

Package ‘EvolutionaryGames’

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Type Package

Title Important Concepts of Evolutionary Game Theory

Version 0.1.2

Maintainer Jochen Staudacher <jochen.staudacher@hs-kempten.de>

Description Evolutionary game theory applies game theory to evolving populations in biology, see e.g. one of the books by Weibull (1994, ISBN:978-0262731218) or by Sandholm (2010, ISBN:978-0262195874) for more details. A comprehensive set of tools to illustrate the core concepts of evolutionary game theory, such as evolutionary stability or various evolutionary dynamics, for teaching and academic research is provided.

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Encoding UTF-8

Imports deSolve (>= 1.14), geometry (>= 0.3-6), ggplot2 (>= 2.2.1),
grDevices (>= 3.2.2), interp (>= 1.0-29), MASS (>= 7.3-43),
reshape2 (>= 1.4.2)

Suggests knitr, rmarkdown, rgl

RoxygenNote 7.1.2

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NeedsCompilation no

Author Daniel Gebele [aut, cph],
Jochen Staudacher [aut, cre, cph]

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Contents

BNN	2
BR	3
ESS	4
ESset	5
ILogit	6

Logit	7
MSReplicator	8
phaseDiagram2S	9
phaseDiagram3S	10
phaseDiagram4S	11
Replicator	12
Smith	13
triangle	14

Index	15
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BNN	<i>Brown-von Neumann-Nash dynamic</i>
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Description

Brown-von Neumann-Nash replicator dynamic as a type of evolutionary dynamics.

Usage

```
BNN(time, state, parameters)
```

Arguments

time	Regular sequence that represents the time sequence under which simulation takes place.
state	Numeric vector that represents the initial state.
parameters	Numeric vector that represents parameters needed by the dynamic.

Value

Numeric list. Each component represents the rate of change depending on the dynamic.

Author(s)

Daniel Gebele <dngebele@gmail.com>

References

Brown, G. W. and von Neumann, J. (1950) "Solutions of games by differential equations", In: Kuhn, Harold William and Tucker, Albert William (Eds.) "Contributions to the Theory of Games I", Princeton University Press, pp. 73–79.

Examples

```
dynamic <- BNN
A <- matrix(c(0, -2, 1, 1, 0, -2, -2, 1, 0), 3, byrow=TRUE)
state <- matrix(c(0.4, 0.3, 0.3), 1, 3, byrow=TRUE)
phaseDiagram3S(A, dynamic, NULL, state, FALSE, FALSE)
```

BR	<i>BR dynamic</i>
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Description

Best response dynamic as a type of evolutionary dynamics.

Usage

```
BR(time, state, parameters)
```

Arguments

time	Regular sequence that represents the time sequence under which simulation takes place.
state	Numeric vector that represents the initial state.
parameters	Numeric vector that represents parameters needed by the dynamic.

Value

Numeric list. Each component represents the rate of change depending on the dynamic.

Author(s)

Daniel Gebele <dngebele@gmail.com>

References

Gilboa, I. and Matsui, A. (1991) "Social Stability and Equilibrium", *Econometrica* 59, pp. 859–867.

Examples

```
dynamic <- BR
A <- matrix(c(0, -2, 1, 1, 0, -2, -2, 1, 0), 3, byrow=TRUE)
state <- matrix(c(0.4, 0.3, 0.3), 1, 3, byrow=TRUE)
phaseDiagram3S(A, dynamic, NULL, state, FALSE, FALSE)
```

ESS

ESS for two-player games with a maximum of three strategies

Description

Computes Evolutionary Stable Strategies of a game with two players and a maximum of three strategies.

Usage

```
ESS(A, strategies = c(), floats = TRUE)
```

Arguments

A	Numeric matrix of size 2x2 or 3x3 representing the number of strategies of a symmetric matrix game.
strategies	String vector of length n that names all strategies whereas n represents the number of strategies.
floats	Logical value that handles number representation. If set to TRUE, floating-point arithmetic will be used, otherwise fractions. Default is TRUE.

Value

Numeric matrix. Each row represents an ESS.

Author(s)

Daniel Gebele <dngebele@gmail.com>

References

Smith, J. M. and Price, G. R. (1973) "The logic of animal conflict", Nature 246, pp. 15–18.

Examples

```
ESS(matrix(c(-1, 4, 0, 2), 2, byrow=TRUE), c("Hawk", "Dove"), FALSE)
ESS(matrix(c(1, 2, 0, 0, 1, 2, 2, 0, 1), 3, byrow=TRUE))
```

ESset

Evolutionarily stable set for two-player games with three strategies

Description

Computes evolutionarily stable sets of a game with two players and three strategies.

Usage

```
ESset(A, strategies = c("1", "2", "3"), floats = TRUE)
```

Arguments

A	Numeric matrix of size 3x3 representing the number of strategies of a symmetric matrix game.
strategies	String vector of length 3 that names all strategies.
floats	Logical value that handles number representation. If set to TRUE, floating-point arithmetic will be used, otherwise fractions. Default is TRUE.

Value

Numeric matrix. Each row represents the start and end point of a line (ESset). In addition, a plot of the ESset in the game will be created.

Author(s)

Daniel Gebele <dngebele@gmail.com>

References

Thomas, B. (1985) "On evolutionarily stable sets", Journal of Mathematical Biology 22, pp. 105–115.

Examples

```
# Please note that the computation of evolutionarily stable sets
# is rather time-consuming.
# Depending on your machine you might need to wait more
# than 10 seconds in order to run the following example.
## Not run:
A <- matrix(c(-2, 5, 10/9, 0, 5/2, 10/9, -10/9, 35/9, 10/9), 3, byrow=TRUE)
strategies <- c("Hawk", "Dove", "Mixed ESS")
ESset(A, strategies)

## End(Not run)
```

 ILogit

ILogit dynamic

Description

Imitative Logit dynamic as a type of evolutionary dynamics.

Usage

```
ILogit(time, state, parameters)
```

Arguments

time	Regular sequence that represents the time sequence under which simulation takes place.
state	Numeric vector that represents the initial state.
parameters	Numeric vector that represents parameters needed by the dynamic.

Value

Numeric list. Each component represents the rate of change depending on the dynamic.

Author(s)

Jochen Staudacher <jochen.staudacher@hs-kempten.de>

References

Weibull, J. W. (1997) "Evolutionary Game Theory", MIT Press.

Examples

```
dynamic <- ILogit
A <- matrix(c(-1, 0, 0, 0, -1, 0, 0, 0, -1), 3, byrow=TRUE)
state <- matrix(c(0.1, 0.2, 0.7, 0.2, 0.7, 0.1, 0.9, 0.05, 0.05), 3, 3, byrow=TRUE)
eta <- 0.7
phaseDiagram3S(A, dynamic, eta, state, TRUE, FALSE)
```

Logit	<i>Logit dynamic</i>
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Description

Logit dynamic as a type of evolutionary dynamics.

Usage

```
Logit(time, state, parameters)
```

Arguments

time	Regular sequence that represents the time sequence under which simulation takes place.
state	Numeric vector that represents the initial state.
parameters	Numeric vector that represents parameters needed by the dynamic.

Value

Numeric list. Each component represents the rate of change depending on the dynamic.

Author(s)

Daniel Gebele <dngebele@gmail.com>

References

Fudenberg, D. and Levine, D. K. (1998) "The Theory of Learning in Games", MIT Press.

Examples

```
dynamic <- Logit
A <- matrix(c(0, -2, 1, 1, 0, -2, -2, 1, 0), 3, byrow=TRUE)
state <- matrix(c(0.4, 0.3, 0.3), 1, 3, byrow=TRUE)
eta <- 0.1
phaseDiagram3S(A, dynamic, eta, state, FALSE, FALSE)
```

MSReplicator	<i>Maynard Smith replicator dynamic</i>
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Description

Maynard Smith replicator dynamic as a type of evolutionary dynamics.

Usage

```
MSReplicator(time, state, parameters)
```

Arguments

time	Regular sequence that represents the time sequence under which simulation takes place.
state	Numeric vector that represents the initial state.
parameters	Numeric vector that represents parameters needed by the dynamic.

Value

Numeric list. Each component represents the rate of change depending on the dynamic.

Author(s)

Daniel Gebele <dngebele@gmail.com>

References

Smith, J. M. (1982) "Evolution and the Theory of Games", Cambridge University Press.

Examples

```
dynamic <- MSReplicator
A <- matrix(c(0, -2, 1, 1, 0, -2, -2, 1, 0), 3, byrow=TRUE)
state <- matrix(c(0.4, 0.3, 0.3), 1, 3, byrow=TRUE)
phaseDiagram3S(A, dynamic, NULL, state, FALSE, FALSE)
```

`phaseDiagram2S`*Phase Diagram for two-player games with two strategies*

Description

Plots phase diagram of a game with two players and two strategies.

Usage

```
phaseDiagram2S(  
  A,  
  dynamic,  
  params = NULL,  
  vectorField = TRUE,  
  strategies = c("1", "2")  
)
```

Arguments

<code>A</code>	Numeric matrix of size 2x2 representing the number of strategies of a symmetric matrix game.
<code>dynamic</code>	Function representing an evolutionary dynamic.
<code>params</code>	Numeric vector representing additional parameters for the evolutionary dynamic.
<code>vectorField</code>	Logical value that handles vector field presentation. If set to TRUE, vector field will be shown, otherwise not. Default is TRUE.
<code>strategies</code>	String vector of length 2 that names all strategies.

Value

None.

Author(s)

Daniel Gebele <dngebele@gmail.com>

Examples

```
A <- matrix(c(-1, 4, 0, 2), 2, 2, byrow=TRUE)  
phaseDiagram2S(A, Replicator, strategies = c("Hawk", "Dove"))
```

`phaseDiagram3S`*Phase Diagram for two-player games with three strategies*

Description

Plots phase diagram of a game with two players and three strategies.

Usage

```
phaseDiagram3S(  
  A,  
  dynamic,  
  params = NULL,  
  trajectories = NULL,  
  contour = FALSE,  
  vectorField = FALSE,  
  strategies = c("1", "2", "3")  
)
```

Arguments

<code>A</code>	Numeric matrix of size 3x3 representing the number of strategies of a symmetric matrix game.
<code>dynamic</code>	Function representing an evolutionary dynamic.
<code>params</code>	Numeric vector with additional parameters for the evolutionary dynamic.
<code>trajectories</code>	Numeric matrix of size mx3. Each row represents the initial values for the trajectory to be examined.
<code>contour</code>	Logical value that handles contour diagram presentation. If set to TRUE, contour diagram will be shown, otherwise not. Default is FALSE.
<code>vectorField</code>	Logical value that handles vector field presentation. If set to TRUE, vector field will be shown, otherwise not. Default is FALSE.
<code>strategies</code>	String vector of length 3 that names all strategies.

Value

None.

Author(s)

Daniel Gebele <dngebele@gmail.com>

Examples

```
A <- matrix(c(0, -2, 1, 1, 0, -2, -2, 1, 0), 3, byrow=TRUE)
state <- matrix(c(0.4, 0.3, 0.3), 1, 3, byrow=TRUE)

phaseDiagram3S(A, Replicator, NULL, state, FALSE, FALSE)
phaseDiagram3S(A, Replicator, NULL, state, TRUE, TRUE)

# Plot two trajectories rather than only one:
A <- matrix(c(0, -2, 1, 1, 0, -2, -2, 1, 0), 3, byrow=TRUE)
state <- matrix(c(0.4, 0.3, 0.3, 0.6, 0.2, 0.2), 2, 3, byrow=TRUE)
phaseDiagram3S(A, Replicator, NULL, state, FALSE, FALSE)
```

phaseDiagram4S

Phase Diagram for two-player games with four strategies

Description

Plots phase diagram of a game with two players and four strategies.

Usage

```
phaseDiagram4S(
  A,
  dynamic,
  params = NULL,
  trajectory = NULL,
  strategies = c("1", "2", "3", "4"),
  noRGL = TRUE
)
```

Arguments

A	Numeric matrix of size 4x4 representing the number of strategies of a symmetric matrix game.
dynamic	Function representing an evolutionary dynamic.
params	Numeric vector with additional parameters for the evolutionary dynamic.
trajectory	Numeric vector of size 4 representing the initial value for the trajectory to be examined.
strategies	String vector of length 4 that names all strategies.
noRGL	Logical value that handles diagram rotation. If set to FALSE, diagram will be rotatable, otherwise not. Default is TRUE.

Value

None.

Author(s)

Daniel Gebele <dngebele@gmail.com>

Examples

```
A <- matrix(c(5, -9, 6, 8, 20, 1, 2, -18, -14, 0, 2, 20, 13, 0, 4, -13),
  4, 4, byrow=TRUE)
state <- c(0.3, 0.2, 0.1, 0.4)
phaseDiagram4S(A, Replicator, NULL, state)
```

Replicator

Replicator dynamic

Description

Replicator dynamic as a type of evolutionary dynamics.

Usage

```
Replicator(time, state, parameters)
```

Arguments

time	Regular sequence that represents the time sequence under which simulation takes place.
state	Numeric vector that represents the initial state.
parameters	Numeric vector that represents parameters needed by the dynamic.

Value

Numeric list. Each component represents the rate of change depending on the dynamic.

Author(s)

Daniel Gebele <dngebele@gmail.com>

References

Taylor, P. D. and Jonker, L. B. (1978) "Evolutionary stable strategies and game dynamics", *Mathematical Biosciences* 40 (1-2), pp. 145–156.

Examples

```
dynamic <- Replicator
A <- matrix(c(0, -2, 1, 1, 0, -2, -2, 1, 0), 3, byrow=TRUE)
state <- matrix(c(0.4, 0.3, 0.3), 1, 3, byrow=TRUE)
phaseDiagram3S(A, dynamic, NULL, state, FALSE, FALSE)
```

Smith	<i>Smith dynamic</i>
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Description

Smith dynamic as a type of evolutionary dynamics.

Usage

```
Smith(time, state, parameters)
```

Arguments

time	Regular sequence that represents the time sequence under which simulation takes place.
state	Numeric vector that represents the initial state.
parameters	Numeric vector that represents parameters needed by the dynamic.

Value

Numeric list. Each component represents the rate of change depending on the dynamic.

Author(s)

Daniel Gebele <dngebele@gmail.com>

References

Smith, M. J. (1984) "The Stability of a Dynamic Model of Traffic Assignment – An Application of a Method of Lyapunov", *Transportation Science* 18, pp. 245–252.

Examples

```
dynamic <- Smith
A <- matrix(c(0, -2, 1, 1, 0, -2, -2, 1, 0), 3, byrow=TRUE)
state <- matrix(c(0.4, 0.3, 0.3), 1, 3, byrow=TRUE)
phaseDiagram3S(A, dynamic, NULL, state, FALSE, FALSE)
```

triangle	<i>Triangle for 2-simplex operations</i>
----------	--

Description

Generates a triangle representing the 2-simplex.

Usage

```
triangle(labels = c("1", "2", "3"))
```

Arguments

labels String vector of length 3 that names the edges of the triangle.

Value

List of size 2 with members coords and canvas. coords holds edge coordinates of the 2-simplex, canvas a ggplot2 plot object of the 2-simplex.

Author(s)

Daniel Gebele <dngebele@gmail.com>

Examples

```
triangle()
```

Index

BNN, [2](#)

BR, [3](#)

ESS, [4](#)

ESset, [5](#)

ILogit, [6](#)

Logit, [7](#)

MSReplicator, [8](#)

phaseDiagram2S, [9](#)

phaseDiagram3S, [10](#)

phaseDiagram4S, [11](#)

Replicator, [12](#)

Smith, [13](#)

triangle, [14](#)