Package 'MixtureFitting'

May 23, 2025

Version 0.6.1

Date 2025-05-20

Title Fitting of Univariate Mixture Distributions to Data using Various Approaches

Depends R (>= 2.0.1)

Description Methods for fitting mixture distributions to univariate data using expectation maximization, HWHM and other methods. Supports Gaussian, Cauchy, Student's t and von Mises mixtures. For more details see Merkys (2018) <a href="https://www.com/doc/ordersed/add/cathods/cat

//www.lvb.lt/permalink/370LABT_NETWORK/1m6ui06/alma9910036312108451>.

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NeedsCompilation yes

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Repository CRAN

Date/Publication 2025-05-23 12:02:06 UTC

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abs_convergence Absolute Convergence Check.

Description

Compare two values to tell whether an optimization process has converged.

Usage

```
abs_convergence( p_now, p_prev, epsilon = 1e-6 )
```

Arguments

p_now	function value of <i>i</i> -th iteration.
p_prev	function value of <i>i</i> -1-th iteration.
epsilon	convergence criterion

Value

TRUE if deemed to have converged, FALSE otherwise

Author(s)

Andrius Merkys

bhattacharyya_dist Bhattacharyya distance for univariate Gaussian distributions.

Description

Measures Bhattacharyya distance for two univariate Gaussian distributions.

Usage

```
bhattacharyya_dist( mu1, mu2, sigma1, sigma2 )
```

Arguments

mu1	mean of the first Gaussian distribution.
mu2	mean of the second Gaussian distribution.
sigma1	standard deviation of the first Gaussian distribution.
sigma2	standard deviation of the second Gaussian distribution.

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Value

Bhattacharyya distance as double.

Author(s)

Andrius Merkys

bic

Bayesian Information Criterion (BIC)

Description

Calculates Bayesian Information Criterion (BIC) for any type of mixture model. Log-likelihood function has to be provided.

Usage

bic(x, p, llf)

Arguments

Х	data vector
р	vector of mixture model parameters
11f	function calculating log-likelihood, called as llf(x,p)

Value

Bayesian Information Criterion value.

Author(s)

Andrius Merkys

cmm_fit_em	Estimate Cauchy Mixture parameters using Expectation Maximiza-	
	tion.	

Description

Estimates parameters for Caucy mixture using Expectation Maximization algorithm.

Usage

Arguments

х	data vector
р	initialization vector of $3*n$ parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, gamma1, gamma2,, gamman), where Ai is the proportion of i-th component, mui is the center of i-th component and gammai is the Cauchy scale of i-th component.$
epsilon	tolerance threshold for convergence. Structure of epsilon is epsilon = $c(epsilon_A, epsilon_mu, epsilon_gamma)$, where epsilon_A is threshold for component proportions, epsilon_mu is threshold for component centers and epsilon_gamma is threshold for component Cauchy scales.
iter.cauchy	number of iterations to fit a single Cauchy component.
debug	flag to turn the debug prints on/off.
implementation	flag to switch between C (default) and R implementations.

Value

Vector of mixture parameters, whose structure is the same as of input parameter's p.

Author(s)

Andrius Merkys

References

Ferenc Nahy. Parameter Estimation of the Cauchy Distribution in Information Theory Approach (2006). Journal of Universal Computer Science

cmm_fit_hwhm_spline_deriv

Estimate Cauchy Mixture Parameters Using Derivatives and Half-Width at Half-Maximum Method.

Description

Estimate Cauchy mixture parameters using derivatives and half-width at half-maximum (HWHM) method. The method smooths the histogram before attempting to locate the modes. Then it describes them using HWHM.

Usage

cmm_fit_hwhm_spline_deriv(x, y)

х	data vector
У	response vector for x

Value

Parameter vector of 3*n parameters, where *n* is number of mixture components. Structure of p vector is p = c(A1, A2, ..., An, mu1, mu2, ..., mun, gamma1, gamma2, ..., gamman), where A*i*is the proportion of*i*-th component, mu*i*is the center of*i*-th component and gamma*i*is the Cauchy scale of*i*-th component.

Author(s)

Andrius Merkys

cmm_init_vector

Estimate Cauchy Mixture parameters using Expectation Maximization.

Description

Estimate an initialization vector for Cauchy mixture fitting via Expectation Maximization. Proportions are set to equal, centers are equispaced through the whole domain of input sample, and scales are set to 1.

Usage

cmm_init_vector(x, m, implementation = "C")

Arguments

х	data vector
m	number of mixture components
implementation	flag to switch between C (default) and R implementations.

Value

Parameter vector of 3*n parameters, where *n* is number of mixture components. Structure of p vector is p = c(A1, A2, ..., An, mu1, mu2, ..., mun, gamma1, gamma2, ..., gamman), where A*i*is the proportion of*i*-th component, mu*i*is the center of*i*-th component and gamma*i*is the Cauchy scale of*i*-th component.

Author(s)

cmm_init_vector_kmeans

Estimate Cauchy Mixture parameters using Expectation Maximization.

Description

Estimate an initialization vector for Cauchy mixture fitting using k-means. R implementation of k-means in kmeans() is used to find data point assignment to clusters. Then several iterations of Cauchy mixture fitting (per Nahy 2006) is used to derive mixture parameters.

Usage

```
cmm_init_vector_kmeans( x, m, iter.cauchy = 20 )
```

Arguments

х	data vector
m	number of mixture components
iter.cauchy	number of iterations to fit a single Cauchy component.

Value

Parameter vector of 3*n parameters, where *n* is number of mixture components. Structure of p vector is p = c(A1, A2, ..., An, mu1, mu2, ..., mun, gamma1, gamma2, ..., gamman), where A*i*is the proportion of*i*-th component, mu*i*is the center of*i*-th component and gamma*i*is the Cauchy scale of*i*-th component.

Author(s)

Andrius Merkys

References

Ferenc Nahy. Parameter Estimation of the Cauchy Distribution in Information Theory Approach (2006). Journal of Universal Computer Science

cmm_intersections Intersection

Description

Finds intersections of two Cauchy distributions by finding roots of a quadratic equation.

Usage

```
cmm_intersections( p )
```

Arguments

```
р
```

parameter vector of 6 parameters. Structure of p vector is p = c(A1, A2, mu1, mu2, gamma1, gamma2), where A*i* is the proportion of *i*-th component, mu*i* is the location of *i*-th component, gamma*i* is the Cauchy scale of *i*-th component.

Value

A vector of x values of intersections (zero, one or two). Returns NaN if both distributions are identical.

Author(s)

Andrius Merkys

dcgmm

Density of The Cauchy-Gaussian Distribution

Description

Density function for the Cauchy-Gaussian distribution, according to Eqn. 2 of Swami (2000).

Usage

dcgmm(x, p)

х	data vector
p	parameter vector of 5^n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, e1, e2,, en, mu1, mu2,, mun, gamma1, gamma2,, gamman, sigma1, sigma2,, sigman), where Ai is the proportion of i-th component, ei is the proportion of Cauchy subcomponent of i-th component, mui is the center of i-th component, gammai is the Cauchy concentration of i-th component and sigmai is the Gaussian standard deviation of i-th component$
	of <i>i</i> -th component.

dcmm

Value

A vector.

Author(s)

Andrius Merkys

References

Swami, A. Non-Gaussian mixture models for detection and estimation in heavy-tailed noise 2000 IEEE International Conference on Acoustics, Speech, and Signal Processing. Proceedings (Cat. No.00CH37100), 2000, 6, 3802-3805

dcmm

Density of The Cauchy Mixture Distribution

Description

Density function for the Cauchy mixture distribution.

Usage

dcmm(x, p, implementation = "C")

Arguments

x	data vector
p	parameter vector of 3^n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, gamma1, gamma2,, gamman), where Ai is the proportion of i-th component, mui is the location of i-th component, gammai is the Cauchy scale of i-th component.$

implementation flag to switch between C (default) and R implementations.

Value

A vector.

Author(s)

Description

Density function for the Gaussian mixture distribution.

Usage

Arguments

x	data vector	
p	parameter vector of 3^n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, sigma1, sigma2,, sigman), where Ai is the proportion of i-th component, mui is the location of i-th component, sigmai is the scale of i-th component.$	
normalise_proportions		
	if TRUE, make component proportions sum up to 1 by dividing each one of them by their sum (R implementation only).	
restrict_sigmas	5	
	if TRUE, skip components with scales less or equal to zero (R implementation only).	
implementation	flag to switch between C (default) and R implementations.	

Value

A vector.

Author(s)

Andrius Merkys

digamma_approx Calculate Approximate Value of The Digamma Function.

Description

Calculates approximate value of the digamma function using first eight non-zero members of asymptotic expression for digamma(x). Implemented according to Wikipedia.

dgmm

Usage

digamma_approx(x)

Arguments

x data vector

Value

Digamma function value.

Author(s)

Andrius Merkys

References

Users of Wikipedia. Digamma function. https://en.wikipedia.org/w/index.php?title= Digamma_function&oldid=708779689

ds

Density of The Student's t Model

Description

Density function for the Student's t Model. Wrapper around R's dt(), supporting center and concentration parameters.

Usage

ds(x, c, s, ni)

Arguments

х	data vector
С	center
S	concentration
ni	degrees of freedom

Value

A vector.

Author(s)

Description

Density function for the Student's t Mixture Model.

Usage

dsmm(x, p)

Arguments

х	data vector
p	parameter vector of 4^*n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,, kn, ni1, ni2,, nin), where Ai is the proportion of i-th component, mui is the center of i-th component, ki is the concentration of i-th component and nii is the degrees of freedom of i-th component.$

Value

A vector.

Author(s)

Andrius Merkys

d∨mm

Density of The von Mises Mixture Model.

Description

Density function for the von Mises Mixture Model.

Usage

dvmm(x, p, implementation = "C")

Arguments

Х	data vector
р	parameter vector of 3^*n parameters, where <i>n</i> is number of mixture components.
	Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,,$
	kn), where Ai is the proportion of i-th component, mui is the center of i-th
	component and ki is the concentration of i-th component.
implementation	flag to switch between C (default) and R implementations.

dsmm

gmm_fit_em

Value

A vector.

Author(s)

Andrius Merkys

gmm_fit_em	Estimate Gaussian Mixture parameters using Expectation Maximiza-
	tion.

Description

Estimates parameters for Gaussian mixture using Expectation Maximization algorithm.

Usage

Arguments

x	data vector
p	initialization vector of 3^n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, sigma1, sigma2,, sigman), where Ai is the proportion of i-th component, mui is the center of i-th component and sigmai is the scale of i-th component.$
W	weights of data points, must have the same length as the data vector; if not given or has different length, equal weights are assumed.
epsilon	tolerance threshold for convergence. Structure of epsilon is epsilon = $c(epsilon_A, epsilon_mu, epsilon_sigma)$, where epsilon_A is threshold for component proportions, epsilon_mu is threshold for component centers and epsilon_sigma is threshold for component scales.
debug	flag to turn the debug prints on/off.
implementation	flag to switch between C (default) and R implementations.
	additional arguments passed to $gmm_fit_em_R()$ when R implementation is used.

Value

Vector of mixture parameters, whose structure is the same as of input parameter's p.

Author(s)

gmm_fit_hwhm

Description

Estimate Gaussian mixture parameters using half-width at half-maximum (HWHM) method. Given a histogram, the method attempts to locate most prominent modes and describe them using HWHM.

Usage

gmm_fit_hwhm(x, y, n)

Arguments

х	data vector
У	response vector for <i>x</i>
n	number of mixture components

Value

Parameter vector of 3*n parameters, where *n* is number of mixture components. Structure of p vector is p = c(A1, A2, ..., An, mu1, mu2, ..., mun, sigma1, sigma2, ..., sigman), where A*i*is the proportion of*i*-th component, mu*i*is the location of*i*-th component, sigma*i*is the scale of*i*-th component.

Author(s)

Andrius Merkys

gmm_fit_hwhm_spline_deriv

Estimate Gaussian Mixture Parameters Using Derivatives and Half-Width at Half-Maximum Method.

Description

Estimate Gaussian mixture parameters using derivatives and half-width at half-maximum (HWHM) method. The method smooths the histogram before attempting to locate the modes. Then it describes them using HWHM.

Usage

gmm_fit_hwhm_spline_deriv(x, y)

Arguments

х	data vector
У	response vector for x

Value

Parameter vector of 3*n parameters, where *n* is number of mixture components. Structure of p vector is p = c(A1, A2, ..., An, mu1, mu2, ..., mun, sigma1, sigma2, ..., sigman), where A*i*is the proportion of*i*-th component, mu*i*is the location of*i*-th component, sigma*i*is the scale of*i*-th component.

Author(s)

Andrius Merkys

gmm_fit_kmeans Estimate Gaussian Mixture parameters from kmeans.

Description

Estimates parameters for Gaussian mixture using kmeans.

Usage

gmm_fit_kmeans(x, n)

Arguments

х	data vector
n	number of mixture components

Value

Vector of 3*n mixture parameters, where *n* is number of mixture components. Structure of p vector is p = c(A1, A2, ..., An, mu1, mu2, ..., mun, sigma1, sigma2, ..., sigman), where A*i* is the proportion of *i*-th component, mu*i* is the location of *i*-th component, sigma*i* is the scale of *i*-th component.

Author(s)

gmm_init_vector

Description

Estimate an initialization vector for Gaussian mixture fitting via Expectation Maximization. Proportions and scales are set to equal, centers are equispaced through the whole domain of input sample.

Usage

```
gmm_init_vector( x, n, implementation = "C" )
```

Arguments

Х	data vector
n	number of mixture components
implementation	flag to switch between C (default) and R implementations.

Value

Parameter vector of 3*n parameters, where *n* is number of mixture components. Structure of p vector is p = c(A1, A2, ..., An, mu1, mu2, ..., mun, sigma1, sigma2, ..., sigman), where A*i*is the proportion of*i*-th component, mu*i*is the location of*i*-th component, sigma*i*is the scale of*i*-th component.

Author(s)

Andrius Merkys

gmm_init_vector_kmeans

Estimate Gaussian Mixture parameters using Expectation Maximization.

Description

Estimate an initialization vector for Gaussian mixture fitting using k-means. R implementation of k-means in kmeans() is used to find data point assignment to clusters.

Usage

```
gmm_init_vector_kmeans( x, m )
```

Arguments

х	data vector
m	number of mixture components

Value

Parameter vector of 3*n parameters, where *n* is number of mixture components. Structure of p vector is p = c(A1, A2, ..., An, mu1, mu2, ..., mun, sigma1, sigma2, ..., sigman), where A*i*is the proportion of*i*-th component, mu*i*is the location of*i*-th component, sigma*i*is the scale of*i*-th component.

Author(s)

Andrius Merkys

gmm_init_vector_quantile

Estimate Gaussian Mixture parameters using Expectation Maximization.

Description

Estimate an initialization vector for Gaussian mixture fitting using (weighted) quantiles. Proportions and scales are set to equal, centers are placed at equispaced quantiles.

Usage

```
gmm_init_vector_quantile( x, m, w = numeric() )
```

Arguments

х	data vector
m	number of mixture components
w	weight vector

Value

Parameter vector of 3*n parameters, where *n* is number of mixture components. Structure of p vector is p = c(A1, A2, ..., An, mu1, mu2, ..., mun, sigma1, sigma2, ..., sigman), where A*i*is the proportion of*i*-th component, mu*i*is the location of*i*-th component, sigma*i*is the scale of*i*-th component.

Author(s)

gmm_intersections Intersections of Two Gaussian Distributions

Description

Finds intersections of two Gaussian distributions by finding roots of a quadratic equation.

Usage

```
gmm_intersections( p )
```

Arguments

р

parameter vector of 6 parameters. Structure of p vector is p = c(A1, A2, mu1, mu2, sigma1, sigma2), where A*i* is the proportion of *i*-th component, mu*i* is the location of *i*-th component, sigma*i* is the scale of *i*-th component.

Value

A vector of x values of intersections (zero, one or two). Returns NaN if both distributions are identical.

Author(s)

Andrius Merkys

gmm_merge_components Merge two Gaussian components into one.

Description

Merges *i*th and *j*th components of Gaussian mixture model. Implemented in the same venue as in mergeparameters of fpc.

Usage

gmm_merge_components(x, p, i, j)

Arguments

х	data vector
р	vector of Gaussian mixture parameters. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,, kn)$, where <i>n</i> is number of mixture components, Ai is the proportion of <i>i</i> -th component, mu <i>i</i> is the center of <i>i</i> -th component, k <i>i</i> is the concentration of <i>i</i> -th component.
i	index of the first component to be merged. Component with this index will be replaced by a merged one in the output.
j	index of the second component to be merged. Component with this index will be removed in the output.

Value

Vector of mixture parameters, whose structure is the same as of input parameter's p.

Author(s)

Andrius Merkys

References

Hennig, C. Methods for merging Gaussian mixture components Advances in Data Analysis and Classification, Springer Nature, 2010, 4, 3-34

gmm_size_probability The Gaussian Mixture Distribution

Description

Calculates the posterior probability of a Gaussian mixture with *n* components. Internally, it attempts to maximize log-likelihood of data by calling optim() and returns the list as received from optim().

Usage

```
gmm_size_probability( x, n, method = "SANN" )
```

Arguments

х	data vector
n	number of mixture components
method	optimization method passed to optim()

Value

List representing the converged optim() run.

Author(s)

Andrius Merkys

gmm_size_probability_nls

The Gaussian Mixture Distribution

Description

Calculates the posterior probability of a Gaussian mixture with n components. Internally, it bins the data vector and calls nls() to optimize the mixture fit. Returns the list of the same form as received from optim().

Usage

```
gmm_size_probability_nls( x, n, bins = 100, trace = FALSE )
```

Arguments

х	data vector
n	number of mixture components
bins	number of bins
trace	should debug trace be printed?

Value

List of the same form as received from optim().

Author(s)

Andrius Merkys

gradient_descent Gradient Descent

Description

Simple implementation of gradient descent method. Given a derivative function, it follows its decrease until convergence criterion is met.

Usage

```
gradient_descent( gradfn, start, gamma = 0.1, ..., epsilon = 0.01 )
```

kldiv

Arguments

gradfn	derivative function
start	starting value
gamma	learning rate
	additional arguments passed to derivative function
epsilon	convergence threshold for absolute squared difference

Value

log-likelihood

Author(s)

Andrius Merkys

kldiv

Kullback–Leibler Divergence of ith Student's t Mixture component.

Description

Measures Kullback–Leibler divergence of *i*th Student's t Mixture component using Dirac's delta function. Implemented according to Chen et al. (2004).

Usage

kldiv(x, p, k)

Arguments

х	data vector
р	vector of Student's t mixture parameters. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,, kn, ni1, ni2,, nin), where n is number of mixture components, Ai is the proportion of i-th component, mui is the center of i-th component, ki is the concentration of i-th component and nii is the degrees of freedom of i-th component.$
k	number of the component.

Value

Kullback-Leibler divergence as double.

Author(s)

References

Chen, S.; Wang, H. & Luo, B. Greedy EM Algorithm for Robust T-Mixture Modeling Third International Conference on Image and Graphics (ICIG'04), Institute of Electrical & Electronics Engineers (IEEE), 2004, 548–551

kmeans_circular K-Means Clustering for Points on Circle

Description

Perform k-means clustering on angular data (in degrees).

Usage

```
kmeans_circular( x, centers, iter.max = 10 )
```

Arguments

х	data vector
centers	vector of initial centers (in degrees)
iter.max	maximum number of iterations

Value

Vector of the same length as *centers* defining cluster centers (in degrees).

Author(s)

Andrius Merkys

llcmm

Log-likelihood for Cauchy Mixture

Description

Calculates log-likelihood for a given data vector using a Cauchy mixture distribution.

Usage

llcmm(x, p, implementation = "C")

llgmm

Arguments

x	data vector
р	parameter vector of $3*n$ parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, gamma1, gamma2,, gamman), where Ai is the proportion of i-th component, mui is the center of i-th component and gammai is the Cauchy scale of i-th component.$
implementation	flag to switch between C (default) and R implementations.

Value

log-likelihood

Author(s)

Andrius Merkys

llgmm

Log-likelihood for Gaussian Mixture

Description

Calculates log-likelihood for a given data vector using a Gaussian mixture distribution.

Usage

llgmm(x, p, implementation = "C")

Arguments

х	data vector
p	parameter vector of 3^n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, sigma1, sigma2,, sigman), where Ai is the proportion of i-th component, mui is the center of i-th component and sigmai is the scale of i-th component.$

implementation flag to switch between C (default) and R implementations.

Value

log-likelihood

Author(s)

llgmm_conservative Log-likelihood for Gaussian Mixture

Description

Calculates log-likelihood for a given data vector using a Gaussian mixture distribution. This is a straightforward implementation, different from llgmm() in that that it does not detect and shortcut edge cases.

Usage

llgmm_conservative(x, p)

Arguments

х	data vector
р	parameter vector of 3^n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, sigma1, sigma2,, sigman), where Ai is the proportion of i-th component, mui is the center of i-th component and sigmai is the scale of i-th component.$

Value

log-likelihood

Author(s)

Andrius Merkys

llgmm_opposite Opposite Log-likelihood for Gaussian Mixture

Description

Calculates opposite log-likelihood for a given data vector using a Gaussian mixture distribution.

Usage

```
llgmm_opposite( x, p )
```

х	data vector
p	parameter vector of 3^n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, sigma1, sigma2,, sigman), where Ai is the proportion of i-th component, mui is the center of i-th component and sigmai is the scale of i-th component.$

11smm

Value

opposite log-likelihood (negated log-likelihood value)

Author(s)

Andrius Merkys

llsmm

Log-likelihood for Student's t Mixture

Description

Calculates log-likelihood for a given data vector using a Student's t mixture distribution.

Usage

llsmm(x, p)

Arguments

х	data vector
p	parameter vector of $4*n$ parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,, kn, ni1, ni2,, nin), where Ai is the proportion of i-th component, mui is the center of i-th component, ki is the concentration of i-th component and nii is the degrees of freedom of i-th component.$

Value

log-likelihood

Author(s)

Description

Calculates log-likelihood for a given data vector using a von Mises mixture distribution.

Usage

llvmm(x, p, implementation = "C")

Arguments

х	data vector
р	parameter vector of 3^*n parameters, where <i>n</i> is number of mixture components.
	Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,,$
	kn), where Ai is the proportion of <i>i</i> -th component, mu <i>i</i> is the center of <i>i</i> -th
	component and ki is the concentration of i-th component.
implementation	flag to switch between C (default) and R implementations.

Value

log-likelihood

Author(s)

Andrius Merkys

11vmm_opposite Opposite Log-likelihood for von Mises Mixture

Description

Calculates opposite log-likelihood for a given data vector using a von Mises mixture distribution.

Usage

```
llvmm_opposite( x, p )
```

х	data vector
p	parameter vector of 3^n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,, kn)$, where A <i>i</i> is the proportion of <i>i</i> -th component, mu <i>i</i> is the center of <i>i</i> -th component and k <i>i</i> is the concentration of <i>i</i> -th component.

mk_fit_images

Value

opposite log-likelihood (negated log-likelihood value)

Author(s)

Andrius Merkys

mk_fit_images Mixture Distribution Modeling

Description

Draw a PNG histogram with a mixture density on top of it for each iteration of mixture optimization process.

Usage

mk_fit_images(h, l, prefix = "img_")

Arguments

h	histogram object, as returned from hist()
1	list containing model vectors
prefix	prefix of file name to write

Author(s)

Andrius Merkys

plot_circular_hist Mixture Distribution Modeling

Description

Plot a circular histogram.

Usage

```
plot_circular_hist( x, breaks = 72, ball = 0.5, ... )
```

х	data vector
breaks	number of breaks in histogram
ball	radius of the drawn circle
	parameters passed to plot()

plot_density

Author(s)

Andrius Merkys

plot_density Mixture Distribution Modeling

Description

Draw a PNG histogram with a mixture density on top of it.

Usage

```
plot_density( x, model, density_f, width, height,
cuts = 400, main = "",
filename = NULL,
obs_good = c(), obs_bad = c(),
scale_density = FALSE )
```

Arguments

х	data vector
cuts	number of breaks in histogram
main	main title of the plot
model	model passed to density_f()
density_f	probability density function
filename	name of the file to write
width	image width, passed to png()
height	image height, passed to png()
obs_good	vector of values to mark with rug() in green color
obs_bad	vector of values to mark with rug() in red color
scale_density	should probability density be scaled?

Author(s)

polyroot_NR

Description

Finds one real polynomial root using Newton-Raphson method, implemented according to Wikipedia.

Usage

```
polyroot_NR( p, init = 0, epsilon = 1e-6, debug = FALSE, implementation = "C" )
```

Arguments

р	vector of polynomial coefficients.
init	initial value.
epsilon	tolerance threshold for convergence.
debug	flag to turn the debug prints on/off.
implementation	flag to switch between C (default) and R implementations.

Value

Real polynomial root.

Author(s)

Andrius Merkys

References

Users of Wikipedia. Newton's method. https://en.wikipedia.org/w/index.php?title=Newton% 27s_method&oldid=710342140

pssd

Penalized Sum of Squared Differences Using Gaussian Mixture Distribution

Description

Given two vectors of same length and a Gaussian mixture, calculate the penalized sum of squared differences (SSD) between the first vector and Gaussian mixture densities measured at points from second vector. Penalties are included for proportions and scales that are less than or equal to 0.

Usage

pssd(x, y, p)

Arguments

х	data vector
У	response vector
p	parameter vector of 3^n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, sigma1, sigma2,, sigman), where Ai is the proportion of i-th component, mui is the location of i-th component, sigmai is the scale of i-th component.$

Value

Penalized sum of squared differences.

Author(s)

Andrius Merkys

pssd_gradient	Penalized Sum of Squared Differences Using Gaussian Mixture Distri- bution

Description

Gradient (derivative) function of pssd().

Usage

```
pssd_gradient( x, y, p )
```

Arguments

х	data vector
У	response vector
p	parameter vector of $3*n$ parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, sigma1, sigma2,, sigman), where Ai is the proportion of i-th component, mui is the location of i-th component, sigmai is the scale of i-th component.$

Value

Gradient values measured at *x*.

Author(s)

ratio_convergence Ratio Convergence Check.

Description

Compare two values to tell whether an optimization process has converged. The absolute difference between values of two iterations is divided by the value of previous iteration and compared to the epsilon value.

Usage

```
ratio_convergence( p_now, p_prev, epsilon = 1e-6 )
```

Arguments

p_now	function value of <i>i</i> -th iteration.
p_prev	function value of <i>i</i> -1-th iteration.
epsilon	convergence criterion

Value

TRUE if deemed to have converged, FALSE otherwise

Author(s)

Andrius Merkys

rcmm

Random Sample of The Cauchy Mixture Distribution

Description

Generates a random sample of the Cauchy mixture distribution.

Usage

rcmm(n, p)

n	sample size
р	parameter vector of 3^n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, gamma1, gamma2,, gamman), where Ai is the proportion of i-th component, mui is the location of i-th component, gammai is the Cauchy scale of i-th component.$

rgmm

Value

A vector.

Author(s)

Andrius Merkys

rgmm

Random Sample of the Gaussian Mixture Distribution

Description

Generates a random sample of the Gaussian mixture distribution.

Usage

rgmm(n, p)

Arguments

n	data vector
р	parameter vector of 3^n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, sigma1, sigma2,, sigman), where Ai is the proportion of i-th component, mui is the location of i-th component, sigmai is the scale of i-th component.$

Value

A vector.

Author(s)

Andrius Merkys

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rsimplex_start

Description

Generate initial simplices for simplex().

Usage

rsimplex_start(seed, n, lower, upper)

Arguments

seed	seed for random number generator
n	number of simplices
lower	vector with lower bounds of each dimension
upper	vector with upper bounds of each dimension

Value

A list with *n* simplices.

Author(s)

Andrius Merkys

rvmm

Random Sample of the von Mises Mixture Model.

Description

Generates a random sample of the von Mises Mixture Model.

Usage

rvmm(n, p)

n	sample size
р	parameter vector of 3^n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,, kn)$, where Ai is the proportion of <i>i</i> -th component, mu <i>i</i> is the center of <i>i</i> -th component and k <i>i</i> is the concentration of <i>i</i> -th component.

simplex

Value

A vector.

Author(s)

Andrius Merkys

References

Best & Fisher. Efficient Simulation of the von Mises Distribution. Journal of the RSS, Series C, 1979, 28, 152-157.

simplex

Nelder-Mead's Simplex Method for Function Minimization.

Description

Nelder-Mead's Simplex Method for Function Minimization.

Usage

simplex(fn, start, ..., epsilon = 0.000001, alpha = 1, gamma = 2, rho = 0.5, delta = 0.5, trace = FALSE)

Arguments

fn	minimized function, has to accept the argmin vector as first parameter
start	start vector
	other parameters passed to the minimized function
epsilon	convergence criterion
alpha	reflection coefficient
gamma	expansion coefficient
rho	contraction coefficient
delta	shrink coefficient
trace	should debug trace be printed?

Value

Vector yielding the minimum value of the minimized function

Author(s)

Andrius Merkys

34

smm_fit_em

References

Nelder, J. A. & Mead, R. A Simplex Method For Function Minimization. The Computer Journal, 1965, 308-313.

Users of Wikipedia. Nelder-Mead method. https://en.wikipedia.org/w/index.php?title= Nelder%E2%80%93Mead_method&oldid=1287347131

smm_fit_em

Estimate Student's t Mixture parameters using Expectation Maximization.

Description

Estimates parameters for Student's t mixture using Expectation Maximization algorithm. Calls smm_fit_em_APK10().

Usage

smm_fit_em(x, p, ...)

Arguments

х	data vector
р	initialization vector of $4*n$ parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,, kn, ni1, ni2,, nin), where Ai is the proportion of i-th component, mui is the center of i-th component, ki is the concentration of i-th component and nii is the degrees of freedom of i-th component.$
	additional arguments passed to smm_fit_em_GNL08().

Value

Vector of mixture parameters, whose structure is the same as of input parameter's p.

Author(s)

smm_fit_em_APK10

Description

Estimates parameters for univariate Student's t mixture using Expectation Maximization algorithm, according to Fig. 2 of Aeschliman et al. (2010).

Usage

Arguments

х	data vector	
þ	initialization vector of $4*n$ parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,, kn, ni1, ni2,, nin), where Ai is the proportion of i-th component, mui is the center of i-th component, ki is the concentration of i-th component and nii is the degrees of freedom of i-th component.$	
epsilon	tolerance threshold for convergence. Structure of epsilon is epsilon = $c(epsilon_A, epsilon_mu, epsilon_k, epsilon_ni)$, where epsilon_A is threshold for component proportions, epsilon_mu is threshold for component centers, epsilon_k is threshold for component concentrations and epsilon_ni is threshold for component degrees of freedom.	
collect.history		
	flag to turn accumulation of estimation history on/off.	
debug	flag to turn the debug prints on/off.	

Value

A list.

Author(s)

Andrius Merkys

References

Aeschliman, C.; Park, J. & Kak, A. C. A Novel Parameter Estimation Algorithm for the Multivariate t-Distribution and Its Application to Computer Vision European Conference on Computer Vision 2010, 2010 https://engineering.purdue.edu/RVL/Publications/Aeschliman2010ANovel.pdf
smm_fit_em_CWL04

Greedily estimate Student's t Mixture parameters using Expectation Maximization.

Description

Estimates (greedily) parameters for univariate Student's t mixture using Expectation Maximization algorithm, implemented according to Chen et al. (2004). The algorithm relies upon smm_fit_em_GNL08() to estimate mixture parameters iteratively.

Usage

Arguments

x	data vector	
р	initialization vector of $4*n$ parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,, kn, ni1, ni2,, nin), where Ai is the proportion of i-th component, mui is the center of i-th component, ki is the concentration of i-th component and nii is the degrees of freedom of i-th component.$	
collect.history		
	logical. If set to TRUE, a list of parameter values of all iterations is returned.	
debug	flag to turn the debug prints on/off.	
	parameters passed to smm_fit_em_GNL08().	

Value

A list.

Author(s)

Andrius Merkys

References

Chen, S.; Wang, H. & Luo, B. Greedy EM Algorithm for Robust T-Mixture Modeling Third International Conference on Image and Graphics (ICIG'04), Institute of Electrical & Electronics Engineers (IEEE), 2004, 548–551 smm_fit_em_GNL08

Description

Estimates parameters for univariate Student's t mixture using Expectation Maximization algorithm, according to Eqns. 12–17 of Gerogiannis et al. (2009).

Usage

Arguments

x data	a vector	
nen , the	tialization vector of $4*n$ parameters, where <i>n</i> is number of mixture compo- nts. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2, kn, ni1, ni2,, nin), where Ai is the proportion of i-th component, mui is center of i-th component, ki is the concentration of i-th component and nii is a degrees of freedom of i-th component.$	
silo con silo	erance threshold for convergence. Structure of epsilon is $epsilon = c(ep-on_A, epsilon_mu, epsilon_k, epsilon_ni), where epsilon_A is threshold formponent proportions, epsilon_mu is threshold for component centers, ep-on_k is threshold for component concentrations and epsilon_ni is thresholdcomponent degrees of freedom.$	
collect.history		
log	cical. If set to TRUE, a list of parameter values of all iterations is returned.	
debug flag	g to turn the debug prints on/off.	
min.sigma mir	nimum value of sigma	
min.ni mir	nimum value of degrees of freedom	
max.df max	ximum value of degrees of freedom	
max.steps max	ximum number of steps, may be infinity	
polyroot.solution		
	lyroot finding method used to approximate digamma function. Possible val- s are 'jenkins_taub' and 'newton_raphson'.	
-	action to use for convergence checking. Must accept function values of the two iterations and return TRUE or FALSE.	
last	t two iterations and return TRUE of FALSE.	

smm_init_vector

Value

A list.

Author(s)

Andrius Merkys

References

Gerogiannis, D.; Nikou, C. & Likas, A. The mixtures of Student's t-distributions as a robust framework for rigid registration. Image and Vision Computing, Elsevier BV, 2009, 27, 1285–1294 https://www.cs.uoi.gr/~arly/papers/imavis09.pdf

Cousineau, D. & Chartier, S. Outliers detection and treatment: a review. International Journal of Psychological Research, 2010, 3, 58–67 https://revistas.usb.edu.co/index.php/IJPR/article/view/844

<pre>smm_init_vector</pre>	Estimate Student's t Mixture parameters using Expectation Maximiza-
	tion.

Description

Estimate an initialization vector for Student's t mixture fitting via Expectation Maximization. Proportions are set to be equal, centers are equispaced through the whole domain of input sample, concentrations and degrees of freedom are set to 1.

Usage

smm_init_vector(x, n)

Arguments

Х	data vector
n	number of mixture components

Value

Parameter vector of 4*n parameters, where *n* is number of mixture components. Structure of p vector is p = c(A1, A2, ..., An, mu1, mu2, ..., mun, k1, k2, ..., kn, ni1, ni2, ..., nin), where A*i*is the proportion of*i*-th component, mu*i*is the center of*i*-th component, k*i*is the concentration of*i*-th component and ni*i*is the degrees of freedom of*i*-th component.

Author(s)

Andrius Merkys

```
smm_init_vector_kmeans
```

Estimate Student's t Mixture parameters using Expectation Maximization.

Description

Estimate an initialization vector for Student's t mixture fitting via Expectation Maximization. R implementation of k-means in kmeans() is used to find data point assignment to clusters. s_fit_primitive() is then used to estimate component parameters for each cluster.

Usage

```
smm_init_vector_kmeans( x, m )
```

Arguments

х	data vector
m	number of mixture components

Value

Parameter vector of 3*n parameters, where *n* is number of mixture components. Structure of p vector is p = c(A1, A2, ..., An, mu1, mu2, ..., mun, sigma1, sigma2, ..., sigman), where A*i*is the proportion of*i*-th component, mu*i*is the location of*i*-th component, sigma*i*is the scale of*i*-th component.

Author(s)

Andrius Merkys

smm_split_component Split a component of Student's t-distribution in two.

Description

Splits a component of Student's t-distribution mixture. Implemented according to Eqns. 30–36 of Chen et al. (2004).

Usage

```
smm_split_component( p, alpha = 0.5, beta = 0.5, u = 0.5 )
```

p	vector of Student's t mixture parameters. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,, kn, ni1, ni2,, nin), where n is number of mixture components, Ai is the proportion of i-th component, mui is the center of i-th component, ki is the concentration of i-th component and nii is the degrees of freedom of i-th component.$
alpha	split proportion for component proportions
beta	split proportion for component concentrations
u	split proportion for component centers

Value

Vector of parameters for resulting two-component mixture, whose structure is the same as of input parameter's p.

Author(s)

Andrius Merkys

References

Chen, S.-B. & Luo, B. Robust t-mixture modelling with SMEM algorithm Proceedings of 2004 International Conference on Machine Learning and Cybernetics (IEEE Cat. No.04EX826), Institute of Electrical & Electronics Engineers (IEEE), 2004, 6, 3689–3694

ssd

Sum of Squared Differences Using Gaussian Mixture Distribution

Description

Given two vectors of same length and a Gaussian mixture, calculate the sum of squared differences (SSD) between the first vector and Gaussian mixture densities measured at points from second vector.

Usage

ssd(x, y, p)

Arguments

x	data vector
У	response vector
р	parameter vector of $3*n$ parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, sigma1, sigma2,, sigman), where Ai is the proportion of i-th component, mui is the location of i-th component, sigmai is the scale of i-th component.$

Value

Sum of squared differences.

Author(s)

Andrius Merkys

ssd_gradient

Sum of Squared Differences Using Gaussian Mixture Distribution

Description

Gradient (derivative) function of ssd().

Usage

ssd_gradient(x, y, p)

Arguments

х	data vector
У	response vector
p	parameter vector of 3^n parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, sigma1, sigma2,, sigman), where Ai is the proportion of i-th component, mui is the location of i-th component, sigmai is the scale of i-th component.$

Value

Gradient values measured at *x*.

Author(s)

Andrius Merkys

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s_fit_primitive

Description

Estimates parameters for univariate Student's t distribution parameters using Batch Approximation Algorithm, according to Fig. 2 of Aeschliman et al. (2010).

Usage

s_fit_primitive(x)

Arguments

x data vector

Value

Vector c(mu, k, ni), where mu is the center, k is the concentration and ni is the degrees of freedom of the distribution.

Author(s)

Andrius Merkys

References

Aeschliman, C.; Park, J. & Kak, A. C. A Novel Parameter Estimation Algorithm for the Multivariate t-Distribution and Its Application to Computer Vision European Conference on Computer Vision 2010, 2010 https://engineering.purdue.edu/RVL/Publications/Aeschliman2010ANovel.pdf

vmm_fit_em

Estimate von Mises Mixture parameters using Expectation Maximization.

Description

Estimates parameters for univariate von Mises mixture using Expectation Maximization algorithm.

Usage

х	data vector
р	initialization vector of $3*n$ parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,, kn)$, where A <i>i</i> is the proportion of <i>i</i> -th component, mu <i>i</i> is the center of <i>i</i> -th component and k <i>i</i> is the concentration of <i>i</i> -th component.
epsilon	tolerance threshold for convergence. Structure of epsilon is epsilon = $c(epsilon_A, epsilon_mu, epsilon_k)$, where epsilon_A is threshold for component proportions, epsilon_mu is threshold for component centers and epsilon_k is threshold for component concentrations.
debug	flag to turn the debug prints on/off.
implementation	flag to switch between C (default) and R implementations.

Value

Vector of mixture parameters, whose structure is the same as of input parameter's p.

Author(s)

Andrius Merkys

References

Banerjee et al. Expectation Maximization for Clustering on Hyperspheres (2003), manuscript, accessible on: https://web.archive.org/web/20130120061240/http://www.lans.ece.utexas.edu/~abanerjee/papers/05/banerjee05a.pdf

vmm_fit_em_by_diff Estimate von Mises Mixture parameters using Expectation Maximization.

Description

Estimates parameters for univariate von Mises mixture using Expectation Maximization algorithm. In this version stopping criterion is the difference between parameters in the subsequent iterations.

Usage

х	data vector
р	initialization vector of $3*n$ parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,, kn)$, where A <i>i</i> is the proportion of <i>i</i> -th component, mu <i>i</i> is the center of <i>i</i> -th component and k <i>i</i> is the concentration of <i>i</i> -th component.
epsilon	tolerance threshold for convergence. Structure of epsilon is epsilon = $c(epsilon_A, epsilon_mu, epsilon_k)$, where epsilon_A is threshold for component proportions, epsilon_mu is threshold for component centers and epsilon_k is threshold for component concentrations.
debug	flag to turn the debug prints on/off.
implementation	flag to switch between C (default) and R implementations.

Value

Vector of mixture parameters, whose structure is the same as of input parameter's p.

Author(s)

Andrius Merkys

References

Banerjee et al. Expectation Maximization for Clustering on Hyperspheres (2003), manuscript, accessible on: https://web.archive.org/web/20130120061240/http://www.lans.ece.utexas.edu/~abanerjee/papers/05/banerjee05a.pdf

vmm_fit_em_by_ll Estimate von Mises Mixture parameters using Expectation Maximization.

Description

Estimates parameters for univariate von Mises mixture using Expectation Maximization algorithm. In this version stopping criterion is the difference between log-likelihood estimates of subsequent iterations.

Usage

x	data vector
p	initialization vector of $3*n$ parameters, where <i>n</i> is number of mixture components. Structure of p vector is $p = c(A1, A2,, An, mu1, mu2,, mun, k1, k2,, kn)$, where A <i>i</i> is the proportion of <i>i</i> -th component, mu <i>i</i> is the center of <i>i</i> -th component and k <i>i</i> is the concentration of <i>i</i> -th component.
epsilon	tolerance threshold for convergence
debug	flag to turn the debug prints on/off.
implementation	flag to switch between C (default) and R implementations.

Value

Vector of mixture parameters, whose structure is the same as of input parameter's p.

Author(s)

Andrius Merkys

References

Banerjee et al. Expectation Maximization for Clustering on Hyperspheres (2003), manuscript, accessible on: https://web.archive.org/web/20130120061240/http://www.lans.ece.utexas.edu/~abanerjee/papers/05/banerjee05a.pdf

vmm_init_vector	Estimate von Mises Mixture parameters using Expectation Maximiza-
	tion.

Description

Estimate an initialization vector for von Mises mixture fitting via Expectation Maximization. Proportions are set to equal, centers are equispaced through the whole domain of input sample, and concentrations are set to $(m/(12*180))^2$.

Usage

```
vmm_init_vector( m, implementation = "C" )
```

Arguments m

number of mixture components

implementation flag to switch between C (default) and R implementations.

wmedian

Value

Parameter vector of 3*n parameters, where *n* is number of mixture components. Structure of p vector is p = c(A1, A2, ..., An, mu1, mu2, ..., mun, k1, k2, ..., kn), where A*i* is the proportion of *i*-th component, mu*i* is the center of *i*-th component and k*i* is the concentration of *i*-th component.

Author(s)

Andrius Merkys

wmedian

Calculate Weighted Median.

Description

Calculated weighted median.

Usage

wmedian(x, w, start = 1, end = length(x))

Arguments

х	sample vector
W	weights vector
start	start index (default: 1)
end	end index (default: last index in <i>x</i>)

Value

Median

Author(s)

Andrius Merkys

References

Users of Wikipedia. Weighted median. https://en.wikipedia.org/w/index.php?title=Weighted_ median&oldid=690896947

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