>

References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

See Also

ultraspherical.recurrences

Examples

```
###
### generate the recurrence relations for
### the normalized ultraspherical polynomials
### of orders 0 to 10
### polynomial parameter value is 1.0
###
normalized.r <- ultraspherical.recurrences( 10, 1, normalized=TRUE )
print( normalized.r )
###
### generate the recurrence relations for
### the normalized ultraspherical polynomials
### of orders 0 to 10
### polynomial parameter value is 1.0
###
unnormalized.r <- ultraspherical.recurrences( 10, 1, normalized=FALSE )
print( unnormalized.r )
```

`ultraspherical.weight` *Weight function for the ultraspherical polynomial*

Description

This function returns the value of the weight function for the order k ultraspherical polynomial, $C_k^{(\alpha)}(x)$.

Usage

```
ultraspherical.weight(x,alpha)
```

Arguments

`x`	the function argument which can be a vector
`alpha`	polynomial parameter

Details

The function takes on non-zero values in the interval $(-1, 1)$. The formula used to compute the weight function is as follows.

$w(x) = \left(1 - x^2\right)^{\alpha - 0.5}$

Value

The value of the weight function

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References

Abramowitz, M. and I. A. Stegun, 1968. *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Dover Publications, Inc., New York.

Courant, R., and D. Hilbert, 1989. *Methods of Mathematical Physics*, John Wiley, New York, NY.

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, 1992. *Numerical Recipes in C*, Cambridge University Press, Cambridge, U.K.

Szego, G., 1939. *Orthogonal Polynomials*, 23, American Mathematical Society Colloquium Publications, Providence, RI.

Examples

```
###
### compute the ultraspherical weight function for arguments between -2 and 2
### polynomial parameter is 1.0
###
x <- seq( -2, 2, .01 )
y <- ultraspherical.weight( x, 1 )
plot( x, y )
```

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