



The Parma Polyhedra Library  
Prolog Language Interface  
User's Manual\*  
(version 0.12.1)

Roberto Bagnara<sup>†</sup>  
Patricia M. Hill<sup>‡</sup>  
Enea Zaffanella<sup>§</sup>  
Abramo Bagnara<sup>¶</sup>

April 16, 2012

---

\*This work has been partly supported by: University of Parma's FIL scientific research project (ex 60%) "Pure and Applied Mathematics"; MURST project "Automatic Program Certification by Abstract Interpretation"; MURST project "Abstract Interpretation, Type Systems and Control-Flow Analysis"; MURST project "Automatic Aggregate- and Number-Reasoning for Computing: from Decision Algorithms to Constraint Programming with Multisets, Sets, and Maps"; MURST project "Constraint Based Verification of Reactive Systems"; MURST project "Abstract Interpretation: Design and Applications"; EPSRC project "Numerical Domains for Software Analysis"; EPSRC project "Geometric Abstractions for Scalable Program Analyzers".

<sup>†</sup>bagnara@cs.unipr.it, Department of Mathematics, University of Parma, Italy, and BUGSENG srl.

<sup>‡</sup>patricia.hill@bugseng.com, BUGSENG srl.

<sup>§</sup>zaffanella@cs.unipr.it, Department of Mathematics, University of Parma, Italy, and BUGSENG srl.

<sup>¶</sup>abramo.bagnara@bugseng.com, BUGSENG srl.



Copyright © 2001–2010 Roberto Bagnara (bagnara@cs.unipr.it)  
Copyright © 2010–2012 BUGSENG srl (<http://bugseng.com>)

This document describes the Parma Polyhedra Library (PPL).

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the **Free Software Foundation**; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled “**GNU Free Documentation License**”.

The PPL is free software; you can redistribute it and/or modify it under the terms of the GNU General Public License as published by the **Free Software Foundation**; either version 3 of the License, or (at your option) any later version. A copy of the license is included in the section entitled “**GNU GENERAL PUBLIC LICENSE**”.

The PPL is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the GNU General Public License for more details.

If you have not received a copy of one or both the above mentioned licenses along with the PPL, write to the Free Software Foundation, Inc., 51 Franklin Street, Fifth Floor, Boston, MA 02111-1307, USA.

For the most up-to-date information see the Parma Polyhedra Library site:

<http://bugseng.com/products/ppl/>





## Contents

<b>1</b>	<b>Prolog Language Interface</b>	<b>1</b>
<b>2</b>	<b>GNU General Public License</b>	<b>1</b>
<b>3</b>	<b>GNU Free Documentation License</b>	<b>11</b>
<b>4</b>	<b>System-Independent Features</b>	<b>16</b>
<b>5</b>	<b>Domains Predicates</b>	<b>26</b>
<b>6</b>	<b>Compilation and Installation</b>	<b>39</b>
<b>7</b>	<b>Prolog Interface System-Dependent Features</b>	<b>39</b>
<b>8</b>	<b>Module Index</b>	<b>41</b>
8.1	Modules . . . . .	42
<b>9</b>	<b>Module Documentation</b>	<b>42</b>
9.1	Prolog Language Interface . . . . .	42





## 1 Prolog Language Interface

The Parma Polyhedra Library comes equipped with a Prolog interface. Despite the lack of standardization of Prolog's foreign language interfaces, the PPL Prolog interface supports several Prolog systems and, to the extent this is possible, provides a uniform view of the library from each such system.

The structure of the Prolog interface manual is as follows:

- System-Independent Features
  - [Overview](#)
  - [Predicate Specifications](#)
  - [Domain Independent Predicates](#)
  - [Predicates for MIP\\_Problem](#)
  - [Predicates for PIP\\_Problem](#)
  - [Predicates for C Polyhedra](#)
  - [Ad hoc Predicates for Other Domains](#)
- [Compilation and Installation](#)
- System-Dependent Features
  - [GNU Prolog](#)
  - [CIAO Prolog](#)
  - [SICStus Prolog](#)
  - [SWI Prolog](#)
  - [XSB](#)
  - [YAP](#)

In all the Prolog interface documentation pages, `prefix` is the prefix under which you have installed the library (typically `/usr` or `/usr/local`).

## 2 GNU General Public License

Version 3, 29 June 2007

Copyright (C) 2007 Free Software Foundation, Inc. <http://fsf.org/>

Everyone is permitted to copy and distribute verbatim copies of this license document, but changing it is not allowed.

### Preamble

The GNU General Public License is a free, copyleft license for software and other kinds of works.

The licenses for most software and other practical works are designed to take away your freedom to share and change the works. By contrast, the GNU General Public License is intended to guarantee your freedom to share and change all versions of a program--to make sure it remains free software for all its users. We, the Free Software Foundation, use the GNU General Public License for most of our software; it applies also to any other work released this way by its authors. You can apply it to your programs, too.



When we speak of free software, we are referring to freedom, not price. Our General Public Licenses are designed to make sure that you have the freedom to distribute copies of free software (and charge for them if you wish), that you receive source code or can get it if you want it, that you can change the software or use pieces of it in new free programs, and that you know you can do these things.

To protect your rights, we need to prevent others from denying you these rights or asking you to surrender the rights. Therefore, you have certain responsibilities if you distribute copies of the software, or if you modify it: responsibilities to respect the freedom of others.

For example, if you distribute copies of such a program, whether gratis or for a fee, you must pass on to the recipients the same freedoms that you received. You must make sure that they, too, receive or can get the source code. And you must show them these terms so they know their rights.

Developers that use the GNU GPL protect your rights with two steps: (1) assert copyright on the software, and (2) offer you this License giving you legal permission to copy, distribute and/or modify it.

For the developers' and authors' protection, the GPL clearly explains that there is no warranty for this free software. For both users' and authors' sake, the GPL requires that modified versions be marked as changed, so that their problems will not be attributed erroneously to authors of previous versions.

Some devices are designed to deny users access to install or run modified versions of the software inside them, although the manufacturer can do so. This is fundamentally incompatible with the aim of protecting users' freedom to change the software. The systematic pattern of such abuse occurs in the area of products for individuals to use, which is precisely where it is most unacceptable. Therefore, we have designed this version of the GPL to prohibit the practice for those products. If such problems arise substantially in other domains, we stand ready to extend this provision to those domains in future versions of the GPL, as needed to protect the freedom of users.

Finally, every program is threatened constantly by software patents. States should not allow patents to restrict development and use of software on general-purpose computers, but in those that do, we wish to avoid the special danger that patents applied to a free program could make it effectively proprietary. To prevent this, the GPL assures that patents cannot be used to render the program non-free.

The precise terms and conditions for copying, distribution and modification follow.

## TERMS AND CONDITIONS

### 0. Definitions.

“This License” refers to version 3 of the GNU General Public License.

“Copyright” also means copyright-like laws that apply to other kinds of works, such as semiconductor masks.

“The Program” refers to any copyrightable work licensed under this License. Each licensee is addressed as “you”. “Licensees” and “recipients” may be individuals or organizations.

To “modify” a work means to copy from or adapt all or part of the work in a fashion requiring copyright permission, other than the making of an exact copy. The resulting work is called a “modified version” of the earlier work or a work “based on” the earlier work.

A “covered work” means either the unmodified Program or a work based on the Program.

To “propagate” a work means to do anything with it that, without permission, would make you directly or secondarily liable for infringement under applicable copyright law, except executing it on a computer or modifying a private copy. Propagation includes copying, distribution (with or without modification), making available to the public, and in some countries other activities as well.

To “convey” a work means any kind of propagation that enables other parties to make or receive copies.





Mere interaction with a user through a computer network, with no transfer of a copy, is not conveying.

An interactive user interface displays “Appropriate Legal Notices” to the extent that it includes a convenient and prominently visible feature that (1) displays an appropriate copyright notice, and (2) tells the user that there is no warranty for the work (except to the extent that warranties are provided), that licensees may convey the work under this License, and how to view a copy of this License. If the interface presents a list of user commands or options, such as a menu, a prominent item in the list meets this criterion.

## 1. Source Code.

The “source code” for a work means the preferred form of the work for making modifications to it. “Object code” means any non-source form of a work.

A “Standard Interface” means an interface that either is an official standard defined by a recognized standards body, or, in the case of interfaces specified for a particular programming language, one that is widely used among developers working in that language.

The “System Libraries” of an executable work include anything, other than the work as a whole, that (a) is included in the normal form of packaging a Major Component, but which is not part of that Major Component, and (b) serves only to enable use of the work with that Major Component, or to implement a Standard Interface for which an implementation is available to the public in source code form. A “Major Component”, in this context, means a major essential component (kernel, window system, and so on) of the specific operating system (if any) on which the executable work runs, or a compiler used to produce the work, or an object code interpreter used to run it.

The “Corresponding Source” for a work in object code form means all the source code needed to generate, install, and (for an executable work) run the object code and to modify the work, including scripts to control those activities. However, it does not include the work’s System Libraries, or general-purpose tools or generally available free programs which are used unmodified in performing those activities but which are not part of the work. For example, Corresponding Source includes interface definition files associated with source files for the work, and the source code for shared libraries and dynamically linked subprograms that the work is specifically designed to require, such as by intimate data communication or control flow between those subprograms and other parts of the work.

The Corresponding Source need not include anything that users can regenerate automatically from other parts of the Corresponding Source.

The Corresponding Source for a work in source code form is that same work.

## 2. Basic Permissions.

All rights granted under this License are granted for the term of copyright on the Program, and are irrevocable provided the stated conditions are met. This License explicitly affirms your unlimited permission to run the unmodified Program. The output from running a covered work is covered by this License only if the output, given its content, constitutes a covered work. This License acknowledges your rights of fair use or other equivalent, as provided by copyright law.

You may make, run and propagate covered works that you do not convey, without conditions so long as your license otherwise remains in force. You may convey covered works to others for the sole purpose of having them make modifications exclusively for you, or provide you with facilities for running those works, provided that you comply with the terms of this License in conveying all material for which you do not control copyright. Those thus making or running the covered works for you must do so exclusively on your behalf, under your direction and control, on terms that prohibit them from making any copies of your copyrighted material outside their relationship with you.

Conveying under any other circumstances is permitted solely under the conditions stated below. Sublicens-



ing is not allowed; section 10 makes it unnecessary.

### **3. Protecting Users' Legal Rights From Anti-Circumvention Law.**

No covered work shall be deemed part of an effective technological measure under any applicable law fulfilling obligations under article 11 of the WIPO copyright treaty adopted on 20 December 1996, or similar laws prohibiting or restricting circumvention of such measures.

When you convey a covered work, you waive any legal power to forbid circumvention of technological measures to the extent such circumvention is effected by exercising rights under this License with respect to the covered work, and you disclaim any intention to limit operation or modification of the work as a means of enforcing, against the work's users, your or third parties' legal rights to forbid circumvention of technological measures.

### **4. Conveying Verbatim Copies.**

You may convey verbatim copies of the Program's source code as you receive it, in any medium, provided that you conspicuously and appropriately publish on each copy an appropriate copyright notice; keep intact all notices stating that this License and any non-permissive terms added in accord with section 7 apply to the code; keep intact all notices of the absence of any warranty; and give all recipients a copy of this License along with the Program.

You may charge any price or no price for each copy that you convey, and you may offer support or warranty protection for a fee.

### **5. Conveying Modified Source Versions.**

You may convey a work based on the Program, or the modifications to produce it from the Program, in the form of source code under the terms of section 4, provided that you also meet all of these conditions:

- a) The work must carry prominent notices stating that you modified it, and giving a relevant date.
- b) The work must carry prominent notices stating that it is released under this License and any conditions added under section 7. This requirement modifies the requirement in section 4 to "keep intact all notices".
- c) You must license the entire work, as a whole, under this License to anyone who comes into possession of a copy. This License will therefore apply, along with any applicable section 7 additional terms, to the whole of the work, and all its parts, regardless of how they are packaged. This License gives no permission to license the work in any other way, but it does not invalidate such permission if you have separately received it.
- d) If the work has interactive user interfaces, each must display Appropriate Legal Notices; however, if the Program has interactive interfaces that do not display Appropriate Legal Notices, your work need not make them do so.

A compilation of a covered work with other separate and independent works, which are not by their nature extensions of the covered work, and which are not combined with it such as to form a larger program, in or on a volume of a storage or distribution medium, is called an "aggregate" if the compilation and its resulting copyright are not used to limit the access or legal rights of the compilation's users beyond what the individual works permit. Inclusion of a covered work in an aggregate does not cause this License to apply to the other parts of the aggregate.



## 6. Conveying Non-Source Forms.

You may convey a covered work in object code form under the terms of sections 4 and 5, provided that you also convey the machine-readable Corresponding Source under the terms of this License, in one of these ways:

- a) Convey the object code in, or embodied in, a physical product (including a physical distribution medium), accompanied by the Corresponding Source fixed on a durable physical medium customarily used for software interchange.
- b) Convey the object code in, or embodied in, a physical product (including a physical distribution medium), accompanied by a written offer, valid for at least three years and valid for as long as you offer spare parts or customer support for that product model, to give anyone who possesses the object code either (1) a copy of the Corresponding Source for all the software in the product that is covered by this License, on a durable physical medium customarily used for software interchange, for a price no more than your reasonable cost of physically performing this conveying of source, or (2) access to copy the Corresponding Source from a network server at no charge.
- c) Convey individual copies of the object code with a copy of the written offer to provide the Corresponding Source. This alternative is allowed only occasionally and noncommercially, and only if you received the object code with such an offer, in accord with subsection 6b.
- d) Convey the object code by offering access from a designated place (gratis or for a charge), and offer equivalent access to the Corresponding Source in the same way through the same place at no further charge. You need not require recipients to copy the Corresponding Source along with the object code. If the place to copy the object code is a network server, the Corresponding Source may be on a different server (operated by you or a third party) that supports equivalent copying facilities, provided you maintain clear directions next to the object code saying where to find the Corresponding Source. Regardless of what server hosts the Corresponding Source, you remain obligated to ensure that it is available for as long as needed to satisfy these requirements.
- e) Convey the object code using peer-to-peer transmission, provided you inform other peers where the object code and Corresponding Source of the work are being offered to the general public at no charge under subsection 6d.

A separable portion of the object code, whose source code is excluded from the Corresponding Source as a System Library, need not be included in conveying the object code work.

A “User Product” is either (1) a “consumer product”, which means any tangible personal property which is normally used for personal, family, or household purposes, or (2) anything designed or sold for incorporation into a dwelling. In determining whether a product is a consumer product, doubtful cases shall be resolved in favor of coverage. For a particular product received by a particular user, “normally used” refers to a typical or common use of that class of product, regardless of the status of the particular user or of the way in which the particular user actually uses, or expects or is expected to use, the product. A product is a consumer product regardless of whether the product has substantial commercial, industrial or non-consumer uses, unless such uses represent the only significant mode of use of the product.

“Installation Information” for a User Product means any methods, procedures, authorization keys, or other information required to install and execute modified versions of a covered work in that User Product from a modified version of its Corresponding Source. The information must suffice to ensure that the continued functioning of the modified object code is in no case prevented or interfered with solely because modification has been made.

If you convey an object code work under this section in, or with, or specifically for use in, a User Product, and the conveying occurs as part of a transaction in which the right of possession and use of the User Product is transferred to the recipient in perpetuity or for a fixed term (regardless of how the transaction



is characterized), the Corresponding Source conveyed under this section must be accompanied by the Installation Information. But this requirement does not apply if neither you nor any third party retains the ability to install modified object code on the User Product (for example, the work has been installed in ROM).

The requirement to provide Installation Information does not include a requirement to continue to provide support service, warranty, or updates for a work that has been modified or installed by the recipient, or for the User Product in which it has been modified or installed. Access to a network may be denied when the modification itself materially and adversely affects the operation of the network or violates the rules and protocols for communication across the network.

Corresponding Source conveyed, and Installation Information provided, in accord with this section must be in a format that is publicly documented (and with an implementation available to the public in source code form), and must require no special password or key for unpacking, reading or copying.

## 7. Additional Terms.

“Additional permissions” are terms that supplement the terms of this License by making exceptions from one or more of its conditions. Additional permissions that are applicable to the entire Program shall be treated as though they were included in this License, to the extent that they are valid under applicable law. If additional permissions apply only to part of the Program, that part may be used separately under those permissions, but the entire Program remains governed by this License without regard to the additional permissions.

When you convey a copy of a covered work, you may at your option remove any additional permissions from that copy, or from any part of it. (Additional permissions may be written to require their own removal in certain cases when you modify the work.) You may place additional permissions on material, added by you to a covered work, for which you have or can give appropriate copyright permission.

Notwithstanding any other provision of this License, for material you add to a covered work, you may (if authorized by the copyright holders of that material) supplement the terms of this License with terms:

- a) Disclaiming warranty or limiting liability differently from the terms of sections 15 and 16 of this License; or
- b) Requiring preservation of specified reasonable legal notices or author attributions in that material or in the Appropriate Legal Notices displayed by works containing it; or
- c) Prohibiting misrepresentation of the origin of that material, or requiring that modified versions of such material be marked in reasonable ways as different from the original version; or
- d) Limiting the use for publicity purposes of names of licensors or authors of the material; or
- e) Declining to grant rights under trademark law for use of some trade names, trademarks, or service marks; or
- f) Requiring indemnification of licensors and authors of that material by anyone who conveys the material (or modified versions of it) with contractual assumptions of liability to the recipient, for any liability that these contractual assumptions directly impose on those licensors and authors.

All other non-permissive additional terms are considered “further restrictions” within the meaning of section 10. If the Program as you received it, or any part of it, contains a notice stating that it is governed by this License along with a term that is a further restriction, you may remove that term. If a license document contains a further restriction but permits relicensing or conveying under this License, you may add to a covered work material governed by the terms of that license document, provided that the further restriction does not survive such relicensing or conveying.



If you add terms to a covered work in accord with this section, you must place, in the relevant source files, a statement of the additional terms that apply to those files, or a notice indicating where to find the applicable terms.

Additional terms, permissive or non-permissive, may be stated in the form of a separately written license, or stated as exceptions; the above requirements apply either way.

## 8. Termination.

You may not propagate or modify a covered work except as expressly provided under this License. Any attempt otherwise to propagate or modify it is void, and will automatically terminate your rights under this License (including any patent licenses granted under the third paragraph of section 11).

However, if you cease all violation of this License, then your license from a particular copyright holder is reinstated (a) provisionally, unless and until the copyright holder explicitly and finally terminates your license, and (b) permanently, if the copyright holder fails to notify you of the violation by some reasonable means prior to 60 days after the cessation.

Moreover, your license from a particular copyright holder is reinstated permanently if the copyright holder notifies you of the violation by some reasonable means, this is the first time you have received notice of violation of this License (for any work) from that copyright holder, and you cure the violation prior to 30 days after your receipt of the notice.

Termination of your rights under this section does not terminate the licenses of parties who have received copies or rights from you under this License. If your rights have been terminated and not permanently reinstated, you do not qualify to receive new licenses for the same material under section 10.

## 9. Acceptance Not Required for Having Copies.

You are not required to accept this License in order to receive or run a copy of the Program. Ancillary propagation of a covered work occurring solely as a consequence of using peer-to-peer transmission to receive a copy likewise does not require acceptance. However, nothing other than this License grants you permission to propagate or modify any covered work. These actions infringe copyright if you do not accept this License. Therefore, by modifying or propagating a covered work, you indicate your acceptance of this License to do so.

## 10. Automatic Licensing of Downstream Recipients.

Each time you convey a covered work, the recipient automatically receives a license from the original licensors, to run, modify and propagate that work, subject to this License. You are not responsible for enforcing compliance by third parties with this License.

An “entity transaction” is a transaction transferring control of an organization, or substantially all assets of one, or subdividing an organization, or merging organizations. If propagation of a covered work results from an entity transaction, each party to that transaction who receives a copy of the work also receives whatever licenses to the work the party’s predecessor in interest had or could give under the previous paragraph, plus a right to possession of the Corresponding Source of the work from the predecessor in interest, if the predecessor has it or can get it with reasonable efforts.

You may not impose any further restrictions on the exercise of the rights granted or affirmed under this License. For example, you may not impose a license fee, royalty, or other charge for exercise of rights granted under this License, and you may not initiate litigation (including a cross-claim or counterclaim in a lawsuit) alleging that any patent claim is infringed by making, using, selling, offering for sale, or importing the Program or any portion of it.



## 11. Patents.

A “contributor” is a copyright holder who authorizes use under this License of the Program or a work on which the Program is based. The work thus licensed is called the contributor’s “contributor version”.

A contributor’s “essential patent claims” are all patent claims owned or controlled by the contributor, whether already acquired or hereafter acquired, that would be infringed by some manner, permitted by this License, of making, using, or selling its contributor version, but do not include claims that would be infringed only as a consequence of further modification of the contributor version. For purposes of this definition, “control” includes the right to grant patent sublicenses in a manner consistent with the requirements of this License.

Each contributor grants you a non-exclusive, worldwide, royalty-free patent license under the contributor’s essential patent claims, to make, use, sell, offer for sale, import and otherwise run, modify and propagate the contents of its contributor version.

In the following three paragraphs, a “patent license” is any express agreement or commitment, however denominated, not to enforce a patent (such as an express permission to practice a patent or covenant not to sue for patent infringement). To “grant” such a patent license to a party means to make such an agreement or commitment not to enforce a patent against the party.

If you convey a covered work, knowingly relying on a patent license, and the Corresponding Source of the work is not available for anyone to copy, free of charge and under the terms of this License, through a publicly available network server or other readily accessible means, then you must either (1) cause the Corresponding Source to be so available, or (2) arrange to deprive yourself of the benefit of the patent license for this particular work, or (3) arrange, in a manner consistent with the requirements of this License, to extend the patent license to downstream recipients. “Knowingly relying” means you have actual knowledge that, but for the patent license, your conveying the covered work in a country, or your recipient’s use of the covered work in a country, would infringe one or more identifiable patents in that country that you have reason to believe are valid.

If, pursuant to or in connection with a single transaction or arrangement, you convey, or propagate by procuring conveyance of, a covered work, and grant a patent license to some of the parties receiving the covered work authorizing them to use, propagate, modify or convey a specific copy of the covered work, then the patent license you grant is automatically extended to all recipients of the covered work and works based on it.

A patent license is “discriminatory” if it does not include within the scope of its coverage, prohibits the exercise of, or is conditioned on the non-exercise of one or more of the rights that are specifically granted under this License. You may not convey a covered work if you are a party to an arrangement with a third party that is in the business of distributing software, under which you make payment to the third party based on the extent of your activity of conveying the work, and under which the third party grants, to any of the parties who would receive the covered work from you, a discriminatory patent license (a) in connection with copies of the covered work conveyed by you (or copies made from those copies), or (b) primarily for and in connection with specific products or compilations that contain the covered work, unless you entered into that arrangement, or that patent license was granted, prior to 28 March 2007.

Nothing in this License shall be construed as excluding or limiting any implied license or other defenses to infringement that may otherwise be available to you under applicable patent law.

## 12. No Surrender of Others’ Freedom.

If conditions are imposed on you (whether by court order, agreement or otherwise) that contradict the conditions of this License, they do not excuse you from the conditions of this License. If you cannot convey a covered work so as to satisfy simultaneously your obligations under this License and any other pertinent obligations, then as a consequence you may not convey it at all. For example, if you agree to terms





that obligate you to collect a royalty for further conveying from those to whom you convey the Program, the only way you could satisfy both those terms and this License would be to refrain entirely from conveying the Program.

### 13. Use with the GNU Affero General Public License.

Notwithstanding any other provision of this License, you have permission to link or combine any covered work with a work licensed under version 3 of the GNU Affero General Public License into a single combined work, and to convey the resulting work. The terms of this License will continue to apply to the part which is the covered work, but the special requirements of the GNU Affero General Public License, section 13, concerning interaction through a network will apply to the combination as such.

### 14. Revised Versions of this License.

The Free Software Foundation may publish revised and/or new versions of the GNU General Public License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns.

Each version is given a distinguishing version number. If the Program specifies that a certain numbered version of the GNU General Public License “or any later version” applies to it, you have the option of following the terms and conditions either of that numbered version or of any later version published by the Free Software Foundation. If the Program does not specify a version number of the GNU General Public License, you may choose any version ever published by the Free Software Foundation.

If the Program specifies that a proxy can decide which future versions of the GNU General Public License can be used, that proxy’s public statement of acceptance of a version permanently authorizes you to choose that version for the Program.

Later license versions may give you additional or different permissions. However, no additional obligations are imposed on any author or copyright holder as a result of your choosing to follow a later version.

### 15. Disclaimer of Warranty.

THERE IS NO WARRANTY FOR THE PROGRAM, TO THE EXTENT PERMITTED BY APPLICABLE LAW. EXCEPT WHEN OTHERWISE STATED IN WRITING THE COPYRIGHT HOLDERS AND/OR OTHER PARTIES PROVIDE THE PROGRAM “AS IS” WITHOUT WARRANTY OF ANY KIND, EITHER EXPRESSED OR IMPLIED, INCLUDING, BUT NOT LIMITED TO, THE IMPLIED WARRANTIES OF MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE. THE ENTIRE RISK AS TO THE QUALITY AND PERFORMANCE OF THE PROGRAM IS WITH YOU. SHOULD THE PROGRAM PROVE DEFECTIVE, YOU ASSUME THE COST OF ALL NECESSARY SERVICING, REPAIR OR CORRECTION.

### 16. Limitation of Liability.

IN NO EVENT UNLESS REQUIRED BY APPLICABLE LAW OR AGREED TO IN WRITING WILL ANY COPYRIGHT HOLDER, OR ANY OTHER PARTY WHO MODIFIES AND/OR CONVEYS THE PROGRAM AS PERMITTED ABOVE, BE LIABLE TO YOU FOR DAMAGES, INCLUDING ANY GENERAL, SPECIAL, INCIDENTAL OR CONSEQUENTIAL DAMAGES ARISING OUT OF THE USE OR INABILITY TO USE THE PROGRAM (INCLUDING BUT NOT LIMITED TO LOSS OF DATA OR DATA BEING RENDERED INACCURATE OR LOSSES SUSTAINED BY YOU OR THIRD PARTIES OR A FAILURE OF THE PROGRAM TO OPERATE WITH ANY OTHER PROGRAMS),



EVEN IF SUCH HOLDER OR OTHER PARTY HAS BEEN ADVISED OF THE POSSIBILITY OF SUCH DAMAGES.

### 17. Interpretation of Sections 15 and 16.

If the disclaimer of warranty and limitation of liability provided above cannot be given local legal effect according to their terms, reviewing courts shall apply local law that most closely approximates an absolute waiver of all civil liability in connection with the Program, unless a warranty or assumption of liability accompanies a copy of the Program in return for a fee.

END OF TERMS AND CONDITIONS

## How to Apply These Terms to Your New Programs

If you develop a new program, and you want it to be of the greatest possible use to the public, the best way to achieve this is to make it free software which everyone can redistribute and change under these terms.

To do so, attach the following notices to the program. It is safest to attach them to the start of each source file to most effectively state the exclusion of warranty; and each file should have at least the “copyright” line and a pointer to where the full notice is found.

```
one line to give the program's name and a brief idea of what it does.
Copyright (C)  year   name of author
```

```
This program is free software: you can redistribute it and/or modify
it under the terms of the GNU General Public License as published by
the Free Software Foundation, either version 3 of the License, or
(at your option) any later version.
```

```
This program is distributed in the hope that it will be useful,
but WITHOUT ANY WARRANTY; without even the implied warranty of
MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
GNU General Public License for more details.
```

```
You should have received a copy of the GNU General Public License
along with this program.  If not, see http://www.gnu.org/licenses/.
```

Also add information on how to contact you by electronic and paper mail.

If the program does terminal interaction, make it output a short notice like this when it starts in an interactive mode:

```
program Copyright (C)  year   name of author
This program comes with ABSOLUTELY NO WARRANTY; for details type `show w'.
This is free software, and you are welcome to redistribute it
under certain conditions; type `show c' for details.
```

The hypothetical commands ‘show w’ and ‘show c’ should show the appropriate parts of the General Public License. Of course, your program’s commands might be different; for a GUI interface, you would use an “about box”.

You should also get your employer (if you work as a programmer) or school, if any, to sign a “copyright disclaimer” for the program, if necessary. For more information on this, and how to apply and follow the GNU GPL, see <http://www.gnu.org/licenses/>.





The GNU General Public License does not permit incorporating your program into proprietary programs. If your program is a subroutine library, you may consider it more useful to permit linking proprietary applications with the library. If this is what you want to do, use the GNU Lesser General Public License instead of this License. But first, please read <http://www.gnu.org/philosophy/why-not-lgpl.html>.

## 3 GNU Free Documentation License

Version 1.2, November 2002

Copyright (C) 2000,2001,2002 Free Software Foundation, Inc.  
51 Franklin St, Fifth Floor, Boston, MA 02110-1301, USA.  
Everyone is permitted to copy and distribute verbatim copies  
of this license document, but changing it is not allowed.

### 0. PREAMBLE

The purpose of this License is to make a manual, textbook, or other functional and useful document "free" in the sense of freedom: to assure everyone the effective freedom to copy and redistribute it, with or without modifying it, either commercially or noncommercially. Secondly, this License preserves for the author and publisher a way to get credit for their work, while not being considered responsible for modifications made by others.

This License is a kind of "copyleft", which means that derivative works of the document must themselves be free in the same sense. It complements the GNU General Public License, which is a copyleft license designed for free software.

We have designed this License in order to use it for manuals for free software, because free software needs free documentation: a free program should come with manuals providing the same freedoms that the software does. But this License is not limited to software manuals; it can be used for any textual work, regardless of subject matter or whether it is published as a printed book. We recommend this License principally for works whose purpose is instruction or reference.

### 1. APPLICABILITY AND DEFINITIONS

This License applies to any manual or other work, in any medium, that contains a notice placed by the copyright holder saying it can be distributed under the terms of this License. Such a notice grants a world-wide, royalty-free license, unlimited in duration, to use that work under the conditions stated herein. The "Document", below, refers to any such manual or work. Any member of the public is a licensee, and is addressed as "you". You accept the license if you copy, modify or distribute the work in a way requiring permission under copyright law.

A "Modified Version" of the Document means any work containing the Document or a portion of it, either copied verbatim, or with modifications and/or translated into another language.

A "Secondary Section" is a named appendix or a front-matter section of the Document that deals exclusively with the relationship of the publishers or authors of the Document to the Document's overall subject (or to related matters) and contains nothing that could fall directly within that overall subject. (Thus, if the Document is in part a textbook of mathematics, a Secondary Section may not explain any mathematics.) The relationship could be a matter of historical connection with the subject or with related matters, or of legal, commercial, philosophical, ethical or political position regarding them.

The "Invariant Sections" are certain Secondary Sections whose titles are designated, as being those of Invariant Sections, in the notice that says that the Document is released under this License. If a section does not fit the above definition of Secondary then it is not allowed to be designated as Invariant. The Document may contain zero Invariant Sections. If the Document does not identify any Invariant Sections then there are none.



The "Cover Texts" are certain short passages of text that are listed, as Front-Cover Texts or Back-Cover Texts, in the notice that says that the Document is released under this License. A Front-Cover Text may be at most 5 words, and a Back-Cover Text may be at most 25 words.

A "Transparent" copy of the Document means a machine-readable copy, represented in a format whose specification is available to the general public, that is suitable for revising the document straightforwardly with generic text editors or (for images composed of pixels) generic paint programs or (for drawings) some widely available drawing editor, and that is suitable for input to text formatters or for automatic translation to a variety of formats suitable for input to text formatters. A copy made in an otherwise Transparent file format whose markup, or absence of markup, has been arranged to thwart or discourage subsequent modification by readers is not Transparent. An image format is not Transparent if used for any substantial amount of text. A copy that is not "Transparent" is called "Opaque".

Examples of suitable formats for Transparent copies include plain ASCII without markup, Texinfo input format, LaTeX input format, SGML or XML using a publicly available DTD, and standard-conforming simple HTML, PostScript or PDF designed for human modification. Examples of transparent image formats include PNG, XCF and JPG. Opaque formats include proprietary formats that can be read and edited only by proprietary word processors, SGML or XML for which the DTD and/or processing tools are not generally available, and the machine-generated HTML, PostScript or PDF produced by some word processors for output purposes only.

The "Title Page" means, for a printed book, the title page itself, plus such following pages as are needed to hold, legibly, the material this License requires to appear in the title page. For works in formats which do not have any title page as such, "Title Page" means the text near the most prominent appearance of the work's title, preceding the beginning of the body of the text.

A section "Entitled XYZ" means a named subunit of the Document whose title either is precisely XYZ or contains XYZ in parentheses following text that translates XYZ in another language. (Here XYZ stands for a specific section name mentioned below, such as "Acknowledgements", "Dedications", "Endorsements", or "History".) To "Preserve the Title" of such a section when you modify the Document means that it remains a section "Entitled XYZ" according to this definition.

The Document may include Warranty Disclaimers next to the notice which states that this License applies to the Document. These Warranty Disclaimers are considered to be included by reference in this License, but only as regards disclaiming warranties: any other implication that these Warranty Disclaimers may have is void and has no effect on the meaning of this License.

## 2. VERBATIM COPYING

You may copy and distribute the Document in any medium, either commercially or noncommercially, provided that this License, the copyright notices, and the license notice saying this License applies to the Document are reproduced in all copies, and that you add no other conditions whatsoever to those of this License. You may not use technical measures to obstruct or control the reading or further copying of the copies you make or distribute. However, you may accept compensation in exchange for copies. If you distribute a large enough number of copies you must also follow the conditions in section 3.

You may also lend copies, under the same conditions stated above, and you may publicly display copies.

## 3. COPYING IN QUANTITY

If you publish printed copies (or copies in media that commonly have printed covers) of the Document, numbering more than 100, and the Document's license notice requires Cover Texts, you must enclose the copies in covers that carry, clearly and legibly, all these Cover Texts: Front-Cover Texts on the front cover, and Back-Cover Texts on the back cover. Both covers must also clearly and legibly identify you as the publisher of these copies. The front cover must present the full title with all words of the title equally prominent and visible. You may add other material on the covers in addition. Copying with changes limited to the covers, as long as they preserve the title of the Document and satisfy these conditions, can be treated as verbatim copying in other respects.



If the required texts for either cover are too voluminous to fit legibly, you should put the first ones listed (as many as fit reasonably) on the actual cover, and continue the rest onto adjacent pages.

If you publish or distribute Opaque copies of the Document numbering more than 100, you must either include a machine-readable Transparent copy along with each Opaque copy, or state in or with each Opaque copy a computer-network location from which the general network-using public has access to download using public-standard network protocols a complete Transparent copy of the Document, free of added material. If you use the latter option, you must take reasonably prudent steps, when you begin distribution of Opaque copies in quantity, to ensure that this Transparent copy will remain thus accessible at the stated location until at least one year after the last time you distribute an Opaque copy (directly or through your agents or retailers) of that edition to the public.

It is requested, but not required, that you contact the authors of the Document well before redistributing any large number of copies, to give them a chance to provide you with an updated version of the Document.

#### 4. MODIFICATIONS

You may copy and distribute a Modified Version of the Document under the conditions of sections 2 and 3 above, provided that you release the Modified Version under precisely this License, with the Modified Version filling the role of the Document, thus licensing distribution and modification of the Modified Version to whoever possesses a copy of it. In addition, you must do these things in the Modified Version:

- **A.** Use in the Title Page (and on the covers, if any) a title distinct from that of the Document, and from those of previous versions (which should, if there were any, be listed in the History section of the Document). You may use the same title as a previous version if the original publisher of that version gives permission.
- **B.** List on the Title Page, as authors, one or more persons or entities responsible for authorship of the modifications in the Modified Version, together with at least five of the principal authors of the Document (all of its principal authors, if it has fewer than five), unless they release you from this requirement.
- **C.** State on the Title page the name of the publisher of the Modified Version, as the publisher.
- **D.** Preserve all the copyright notices of the Document.
- **E.** Add an appropriate copyright notice for your modifications adjacent to the other copyright notices.
- **F.** Include, immediately after the copyright notices, a license notice giving the public permission to use the Modified Version under the terms of this License, in the form shown in the Addendum below.
- **G.** Preserve in that license notice the full lists of Invariant Sections and required Cover Texts given in the Document's license notice.
- **H.** Include an unaltered copy of this License.
- **I.** Preserve the section Entitled "History", Preserve its Title, and add to it an item stating at least the title, year, new authors, and publisher of the Modified Version as given on the Title Page. If there is no section Entitled "History" in the Document, create one stating the title, year, authors, and publisher of the Document as given on its Title Page, then add an item describing the Modified Version as stated in the previous sentence.
- **J.** Preserve the network location, if any, given in the Document for public access to a Transparent copy of the Document, and likewise the network locations given in the Document for previous versions it was based on. These may be placed in the "History" section. You may omit a network location for a work that was published at least four years before the Document itself, or if the original publisher of the version it refers to gives permission.



- **K.** For any section Entitled "Acknowledgements" or "Dedications", Preserve the Title of the section, and preserve in the section all the substance and tone of each of the contributor acknowledgements and/or dedications given therein.
- **L.** Preserve all the Invariant Sections of the Document, unaltered in their text and in their titles. Section numbers or the equivalent are not considered part of the section titles.
- **M.** Delete any section Entitled "Endorsements". Such a section may not be included in the Modified Version.
- **N.** Do not retitle any existing section to be Entitled "Endorsements" or to conflict in title with any Invariant Section.
- **O.** Preserve any Warranty Disclaimers.

If the Modified Version includes new front-matter sections or appendices that qualify as Secondary Sections and contain no material copied from the Document, you may at your option designate some or all of these sections as invariant. To do this, add their titles to the list of Invariant Sections in the Modified Version's license notice. These titles must be distinct from any other section titles.

You may add a section Entitled "Endorsements", provided it contains nothing but endorsements of your Modified Version by various parties--for example, statements of peer review or that the text has been approved by an organization as the authoritative definition of a standard.

You may add a passage of up to five words as a Front-Cover Text, and a passage of up to 25 words as a Back-Cover Text, to the end of the list of Cover Texts in the Modified Version. Only one passage of Front-Cover Text and one of Back-Cover Text may be added by (or through arrangements made by) any one entity. If the Document already includes a cover text for the same cover, previously added by you or by arrangement made by the same entity you are acting on behalf of, you may not add another; but you may replace the old one, on explicit permission from the previous publisher that added the old one.

The author(s) and publisher(s) of the Document do not by this License give permission to use their names for publicity for or to assert or imply endorsement of any Modified Version.

## 5. COMBINING DOCUMENTS

You may combine the Document with other documents released under this License, under the terms defined in section 4 above for modified versions, provided that you include in the combination all of the Invariant Sections of all of the original documents, unmodified, and list them all as Invariant Sections of your combined work in its license notice, and that you preserve all their Warranty Disclaimers.

The combined work need only contain one copy of this License, and multiple identical Invariant Sections may be replaced with a single copy. If there are multiple Invariant Sections with the same name but different contents, make the title of each such section unique by adding at the end of it, in parentheses, the name of the original author or publisher of that section if known, or else a unique number. Make the same adjustment to the section titles in the list of Invariant Sections in the license notice of the combined work.

In the combination, you must combine any sections Entitled "History" in the various original documents, forming one section Entitled "History"; likewise combine any sections Entitled "Acknowledgements", and any sections Entitled "Dedications". You must delete all sections Entitled "Endorsements."

## 6. COLLECTIONS OF DOCUMENTS

You may make a collection consisting of the Document and other documents released under this License, and replace the individual copies of this License in the various documents with a single copy that is included in the collection, provided that you follow the rules of this License for verbatim copying of each of the documents in all other respects.

You may extract a single document from such a collection, and distribute it individually under this License, provided you insert a copy of this License into the extracted document, and follow this License in all other respects regarding verbatim copying of that document.



## 7. AGGREGATION WITH INDEPENDENT WORKS

A compilation of the Document or its derivatives with other separate and independent documents or works, in or on a volume of a storage or distribution medium, is called an "aggregate" if the copyright resulting from the compilation is not used to limit the legal rights of the compilation's users beyond what the individual works permit. When the Document is included in an aggregate, this License does not apply to the other works in the aggregate which are not themselves derivative works of the Document.

If the Cover Text requirement of section 3 is applicable to these copies of the Document, then if the Document is less than one half of the entire aggregate, the Document's Cover Texts may be placed on covers that bracket the Document within the aggregate, or the electronic equivalent of covers if the Document is in electronic form. Otherwise they must appear on printed covers that bracket the whole aggregate.

## 8. TRANSLATION

Translation is considered a kind of modification, so you may distribute translations of the Document under the terms of section 4. Replacing Invariant Sections with translations requires special permission from their copyright holders, but you may include translations of some or all Invariant Sections in addition to the original versions of these Invariant Sections. You may include a translation of this License, and all the license notices in the Document, and any Warranty Disclaimers, provided that you also include the original English version of this License and the original versions of those notices and disclaimers. In case of a disagreement between the translation and the original version of this License or a notice or disclaimer, the original version will prevail.

If a section in the Document is Entitled "Acknowledgements", "Dedications", or "History", the requirement (section 4) to Preserve its Title (section 1) will typically require changing the actual title.

## 9. TERMINATION

You may not copy, modify, sublicense, or distribute the Document except as expressly provided for under this License. Any other attempt to copy, modify, sublicense or distribute the Document is void, and will automatically terminate your rights under this License. However, parties who have received copies, or rights, from you under this License will not have their licenses terminated so long as such parties remain in full compliance.

## 10. FUTURE REVISIONS OF THIS LICENSE

The Free Software Foundation may publish new, revised versions of the GNU Free Documentation License from time to time. Such new versions will be similar in spirit to the present version, but may differ in detail to address new problems or concerns. See <http://www.gnu.org/copyleft/>.

Each version of the License is given a distinguishing version number. If the Document specifies that a particular numbered version of this License "or any later version" applies to it, you have the option of following the terms and conditions either of that specified version or of any later version that has been published (not as a draft) by the Free Software Foundation. If the Document does not specify a version number of this License, you may choose any version ever published (not as a draft) by the Free Software Foundation.

### How to use this License for your documents

To use this License in a document you have written, include a copy of the License in the document and put the following copyright and license notices just after the title page:

```
Copyright (c)  YEAR  YOUR NAME.
Permission is granted to copy, distribute and/or modify this document
under the terms of the GNU Free Documentation License, Version 1.2
or any later version published by the Free Software Foundation;
with no Invariant Sections, no Front-Cover Texts, and no Back-Cover
Texts.  A copy of the license is included in the section entitled
```



"GNU Free Documentation License".

If you have Invariant Sections, Front-Cover Texts and Back-Cover Texts, replace the "with...Texts." line with this:

```
with the Invariant Sections being LIST THEIR TITLES, with the
Front-Cover Texts being LIST, and with the Back-Cover Texts being LIST.
```

If you have Invariant Sections without Cover Texts, or some other combination of the three, merge those two alternatives to suit the situation.

If your document contains nontrivial examples of program code, we recommend releasing these examples in parallel under your choice of free software license, such as the GNU General Public License, to permit their use in free software.

## 4 System-Independent Features

### System-Independent Features

The Prolog interface provides access to the numerical abstractions (convex polyhedra, BD shapes, octagonal shapes, etc.) implemented by the PPL library. A general introduction to the numerical abstractions, their representation in the PPL and the operations provided by the PPL is given in the main *PPL user manual*. Here we just describe those aspects that are specific to the Prolog interface.

#### Overview

First, here is a list of notes with general information and advice on the use of the interface.

- The numerical abstract domains available to the Prolog user consist of the *simple* domains, *powersets* of a simple domain and *products* of simple domains.
  - The simple domains are:
    - \* convex polyhedra, which consist of `C_Polyhedron` and `NNC_Polyhedron`;
    - \* weakly relational, which consist of `BD_Shape_N` and `Octagonal_Shape_N` where `N` is one of the numeric types `int8`, `int16`, `int32`, `int64`, `mpz_class`, `mpq_class`, `float`, `double`, `long_double`;
    - \* boxes which consist of `Int8_Box`, `Int16_Box`, `Int32_Box`, `Int64_Box`, `UInt8_Box`, `UInt16_Box`, `UInt32_Box`, `UInt64_Box`, `Double_Box`, `Long_Double_Box`, `Z_Box`, `Rational_Box`, `Float_Box`; and
    - \* the Grid domain.
  - The powerset domains are `Pointset_Powerset_S` where `S` is a simple domain.
  - The product domains consist of `Direct_Product_S_T`, `Smash_Product_S_T`, `Constraints_Product_S_T` and `Shape_Preserving_Product_S_T` where `S` and `T` are simple domains.
- In the following, any of the above numerical abstract domains is called a PPL *domain* and any element of a PPL domain is called a *PPL object*.
- The Prolog interface to the PPL is initialized and finalized by the predicates `ppl_initialize/0` and `ppl_finalize/0`. Thus the only interface predicates callable after `ppl_finalize/0` are `ppl_finalize/0` itself (this further call has no effect) and `ppl_initialize/0`, after which





the interface's services are usable again. Some Prolog systems allow the specification of initialization and deinitialization functions in their foreign language interfaces. The corresponding incarnations of the Prolog interface have been written so that `ppl_initialize/0` and/or `ppl_finalize/0` are called automatically. Section [System-Dependent Features](#) will detail in which cases initialization and finalization is automatically performed or is left to the Prolog programmer's responsibility. However, for portable applications, it is best to invoke `ppl_initialize/0` and `ppl_finalize/0` explicitly: since they can be called multiple times without problems, this will result in enhanced portability at a cost that is, by all means, negligible.

- A PPL object such as a polyhedron can only be accessed by means of a Prolog term called a *handle*. Note, however, that the data structure of a handle, is implementation-dependent, system-dependent and version-dependent, and, for this reason, deliberately left unspecified. What we do guarantee is that the handle requires very little memory.
- A Prolog term can be bound to a valid handle for a PPL object by using predicates such as

```
ppl_new_C_Polyhedron_from_space_dimension/3,
ppl_new_C_Polyhedron_from_C_Polyhedron/2,
ppl_new_C_Polyhedron_from_constraints/2,
ppl_new_C_Polyhedron_from_generators/2,
```

These predicates will create or copy a PPL polyhedron and construct a valid handle for referencing it. The last argument is a Prolog term that is unified with a new valid handle for accessing this polyhedron.

- As soon as a PPL object is no longer required, the memory occupied by it should be released using the PPL predicate such as `ppl_delete_Polyhedron/1`. To understand why this is important, consider a Prolog program and a variable that is bound to a Herbrand term. When the variable dies (goes out of scope) or is uninstantiated (on backtracking), the term it is bound to is amenable to garbage collection. But this only applies for the standard domain of the language: Herbrand terms. In Prolog+PPL, when, for example, a variable bound to a handle for a Polyhedron dies or is uninstantiated, the handle can be garbage-collected, but the polyhedron to which the handle refers will not be released. Once a handle has been used as an argument in `ppl_delete_Polyhedron/1`, it becomes invalid.
- For a PPL object with space dimension  $k$ , the identifiers used for the PPL variables must lie between 0 and  $k - 1$  and correspond to the indices of the associated Cartesian axes. For example, when using the predicates that combine PPL polyhedra or add constraints or generators to a representation of a PPL polyhedron, the polyhedra referenced and any constraints or generators in the call should follow all the (space) dimension-compatibility rules stated in Section *Representations of Convex Polyhedra* of the main *PPL user manual*.
- As explained above, a polyhedron has a fixed topology  $C$  or  $NNC$ , that is determined at the time of its initialization. All subsequent operations on the polyhedron must respect all the topological compatibility rules stated in Section *Representations of Convex Polyhedra* of the main *PPL user manual*.
- Any application using the PPL should make sure that only the intended version(s) of the library are ever used. Predicates

```
ppl_version_major/1,
ppl_version_minor/1,
ppl_version_revision/1,
ppl_version_beta/1,
ppl_version/1,
ppl_banner.
```

allow run-time checking of information about the version being used.



### Predicate Specifications

The PPL predicates provided by the Prolog interface are specified below. The specification uses the following grammar rules:

Number	--> unsigned integer	ranging from 0 to an upper bound depending on the actual Prolog system.
C_int	--> Number   - Number	C integer
C_unsigned	--> Number	C unsigned integer
Coeff	--> Number	used in linear expressions; the upper bound will depend on how the PPL has been configured
Dimension_Type	--> Number	used for the number of affine and space dimensions and the names of the dimensions; the upper bound will depend on the maximum number of dimensions allowed by the PPL (see ppl_max_space_dimensions/1)
Boolean	--> true   false	
Handle	--> Prolog term	used to identify a Polyhedron
Topology	--> c   nnc	Polyhedral kind; c is closed and nnc is NNC
VarId	--> Dimension_Type	variable identifier
PPL_Var	--> '\$VAR' (VarId)	PPL variable
Lin_Expr	--> PPL_Var   Coeff   Lin_Expr   - Lin_Expr   Lin_Expr + Lin_Expr   Lin_Expr - Lin_Expr   Coeff * Lin_Expr   Lin_Expr * Coeff	PPL variable  unary plus unary minus addition subtraction multiplication multiplication
Relation_Symbol	--> =   <=   >=   <   >	equals less than or equal greater than or equal strictly less than strictly greater than
Constraint	--> Lin_Expr Relation_Symbol Lin_Expr	constraint
Constraint_System	--> []   [Constraint   Constraint_System]	list of constraints
Modulus	--> Coeff   - Coeff	
Congruence	--> Lin_Expr == Lin_Expr   (Lin_Expr == Lin_Expr) / Modulus	congruence with modulo 1 congruence with modulo Modulus
Congruence_System	--> []   [Congruence   Congruence_System]	list of congruences





```

Generator_Denominator --> Coeff          must be non-zero
    | - Coeff

Generator    --> point(Lin_Expr)          point
    | point(Lin_Expr, Generator_Denominator)
    |                                     point
    | closure_point(Lin_Expr)            closure point
    | closure_point(Lin_Expr, Generator_Denominator)
    |                                     closure point
    | ray(Lin_Expr)                      ray
    | line(Lin_Expr)                     line

Generator_System          list of generators
    --> []
    | [Generator | Generator_System]

Grid_Generator
    --> grid_point(Lin_Expr)  grid point
    | grid_point(Lin_Expr, Generator_Denominator)
    |                                     grid point
    | parameter(Lin_Expr)      parameter
    | parameter(Lin_Expr, Generator_Denominator)
    |                                     parameter
    | grid_line(Lin_Expr)      grid line

Grid_Generator_System          list of grid generators
    --> []
    | [Grid_Generator | Grid_Generator_System]

Artificial_Parameter  --> Lin_Expr / Coeff

Artificial_Parameter_List --> []
    | [Artificial_Parameter | Artificial_Parameter_List]

Atom    --> Prolog atom

Universe_or_Empty          PPL object
    --> universe | empty

Poly_Relation --> is_disjoint          with a constraint or congruence
    | strictly_intersects          with a constraint or congruence
    | is_included                  with a constraint or congruence
    | saturates                    with a constraint or congruence
    | subsumes                     with a (grid) generator

Relation_List --> []
    | [Poly_Relation | Relation_List]

Complexity  --> polynomial | simplex | any

Vars_Pair   --> PPL_Var - PPL_Var      map relation

P_Func      --> []                    list of map relations
    | [Vars_Pair | P_Func].

Width  --> bits_8 | bits_16 | bits_32 | bits_64 | bits_128

Representation  --> unsigned | signed_2_complement

Overflow  --> overflow_wraps | overflow_undefined | overflow_impossible

Optimization_Mode --> max | min

Problem_Status --> unfeasible
    | unbounded
    | optimized

```



```

Control_Parameter_Name --> pricing          for MIP problems
                        | control_strategy   for PIP problems
                        | pivot_row_strategy for PIP problems

Control_Parameter_Value
    --> pricing_steepest_edge_float
    | pricing_steepest_edge_exact
    | pricing_textbook
    | control_strategy_first
    | control_strategy_deepest
    | control_strategy_all
    | pivot_row_strategy_first
    | pivot_row_strategy_max_column

Vars_List    --> []                list of PPL variables
              | [PPL_Var | Vars_List].

```

## Predicate Descriptions

Below is a short description of many of the interface predicates. For full definitions of terminology used here, see the main *PPL user manual*.

### Domain Independent Predicates

First we describe the domain independent predicates that are included with all instantiations of the Prolog interfaces.

`ppl_version_major(?C_int)`

*Unifies C\_int with the major number of the PPL version.*

`ppl_version_minor(?C_int)`

*Unifies C\_int with the minor number of the PPL version.*

`ppl_version_revision(?C_int)`

*Unifies C\_int with the revision number of the PPL version.*

`ppl_version_beta(?C_int)`

*Unifies C\_int with the beta number of the PPL version.*

`ppl_version(?Atom)`

*Unifies Atom with the PPL version.*

`ppl_banner(?Atom)`

*Unifies Atom with information about the PPL version, the licensing, the lack of any warranty whatsoever, the C++ compiler used to build the library, where to report bugs and where to look for further information.*

`ppl_Coefficient_bits(?Bits)`

*Unifies Bits with the number of bits used to encode a Coefficient in the C++ interface; 0 if unbounded.*

`ppl_Coefficient_is_bounded`

*Succeeds if and only if the Coefficients in the C++ interface are bounded.*

`ppl_Coefficient_max(Max)`

*If the Coefficients in the C++ interface are bounded, then the maximum coefficient the C++ interface can handle is unified with Max. If the Prolog system cannot handle this coefficient, then an exception is thrown. It fails if the Coefficients in the C++ interface are unbounded.*



`ppl_Coefficient_min (Min)`

*If the Coefficients in the C++ interface are bounded, then the minimum coefficient the C++ interface can handle is unified with Min. If the Prolog system cannot handle this coefficient, then an exception is thrown. It fails if the Coefficients in the C++ interface are unbounded.*

`ppl_max_space_dimension (?Dimension_Type)`

*Unifies Dimension\_Type with the maximum space dimension this library can handle.*

`ppl_initialize`

*Initializes the PPL interface. Multiple calls to `ppl_initialize` does no harm.*

`ppl_finalize`

*Finalizes the PPL interface. Once this is executed, the next call to an interface predicate must either be to `ppl_initialize` or to `ppl_finalize`. Multiple calls to `ppl_finalize` does no harm.*

`ppl_set_timeout_exception_atom (+Atom)`

*Sets the atom to be thrown by timeout exceptions to Atom. The default value is `time_out`.*

`ppl_timeout_exception_atom (?Atom)`

*The atom to be thrown by timeout exceptions is unified with Atom.*

`ppl_set_timeout (+Csecs)`

*Computations taking exponential time will be interrupted some time after Csecs centiseconds after that call. If the computation is interrupted that way, the current timeout exception atom will be thrown. Csecs must be strictly greater than zero.*

`ppl_reset_timeout`

*Resets the timeout time so that the computation is not interrupted.*

`ppl_set_deterministic_timeout (+Unscaled_Weight, +Scale)`

*Computations taking exponential time will be interrupted some time after reaching the complexity threshold  $\text{Weight} = \text{Unscaled\_Weight} \cdot 2^{\text{Scale}}$ . If the computation is interrupted that way, the current timeout exception atom will be thrown. Unscaled\_Weight must be strictly greater than zero; Scale must be non-negative; an exception is thrown if the computed weight threshold exceeds the maximum allowed value.*

**NOTE:** This "timeout" checking functionality is said to be *deterministic* because it is not based on actual elapsed time. Its behavior will only depend on (some of the) computations performed in the PPL library and it will be otherwise independent from the computation environment (CPU, operating system, compiler, etc.). The weight mechanism is under beta testing: client applications should be ready to reconsider the tuning of these weight thresholds when upgrading to newer version of the PPL.

`ppl_reset_deterministic_timeout`

*Resets the deterministic timeout so that the computation is not interrupted.*

`ppl_set_rounding_for_PPL`

*Sets the FPU rounding mode so that the PPL abstractions based on floating point numbers work correctly. This is performed automatically at initialization-time. Calling this function is needed only if `restore_pre_PPL_rounding()` has previously been called.*

`ppl_restore_pre_PPL_rounding`

*Sets the FPU rounding mode as it was before initialization of the PPL. After calling this function it is absolutely necessary to call `set_rounding_for_PPL()` before using any PPL abstractions based on floating point numbers. This is performed automatically at finalization-time.*



```
ppl_irrational_precision(?Precision)
```

*Unifies Precision with the precision parameter for irrational calculations.*

```
ppl_set_irrational_precision(+Precision)
```

*Sets the precision parameter for irrational calculations to Precision. In the following irrational calculations returning an unbounded rational (e.g., when computing a square root), the lesser between numerator and denominator will be limited to 2\*\*Precision.*

### Predicates for MIP\_Problem

Here we describe the predicates available for PPL objects defining mixed integer (linear) programming problems.

```
ppl_new_MIP_Problem_from_space_dimension(+Dimension_Type, -Handle)
```

*Creates an MIP Problem MIP with the feasible region the vector space of dimension Dimension\_Type, objective function 0 and optimization mode max. Handle is unified with the handle for MIP.*

```
ppl_new_MIP_Problem(+Dimension_Type, +Constraint_System, +Lin_Expr, +Optimization_Mode, -Handle)
```

*Creates an MIP Problem MIP with the feasible region the vector space of dimension Dimension\_Type, represented by Constraint\_System, objective function Lin\_Expr and optimization mode Optimization\_Mode. Handle is unified with the handle for MIP.*

```
ppl_new_MIP_Problem_from_MIP_Problem(+Handle_1, -Handle_2)
```

*Creates an MIP Problem MIP from the MIP Problem referenced by Handle\_1. Handle\_2 is unified with the handle for MIP.*

```
ppl_MIP_Problem_swap(+Handle_1, +Handle_2)
```

*Swaps the MIP Problem referenced by Handle\_1 with the one referenced by Handle\_2.*

```
ppl_delete_MIP_Problem(+Handle)
```

*Deletes the MIP Problem referenced by Handle. After execution, Handle is no longer a valid handle for a PPL MIP Problem.*

```
ppl_MIP_Problem_space_dimension(+Handle, ?Dimension_Type)
```

*Unifies the dimension of the vector space in which the MIP Problem referenced by Handle is embedded with Dimension\_Type.*

```
ppl_MIP_Problem_integer_space_dimensions(+Handle, ?Vars_List)
```

*Unifies Vars\_List with a list of variables representing the integer space dimensions of the MIP Problem referenced by Handle.*

```
ppl_MIP_Problem_constraints(+Handle, -Constraint_System)
```

*Unifies Constraint\_System with a list of the constraints in the constraints system representing the feasible region for the MIP Problem referenced by Handle.*

```
ppl_MIP_Problem_objective_function(+Handle, ?Lin_Expr)
```

*Unifies Lin\_Expr with the objective function for the MIP Problem referenced by Handle.*

```
ppl_MIP_Problem_optimization_mode(+Handle, ?Optimization_Mode)
```

*Unifies Optimization\_Mode with the optimization mode for the MIP Problem referenced by Handle.*

```
ppl_MIP_Problem_clear(+Handle)
```



*Resets the MIP problem referenced by Handle to be the trivial problem with the feasible region the 0-dimensional universe, objective function 0 and optimization mode max.*

```
ppl_MIP_Problem_add_space_dimensions_and_embed( +Handle, +Dimension_Type)
```

*Embeds the MIP problem referenced by Handle in a space that is enlarged by Dimension\_Type dimensions.*

```
ppl_MIP_Problem_add_to_integer_space_dimensions(+Handle, +Vars_List)
```

*Updates the MIP Problem referenced by Handle so that the variables in Vars\_List are added to the set of integer space dimensions.*

```
ppl_MIP_Problem_add_constraint(+Handle, +Constraint)
```

*Updates the MIP Problem referenced by Handle so that the feasible region is represented by the original constraint system together with the constraint Constraint.*

```
ppl_MIP_Problem_add_constraints(+Handle, +Constraint_System)
```

*Updates the MIP Problem referenced by Handle so that the feasible region is represented by the original constraint system together with all the constraints in Constraint\_System.*

```
ppl_MIP_Problem_set_objective_function(+Handle, +Lin_Expr)
```

*Updates the MIP Problem referenced by Handle so that the objective function is changed to Lin\_Expr.*

```
ppl_MIP_Problem_set_control_parameter(+Handle, +Control_Parameter_Value)
```

*Updates the MIP Problem referenced by Handle so that the value for the relevant control parameter name is changed to Control\_Parameter\_Value.*

```
ppl_MIP_Problem_get_control_parameter(+Handle, +Control_Parameter_Name,  
?Control_Parameter_Value)
```

*Unifies Control\_Parameter\_Value with the value of the control parameter Control\_Parameter\_Name.*

```
ppl_MIP_Problem_set_optimization_mode(+Handle, +Optimization_Mode)
```

*Updates the MIP Problem referenced by Handle so that the optimization mode is changed to Optimization\_Mode.*

```
ppl_MIP_Problem_is_satisfiable(+Handle)
```

*Succeeds if and only if the MIP Problem referenced by Handle is satisfiable.*

```
ppl_MIP_Problem_solve(+Handle, ?MIP_Problem_Status)
```

*Solves the MIP problem referenced by Handle and unifies MIP\_Problem\_Status with: unfeasible, if the MIP problem is not satisfiable; unbounded, if the MIP problem is satisfiable but there is no finite bound to the value of the objective function; optimized, if the MIP problem admits an optimal solution.*

```
ppl_MIP_Problem_feasible_point(+Handle, ?Generator)
```

*Unifies Generator with a feasible point for the MIP problem referenced by Handle.*

```
ppl_MIP_Problem_optimizing_point(+Handle, ?Generator)
```

*Unifies Generator with an optimizing point for the MIP problem referenced by Handle.*

```
ppl_MIP_Problem_optimal_value(+Handle, ?Coeff_1, ?Coeff_2)
```

*Unifies Coeff\_1 and Coeff\_2 with the numerator and denominator, respectively, for the optimal value for the MIP problem referenced by Handle.*

```
ppl_MIP_Problem_evaluate_objective_function(+Handle, +Generator, ?Coeff_  
_1, ?Coeff_2)
```



*Evaluates the objective function of the MIP problem referenced by Handle at point Generator. Coefficient\_1 is unified with the numerator and Coefficient\_2 is unified with the denominator of the objective function value at Generator.*

`ppl_MIP_Problem_OK(+Handle)`

*Succeeds only if the MIP Problem referenced by Handle is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

`ppl_MIP_Problem_ascii_dump(+Handle)`

*Dumps an ascii representation of the PPL internal state for the MIP problem referenced by Handle on the standard output.*

### Predicates for PIP\_Problem

Here we describe some functions available for PPL objects defining parametric integer programming problems.

`ppl_new_PIP_Problem_from_space_dimension(+Dimension_Type, -Handle)`

*Creates a PIP Problem PIP with the feasible region the vector space of dimension dimension, empty constraint\_system and empty set of parametric variables. Handle is unified with the handle for PIP.*

`ppl_new_PIP_Problem_from_PIP_Problem(+Handle_1, -Handle_2)`

*Creates a PIP Problem PIP from the PIP Problem referenced by Handle\_1. Handle\_2 is unified with the handle for PIP.*

`ppl_new_PIP_Problem(+Dimension_Type, +Constraint_System, +Vars_List, --Handle)`

*Creates a PIP Problem PIP having space dimension dimension, a feasible region represented by constraint\_system and parametric variables represented by Vars\_List. Handle is unified with the handle for PIP.*

`ppl_PIP_Problem_swap(+Handle_1, +Handle_2)`

*Swaps the PIP Problem referenced by Handle\_1 with the one referenced by Handle\_2.*

`ppl_delete_PIP_Problem(+Handle)`

*Deletes the PIP Problem referenced by Handle. After execution, Handle is no longer a valid handle for a PPL PIP Problem.*

`ppl_PIP_Problem_space_dimension(+Handle, ?Dimension_Type)`

*Unifies the dimension of the vector space in which the PIP Problem referenced by Handle is embedded with Dimension\_Type.*

`ppl_PIP_Problem_parameter_space_dimensions(+Handle, ?Vars_List)`

*Unifies Vars\_List with a list of variables representing the parameter space dimensions of the PIP Problem referenced by Handle.*

`ppl_PIP_Problem_constraints(+Handle, ?Constraint_System)`

*Unifies Constraint\_System with a list of the constraints in the constraints system representing the feasible region for the PIP Problem referenced by Handle.*

`ppl_PIP_Problem_get_control_parameter(+Handle, +Control_Parameter_Name, ?Control_Parameter_Value)`

*Unifies Control\_Parameter\_Value with the value of the control parameter Control\_Parameter\_Name.*



```
ppl_PIP_Problem_clear(+Handle)
```

*Resets the PIP problem referenced by Handle to be the trivial problem with the feasible region the 0-dimensional universe.*

```
ppl_PIP_Problem_add_space_dimensions_and_embed( +Handle, +Dimension_Type1,
+Dimension_Type2)
```

*Embeds the PIP problem referenced by handle in a space that is enlarged by dimension1 non-parameter dimensions and dimension2 parameter dimensions.*

```
ppl_PIP_Problem_add_to_parameter_space_dimensions(+Handle, +Vars_List)
```

*Updates the PIP Problem referenced by Handle so that the variables in Vars\_List are added to the set of parameter space dimensions.*

```
ppl_PIP_Problem_add_constraint(+Handle, +Constraint)
```

*Updates the PIP Problem referenced by Handle so that the feasible region is represented by the original constraint system together with the constraint Constraint.*

```
ppl_PIP_Problem_add_constraints(+Handle, +Constraint_System)
```

*Updates the PIP Problem referenced by Handle so that the feasible region is represented by the original constraint system together with all the constraints in Constraint\_System.*

```
ppl_PIP_Problem_set_control_parameter(+Handle, +Control_Parameter_Value)
```

*Updates the PIP Problem referenced by Handle so that the value for the relevant control parameter name is changed to Control\_Parameter\_Value.*

```
ppl_PIP_Problem_is_satisfiable(+Handle)
```

*Succeeds if and only if the PIP Problem referenced by Handle is satisfiable.*

```
ppl_PIP_Problem_solve(+Handle, ?PIP_Problem_Status)
```

*Solves the PIP problem referenced by Handle and unifies PIP\_Problem\_Status with: unfeasible, if the PIP problem is not satisfiable; optimized, if the PIP problem admits an optimal solution.*

```
ppl_PIP_Problem_solution(+Handle1, ?Handle2)
```

*Solves the PIP problem referenced by Handle1 and creates a PIP tree node Node representing this a solution if it exists and bottom otherwise Handle\_2 is unified with the handle for Sol.*

```
ppl_PIP_Problem_optimizing_solution(+Handle, ?PIP_Tree_Node)
```

*Solves the PIP problem referenced by Handle1 and creates a PIP tree node Node representing this an optimizing solution if a solution exists and bottom otherwise Handle\_2 is unified with the handle for Sol.*

```
ppl_PIP_Problem_has_big_parameter_dimension(+Handle, +Dimension_Type)
```

*Succeeds if and only if the PIP Problem referenced by Handle has a dimension dim for the big parameter and Dimension\_Type unifies with dim.*

```
ppl_PIP_Problem_set_big_parameter_dimension(+Handle, +Dimension_Type)
```

*Updates the PIP Problem referenced by Handle so that the dimension for the big parameter is Dimension\_Type.*

```
ppl_PIP_Problem_OK(+Handle)
```

*Succeeds only if the PIP Problem referenced by Handle is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_PIP_Problem_ascii_dump(+Handle)
```



*Dumps an ascii representation of the PPL internal state for the PIP problem referenced by `Handle` on the standard output.*

```
ppl_PIP_Tree_Node_constraints(+Handle, ?Constraint_System)
```

*Unifies `Constraint_System` with a list of the parameter constraints in the PIP tree node referenced by `Handle`.*

```
ppl_PIP_Tree_Node_is_solution(+Handle)
```

*Succeeds if and only if `handle` represents a solution node.*

```
ppl_PIP_Tree_Node_is_decision(+Handle)
```

*Succeeds if and only if `handle` represents a decision node.*

```
ppl_PIP_Tree_Node_is_bottom(+Handle)
```

*Succeeds if and only if `handle` represents bottom.*

```
ppl_PIP_Tree_Node_artificials(+Handle, ?Artificial_Parameter_List)
```

*Unifies `Artificial_Parameter_List` with a list of the artificial parameters in the PIP tree node referenced by `Handle`.*

```
ppl_PIP_Tree_Node_OK(+Handle)
```

*Succeeds only if the PIP tree node referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_PIP_Tree_Node_parametric_values(+Handle, +Var, ?Lin_Expr)
```

*Unifies `Lin_Expr` with a linear expression representing the values of problem variable `Var` in the solution node represented by `Handle`. The linear expression may involve problem parameters as well as artificial parameters.*

```
ppl_PIP_Tree_Node_true_child(+Handle1, ?Handle2)
```

*If the `PIP_Tree_Node` represented by `Handle1` is a decision node unifies the PIP tree node referenced by `Handle2` with the child on the true branch of the PIP tree node represented by `Handle1`. An exception is thrown if this is not a decision node.*

```
ppl_PIP_Tree_Node_false_child(+Handle1, ?Handle2)
```

*If the `PIP_Tree_Node` represented by `Handle1` is a decision node unifies the PIP tree node referenced by `Handle2` with the child on the false branch of the PIP tree node represented by `Handle1`. An exception is thrown if this is not a decision node.*

## 5 Domains Predicates

The structure of this section is as follows:

- [Predicates for C Polyhedra](#)
- [Ad hoc Predicates for Other Domains](#)

### Predicates for the C Polyhedron Domain

Here we provide a short description for each of the predicates available for the domain of C polyhedra. Note that predicates for other domains will follow a similar pattern.





### Constructor, copy, conversion and destructor predicates

#### Constructor predicates for C polyhedra

The constructor predicates build a C polyhedron from a specification and binds the given variable to a handle for future referencing. The specification can be:

- the number of space dimensions and an atom indicating if it is to be the universe or empty element.
- a representation for the particular class of semantic geometric descriptors to which the element being built belongs. For example, a C Polyhedron can be built from a list of non-strict inequality or equality constraints or a list of equality congruences or a list of generators that contains no closure points.

```
ppl_new_C_Polyhedron_from_space_dimension(+Dimension_Type, +Universe_or-
_Empty, -Handle)
```

*Builds a new C polyhedron  $\mathcal{P}$  with *Dimension\_Type* dimensions; it is empty or the universe depending on whether *Atom* is empty or universe, respectively. *Handle* is unified with the handle for  $\mathcal{P}$ . Thus the query*

```
?- ppl_new_C_Polyhedron_from_space_dimension(3, universe, X).
```

*creates the C polyhedron defining the 3-dimensional vector space  $\mathbb{R}^3$  with *X* bound to a valid handle for accessing it.*

```
ppl_new_C_Polyhedron_from_constraints(+Constraint_System, -Handle)
```

*Builds a new C polyhedron  $\mathcal{P}$  from *Constraint\_System*. *Handle* is unified with the handle for  $\mathcal{P}$ .*

```
ppl_new_C_Polyhedron_from_congruences(+Congruence_System, -Handle)
```

*Builds a new C polyhedron  $\mathcal{P}$  from *Congruence\_System*. *Handle* is unified with the handle for  $\mathcal{P}$ .*

```
ppl_new_C_Polyhedron_from_generators(+Generator_System, -Handle)
```

*Builds a new C polyhedron  $\mathcal{P}$  from *Generator\_System*. *Handle* is unified with the handle for  $\mathcal{P}$ .*

#### Predicates that build new C polyhedra by copying or converting from other semantic geometric descriptions

Besides the constructors listed above, the library also provides:

- copy constructors that will copy an element belonging to the same class of semantic geometric descriptions
- conversion operators that build a new semantic geometric description starting from a **friend**; that is, a semantic geometric description in different class (e.g., `ppl_new_Grid_from_C_Polyhedron`, `ppl_new_C_Polyhedron_from_BD_Shape_mpq_class`, etc.).

The copy and conversion predicates have two versions, one with arity 2 for the source and target handles and one with an extra argument denoting the maximum complexity to be used in the conversion; this complexity argument is ignored when the the friend and the element being built are in the same class.

```
ppl_new_C_Polyhedron_from_C_Polyhedron(+Handle_1, -Handle_2)
```

*Builds a new C polyhedron  $\mathcal{P}_1$  from the c polyhedron referenced by handle *Handle\_1*. *Handle\_2* is unified with the handle for  $\mathcal{P}_1$ .*

```
ppl_new_C_Polyhedron_from_NNC_Polyhedron(+Handle_1, -Handle_2)
```

*Builds a new C polyhedron  $\mathcal{P}_1$  from the nnc polyhedron referenced by handle *Handle\_1*. *Handle\_2* is unified with the handle for  $\mathcal{P}_1$ .*



```
ppl_new_C_Polyhedron_from_C_Polyhedron_with_complexity(+Handle, +Complexity,
-Handle)
```

*Builds a new C polyhedron  $P_1$  from the c polyhedron referenced by handle  $Handle_1$  using an algorithm whose complexity does not exceed  $Complexity$ ;  $Handle_2$  is unified with the handle for  $P_1$ .*

```
ppl_new_C_Polyhedron_from_NNC_Polyhedron_with_complexity(+Handle, +Complexity,
-Handle)
```

*Builds a new C polyhedron  $P_1$  from the nnc polyhedron referenced by handle  $Handle_1$  using an algorithm whose complexity does not exceed  $Complexity$ ;  $Handle_2$  is unified with the handle for  $P_1$ .*

### Destructor predicate

Below is the destructor predicate for the Polyhedron domain.

```
ppl_delete_Polyhedron(+Handle)
```

*Invalidates the handle referenced by  $Handle$ : this makes sure the corresponding resources will eventually be released.*

### Predicates that do not change the polyhedron

#### Test Predicates

These predicates test the polyhedron for different properties and succeed or fail depending on the outcome.

```
ppl_Polyhedron_is_empty(+Handle)
```

*Succeeds if and only if the polyhedron referenced by  $Handle$  is empty.*

```
ppl_Polyhedron_is_universe(+Handle)
```

*Succeeds if and only if the polyhedron referenced by  $Handle$  is the universe.*

```
ppl_Polyhedron_is_bounded(+Handle)
```

*Succeeds if and only if the polyhedron referenced by  $Handle$  is bounded.*

```
ppl_Polyhedron_contains_integer_point(+Handle)
```

*Succeeds if and only if the polyhedron referenced by  $Handle$  contains an integer point.*

```
ppl_Polyhedron_is_topologically_closed(+Handle)
```

*Succeeds if and only if the polyhedron referenced by  $Handle$  is topologically closed.*

```
ppl_Polyhedron_is_discrete(+Handle)
```

*Succeeds if and only if the polyhedron referenced by  $Handle$  is discrete.*

```
ppl_Polyhedron_bounds_from_above(+Handle, +Lin_Expr)
```

*Succeeds if and only if  $Lin\_Expr$  is bounded from above in the polyhedron referenced by  $Handle$ .*

```
ppl_Polyhedron_bounds_from_below(+Handle, +Lin_Expr)
```

*Succeeds if and only if  $Lin\_Expr$  is bounded from below in the polyhedron referenced by  $Handle$ .*

```
ppl_Polyhedron_contains_Polyhedron(+Handle_1, +Handle_2)
```

*Succeeds if and only if the polyhedron referenced by  $Handle_2$  is included in or equal to the polyhedron referenced by  $Handle_1$ .*

```
ppl_Polyhedron_strictly_contains_Polyhedron(+Handle_1, +Handle_2)
```



*Succeeds if and only if the polyhedron referenced by `Handle_2` is included in but not equal to the polyhedron referenced by `Handle_1`.*

```
ppl_Polyhedron_is_disjoint_from_Polyhedron(+Handle_1, +Handle_2)
```

*Succeeds if and only if the polyhedron referenced by `Handle_2` is disjoint from the polyhedron referenced by `Handle_1`.*

```
ppl_Polyhedron_equals_Polyhedron(+Handle_1, +Handle_2)
```

*Succeeds if and only if the polyhedron referenced by `Handle_1` is equal to the polyhedron referenced by `Handle_2`.*

```
ppl_Polyhedron_OK(+Handle)
```

*Succeeds only if the polyhedron referenced by `Handle` is well formed, i.e., if it satisfies all its implementation invariants. Useful for debugging purposes.*

```
ppl_Polyhedron_constrains(+Handle, +PPL_Var)
```

*Succeeds if and only if the polyhedron referenced by `Handle` constrains the dimension `PPL_Var`.*

### Predicates that return information about the polyhedron

These predicates will obtain more detailed information about the polyhedron unifying some of their arguments with the results.

```
ppl_Polyhedron_space_dimension(+Handle, ?Dimension_Type)
```

*Unifies `Dimension_Type` with the dimension of the vector space enclosing the polyhedron referenced by `Handle`.*

```
ppl_Polyhedron_affine_dimension(+Handle, ?Dimension_Type)
```

*Unifies `Dimension_Type` with the affine dimension of the polyhedron referenced by `Handle`.*

```
ppl_Polyhedron_relation_with_constraint(+Handle, +Constraint, ?Relation_List)
```

*Unifies `Relation_List` with the list of relations the polyhedron referenced by `Handle` has with `Constraint`. The possible relations are listed in the grammar rules above.*

```
ppl_Polyhedron_relation_with_generator(+Handle, +Generator, ?Relation_List)
```

*Unifies `Relation_List` with the list of relations the polyhedron referenced by `Handle` has with `Generator`. The possible relations are listed in the grammar rules above.*

```
ppl_Polyhedron_relation_with_congruence(+Handle, +Congruence, ?Relation_List)
```

*Unifies `Relation_List` with the list of relations the polyhedron referenced by `Handle` has with `Congruence`. The possible relations are listed in the grammar rules above.*

```
ppl_Polyhedron_get_constraints(+Handle, ?Constraint_System)
```

*Unifies `Constraint_System` with the constraints (in the form of a list) in the constraint system satisfied by the polyhedron referenced by `Handle`.*

```
ppl_Polyhedron_get_congruences(+Handle, ?Congruence_System)
```

*Unifies `Congruence_System` with the congruences (in the form of a list) in the congruence system satisfied by the polyhedron referenced by `Handle`.*

```
ppl_Polyhedron_get_generators(+Handle, ?Generator_System)
```



*Unifies Generator\_System with the generators (in the form of a list) in the generator system for the polyhedron referenced by Handle.*

`ppl_Polyhedron_get_minimized_constraints(+Handle, ?Constraint_System)`

*Unifies Constraint\_System with the constraints (in the form of a list) in the minimized constraint system satisfied by the polyhedron referenced by Handle.*

`ppl_Polyhedron_get_minimized_congruences(+Handle, ?Congruence_System)`

*Unifies Congruence\_System with the congruences (in the form of a list) in the minimized congruence system for the polyhedron referenced by Handle.*

`ppl_Polyhedron_get_minimized_generators(+Handle, ?Generator_System)`

*Unifies Generator\_System with the generators (in the form of a list) in the minimized generator system satisfied by the polyhedron referenced by Handle.*

`ppl_Polyhedron_maximize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

*Succeeds if and only if polyhedron P referenced by Handle is not empty and Lin\_Expr is bounded from above in P.*

*Coeff\_1 is unified with the numerator of the supremum value and Coeff\_2 with the denominator of the supremum value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.*

`ppl_Polyhedron_minimize(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean)`

*Succeeds if and only if polyhedron P referenced by Handle is not empty and Lin\_Expr is bounded from below in P.*

*Coeff\_1 is unified with the numerator of the infimum value and Coeff\_2 with the denominator of the infimum value. If the infimum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.*

`ppl_Polyhedron_maximize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

*Succeeds if and only if polyhedron P referenced by Handle is not empty and Lin\_Expr is bounded from above in P.*

*Coeff\_1 is unified with the numerator of the supremum value and Coeff\_2 with the denominator of the supremum value and Point with a point or closure point where Lin\_Expr reaches this value. If the supremum is also the maximum, Boolean is unified with the atom true and, otherwise, unified with the atom false.*

`ppl_Polyhedron_minimize_with_point(+Handle, +Lin_Expr, ?Coeff_1, ?Coeff_2, ?Boolean, ?Point)`

*Succeeds if and only if polyhedron P referenced by Handle is not empty and Lin\_Expr is bounded from below in P.*

*Coeff\_1 is unified with the numerator of the infimum value and Coeff\_2 with the denominator of the infimum value and Point with a point or closure point where Lin\_Expr reaches this value. If the infimum is also the minimum, Boolean is unified with the atom true and, otherwise, unified with the atom false.*

`ppl_Polyhedron_external_memory_in_bytes(+Handle, ?Number)`

*Unifies Number with the size of the total memory in bytes occupied by the polyhedron referenced by Handle.*

`ppl_Polyhedron_total_memory_in_bytes(+Handle, ?Number)`



*Unifies `Number` with the size of the external memory in bytes occupied by the polyhedron referenced by `Handle`.*

### Ascii dump predicate

This output predicate is useful for debugging.

```
ppl_Polyhedron_ascii_dump(+Handle)
```

*Dumps an ascii representation of the PPL internal state for the polyhedron referenced by `Handle` on the standard output.*

### Space-dimension preserving predicates that may change the polyhedron

These predicates may modify the polyhedron referred to by the handle in first argument; the (dimension of the) vector space in which it is embedded is unchanged.

#### Predicates that may change the polyhedron by adding to its constraint or generator descriptions

Note that there are two forms of these predicates differentiated in the names by the words "add" or "refine with"; see Section *Generic Operations on Semantic Geometric Descriptors* in the main *PPL User Manual* for the differences in the semantics and therefore, the expected behavior, between these forms.

```
ppl_Polyhedron_add_constraint(+Handle, +Constraint)
```

*Updates the polyhedron referenced by `Handle` to one obtained by adding `Constraint` to its constraint system. For a C polyhedron, `Constraint` must be an equality or a non-strict inequality.*

```
ppl_Polyhedron_add_congruence(+Handle, +Congruence)
```

*Updates the polyhedron referenced by `Handle` to one obtained by adding `Congruence` to its congruence system. For a C polyhedron, `Congruence` must be an equality.*

```
ppl_Polyhedron_add_generator(+Handle, +Generator)
```

*Updates the polyhedron referenced by `Handle` to one obtained by adding `Generator` to its generator system. For a C polyhedron, `Generator` must be a line, ray or point.*

```
ppl_Polyhedron_add_constraints(+Handle, +Constraint_System)
```

*Updates the polyhedron referenced by `Handle` to one obtained by adding to its constraint system the constraints in `Constraint_System`. For a C polyhedron, `Constraints` must be a list of equalities and non-strict inequalities.*

```
ppl_Polyhedron_add_congruences(+Handle, +Congruence_System)
```

*Updates the polyhedron referenced by `Handle` to one obtained by adding to its congruence system the congruences in `Congruence_System`. For a C polyhedron, `Congruences` must be a list of equalities.*

```
ppl_Polyhedron_add_generators(+Handle, +Generator_System)
```

*Updates the polyhedron referenced by `Handle` to one obtained by adding to its generator system the generators in `Generator_System`. For a C polyhedron, `Generators` must be a list of lines, rays and points.*

```
ppl_Polyhedron_refine_with_constraint(+Handle, +Constraint)
```

*Updates the polyhedron referenced by `Handle` to one obtained by refining its constraint system with `Constraint`.*

```
ppl_Polyhedron_refine_with_congruence(+Handle, +Congruence)
```



*Updates the polyhedron referenced by Handle to one obtained by refining its congruence system with Congruence.*

```
ppl_Polyhedron_refine_with_constraints( +Handle, +Constraint_System)
```

*Updates the polyhedron referenced by Handle to one obtained by refining its constraint system with the constraints in Constraint\_System.*

```
ppl_Polyhedron_refine_with_congruences( +Handle, +Congruence_System)
```

*Updates the polyhedron referenced by Handle to one obtained by refining its congruence system with the congruences in Congruence\_System.*

### Predicates that transform the polyhedron

These predicates enable transformations such as taking the topological closure (which for the domain of C polyhedron is the identity transformation), unconstraining a specified dimension as explained in the main *PPL User Manual* in Section *Cylindrification Operator* and several different image and preimage affine transfer relations; for details of the latter see Sections *Images and Preimages of Affine Transfer Relations* and *Generalized Affine Relations*

```
ppl_Polyhedron_topological_closure_assign(+Handle)
```

*Assigns to the polyhedron referenced by Handle its topological closure.*

```
ppl_Polyhedron_unconstrain_space_dimension(+Handle, +PPL_Var)
```

*Modifies the polyhedron P referenced by Handle by unconstraining the space dimension PPL\_Var.*

```
ppl_Polyhedron_unconstrain_space_dimensions(+Handle, +List_of_PPL_Var)
```

*Modifies the polyhedron P referenced by Handle by unconstraining the space dimensions that are specified in List\_of\_PPL\_Var. The presence of duplicates in List\_of\_PPL\_Var is a waste but an innocuous one.*

```
ppl_Polyhedron_affine_image(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

*Transforms the polyhedron referenced by Handle assigning the affine expression for Lin\_Expr/Coeff to PPL\_Var.*

```
ppl_Polyhedron_affine_preimage(+Handle, +PPL_Var, +Lin_Expr, +Coeff)
```

*Transforms the polyhedron referenced by Handle substituting the affine expression for Lin\_Expr/Coeff to PPL\_Var.*

```
ppl_Polyhedron_bounded_affine_image(+Handle, +PPL_Var, +Lin_Expr_1, +Lin_Expr_2, +Coeff)
```

*Assigns to polyhedron P referenced by Handle the generalized image with respect to the generalized affine transfer relation  $Lin\_Expr\_1/Coeff \leq PPL\_Var \leq Lin\_Expr\_2/Coeff$ .*

```
ppl_Polyhedron_bounded_affine_preimage(+Handle, +PPL_Var, +Lin_Expr_1, +-Lin_Expr_2, +Coeff)
```

*Assigns to polyhedron P referenced by Handle the generalized preimage with respect to the generalized affine transfer relation  $Lin\_Expr\_1/Coeff \leq PPL\_Var \leq Lin\_Expr\_2/Coeff$ .*

```
ppl_Polyhedron_generalized_affine_image(+Handle, +PPL_Var, +Relation_Symbol, +Lin_Expr, +Coeff)
```

*Assigns to polyhedron P referenced by Handle the generalized image with respect to the generalized affine transfer relation  $PPL\_Var \bowtie Lin\_Expr/Coeff$ , where  $\bowtie$  is the symbol represented by Relation\_Symbol.*



```
ppl_Polyhedron_generalized_affine_preimage(+Handle, +PPL_Var, +Relation-
_Symbol, +Lin_Expr, +Coeff)
```

*Assigns to polyhedron  $P$  referenced by  $Handle$  the generalized preimage with respect to the generalized affine transfer relation  $PPL\_Var \bowtie Lin\_Expr/Coeff$ , where  $\bowtie$  is the symbol represented by  $Relation\_Symbol$ .*

```
ppl_Polyhedron_generalized_affine_image_lhs_rhs(+Handle, +Lin_Expr_1, +-
Relation_Symbol, +Lin_Expr_2)
```

*Assigns to polyhedron  $P$  referenced by  $Handle$  the generalized image with respect to the generalized affine transfer relation  $Lin\_Expr_1 \bowtie Lin\_Expr_2$ , where  $\bowtie$  is the symbol represented by  $Relation\_Symbol$ .*

```
ppl_Polyhedron_generalized_affine_preimage_lhs_rhs(+Handle, +Lin_Expr_-
1, +Relation_Symbol, +Lin_Expr_2)
```

*Assigns to polyhedron  $P$  referenced by  $Handle$  the generalized preimage with respect to the generalized affine transfer relation  $Lin\_Expr_1 \bowtie Lin\_Expr_2$ , where  $\bowtie$  is the symbol represented by  $Relation\_Symbol$ .*

#### Predicates whose results depend on more than one polyhedron

These predicates include the binary operators which will assign to the polyhedron referred to by the first argument its combination with the polyhedron referred to by the second argument as described in the main *PPL User Manual* in Sections *Intersection and Convex Polyhedral Hull* and *Convex Polyhedral Difference*; and a linear partitioning operator described below.

```
ppl_Polyhedron_intersection_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron  $P$  referenced by  $Handle\_1$  the intersection of  $P$  and the polyhedron referenced by  $Handle\_2$ .*

```
ppl_Polyhedron_upper_bound_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron  $P$  referenced by  $Handle\_1$  the upper bound of  $P$  and the polyhedron referenced by  $Handle\_2$ .*

```
ppl_Polyhedron_difference_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron  $P$  referenced by  $Handle\_1$  the difference of  $P$  and the polyhedron referenced by  $Handle\_2$ .*

```
ppl_Polyhedron_time_elapse_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron  $P$  referenced by  $Handle\_1$  the time elapse of  $P$  and the polyhedron referenced by  $Handle\_2$ .*

```
ppl_Polyhedron_poly_hull(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron  $P$  referenced by  $Handle\_1$  the poly-hull of  $P$  and the polyhedron referenced by  $Handle\_2$ .*

```
ppl_Polyhedron_poly_difference(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron  $P$  referenced by  $Handle\_1$  the poly-difference of  $P$  and the polyhedron referenced by  $Handle\_2$ .*

```
ppl_Polyhedron_upper_bound_assign_if_exact(+Handle_1, +Handle_2)
```

*Succeeds if the least upper bound of the polyhedron  $P\_1$  referenced by  $Handle\_1$  with the polyhedron referenced by  $Handle\_2$  is exact; in which case the least upper bound is assigned to  $P\_1$ ; fails otherwise.*

```
ppl_Polyhedron_poly_hull_assign_if_exact(+Handle_1, +Handle_2)
```





*Succeeds if the least upper bound of the polyhedron  $P_1$  referenced by  $Handle_1$  with the polyhedron referenced by  $Handle_2$  is exact; in which case the least upper bound is assigned to  $P_1$ ; fails otherwise.*

```
ppl_Polyhedron_simplify_using_context_assign(+Handle_1, +Handle_2, ?Boolean)
```

*Succeeds if and only if the intersection of polyhedron  $P_1$  referenced by  $Handle_1$  and the polyhedron  $P_2$  referenced by  $Handle_2$  is non-empty. Assigns to  $P_1$  its meet-preserving simplification with respect to  $P_2$ .*

```
ppl_Polyhedron_linear_partition(+Handle_1, +Handle_2, -Handle_3, -Handle_4)
```

*$Handle_1$  and  $Handle_2$  are handles for elements  $P_1$  and  $P_2$  in the Polyhedron domain. The predicate unifies handle  $Handle_3$  to a reference to the intersection of  $P_1$  and  $P_2$  and  $Handle_4$  to a reference to a pointset powerset of nnc polyhedra  $P_4$ ; where  $P_4$  is the linear partition of  $P_1$  with respect to  $P_2$ . This predicate is only provided if the class `Pointset_Powerset_NNC_Polyhedron` has been enabled when configuring the library.*

### Predicates for widening and extrapolation

In addition to the above binary operators, there are also a number of widening, extrapolation and narrowing operators as described in the main *PPL User Manual* in Sections *Widening Operators*, *Widening with Tokens* and *Extrapolation Operators*. Note that for all these widening and extrapolation predicates to behave as specified the polyhedron referred to by the second argument has to be contained in (or equal to) the polyhedron referred to by the first argument.

```
ppl_Polyhedron_BHRZ03_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the BHRZ03-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$ . The widening with tokens delay technique is applied with  $C\_unsigned\_1$  tokens;  $C\_unsigned\_2$  is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Polyhedron_H79_widening_assign_with_tokens(+Handle_1, +Handle_2, +-C_unsigned_1, ?C_unsigned_2)
```

*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the H79-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$ . The widening with tokens delay technique is applied with  $C\_unsigned\_1$  tokens;  $C\_unsigned\_2$  is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Polyhedron_BHRZ03_widening_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the BHRZ03-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$ .*

```
ppl_Polyhedron_H79_widening_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the H79-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$ .*

```
ppl_Polyhedron_widening_assign_with_tokens(+Handle_1, +Handle_2, +C_unsigned_1, ?C_unsigned_2)
```

*Same as predicate `ppl_Polyhedron_H79_widening_assign_with_tokens/4`*

```
ppl_Polyhedron_widening_assign(+Handle_1, +Handle_2)
```

*Same as predicate `ppl_Polyhedron_H79_widening_assign/2`*

```
ppl_Polyhedron_limited_BHRZ03_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```





*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the BHRZ03-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$  intersected with the constraints in  $Constraint\_System$  that are satisfied by all the points of  $P_1$ . The widening with tokens delay technique is applied with  $C\_unsigned\_1$  tokens;  $C\_unsigned\_2$  is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Polyhedron_bounded_BHRZ03_extrapolation_assign_with_tokens(+Handle_1, +Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the BHRZ03-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$  intersected with the constraints in  $Constraint\_System$  that are satisfied by all the points of  $P_1$ , further intersected with the smallest box containing  $P_1$ . The widening with tokens delay technique is applied with  $C\_unsigned\_1$  tokens;  $C\_unsigned\_2$  is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Polyhedron_limited_H79_extrapolation_assign_with_tokens(+Handle_1, +-Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the H79-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$  intersected with the constraints in  $Constraint\_System$  that are satisfied by all the points of  $P_1$ . The widening with tokens delay technique is applied with  $C\_unsigned\_1$  tokens;  $C\_unsigned\_2$  is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Polyhedron_bounded_H79_extrapolation_assign_with_tokens(+Handle_1, +-Handle_2, +Constraint_System, +C_unsigned_1, ?C_unsigned_2)
```

*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the H79-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$  intersected with the constraints in  $Constraint\_System$  that are satisfied by all the points of  $P_1$ , further intersected with the smallest box containing  $P_1$ . The widening with tokens delay technique is applied with  $C\_unsigned\_1$  tokens;  $C\_unsigned\_2$  is unified with the number of tokens remaining at the end of the operation.*

```
ppl_Polyhedron_limited_BHRZ03_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)
```

*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the BHRZ03-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$  intersected with the constraints in  $Constraint\_System$  that are satisfied by all the points of  $P_1$ .*

```
ppl_Polyhedron_bounded_BHRZ03_extrapolation_assign(+Handle_1, +Handle_2, +Constraint_System)
```

*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the BHRZ03-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$  intersected with the constraints in  $Constraint\_System$  that are satisfied by all the points of  $P_1$ , further intersected with the smallest box containing  $P_1$ .*

```
ppl_Polyhedron_limited_H79_extrapolation_assign(+Handle_1, +Handle_2, +-Constraint_System)
```

*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the H79-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$  intersected with the constraints in  $Constraint\_System$  that are satisfied by all the points of  $P_1$ .*

```
ppl_Polyhedron_bounded_H79_extrapolation_assign(+Handle_1, +Handle_2, +-Constraint_System)
```

*Assigns to the polyhedron  $P_1$  referenced by  $Handle_1$  the H79-widening of  $P_1$  with the polyhedron referenced by  $Handle_2$  intersected with the constraints in  $Constraint\_System$  that are satisfied by all the points of  $P_1$ , further intersected with the smallest box containing  $P_1$ .*



### Predicates that may modify the vector space

These predicates enable the modification of the vector space of the polyhedron referred to in the first argument.

#### Predicate for concatenation

For more information on this operation, see Section *Concatenating Polyhedra*, of the main *PPL User Manual*.

```
ppl_Polyhedron_concatenate_assign(+Handle_1, +Handle_2)
```

*Assigns to the polyhedron  $P$  referenced by  $Handle\_1$  the concatenation of  $P$  and the polyhedron referenced by  $Handle\_2$ .*

#### Predicates for mapping dimensions or changing the vector space

These predicates enable the modification of the vector space of the polyhedron referred to in the first argument. These predicates enable the modification of the vector space of the polyhedron referred to in the first argument. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Adding New Dimensions to the Vector Space*, *Removing Dimensions from the Vector Space*, *Mapping the Dimensions of the Vector Space*, *Expanding One Dimension of the Vector Space to Multiple Dimensions* and *Folding Multiple Dimensions of the Vector Space into One Dimension*.

```
ppl_Polyhedron_add_space_dimensions_and_embed(+Handle, +Dimension_Type)
```

*Adds  $Dimension\_Type$  new dimensions to the space enclosing the polyhedron  $P$  referenced by  $Handle$  and and embeds  $P$  in this space.*

```
ppl_Polyhedron_add_space_dimensions_and_project(+Handle, +Dimension_Type)
```

*Adds  $Dimension\_Type$  new dimensions to the space enclosing the polyhedron  $P$  referenced by  $Handle$  and and projects  $P$  in this space.*

```
ppl_Polyhedron_remove_space_dimensions(+Handle, +List_of_PPL_Vars)
```

*Removes from the vector space enclosing the polyhedron  $P$  referenced by  $Handle$  the space dimensions that are specified in  $List\_of\_PPL\_Var$ . The presence of duplicates in  $List\_of\_PPL\_Var$  is a waste but an innocuous one.*

```
ppl_Polyhedron_remove_higher_space_dimensions(+Handle, +Dimension_Type)
```

*Removes the higher dimensions from the vector space enclosing the polyhedron  $P$  referenced by  $Handle$  so that, upon successful return, the new space dimension is  $Dimension\_Type$ .*

```
ppl_Polyhedron_expand_space_dimension(+Handle, +PPL_Var, +Dimension_Type)
```

*Expands the  $PPL\_Var$ -th dimension of the vector space enclosing the polyhedron referenced by  $Handle$  to  $Dimension\_Type$  new space dimensions.*

```
ppl_Polyhedron_fold_space_dimensions(+Handle, +List_of_PPL_Vars, +PPL_Var)
```

*Modifies the polyhedron referenced by  $Handle$  by folding the space dimensions contained in  $List\_of\_PPL\_Vars$  into dimension  $PPL\_Var$ . The presence of duplicates in  $List\_of\_PPL\_Vars$  is a waste but an innocuous one.*

```
ppl_Polyhedron_map_space_dimensions(+Handle, +P_Func)
```

*Remaps the dimensions of the vector space according to a partial function. This function is specified by means of the  $P\_Func$ , which has  $n$  entries. The result is undefined if  $P\_Func$  does not encode a partial function.*



## Ad hoc Predicates for Other Domains

### Extra Predicates Specifically for the Pointset Powerset Domains

The powerset domains can be instantiated by taking as a base domain any fixed semantic geometric description (C and NNC polyhedra, BD and octagonal shapes, boxes and grids). An element of the powerset domain represents a disjunctive collection of base objects (its disjuncts), all having the same space dimension. For more information on this construct, see Section *The Powerset Domain* in the main *PPL User Manual*.

Besides the predicates that are available in all semantic geometric descriptions (whose documentation is not repeated here), the powerset domain also provides several ad hoc predicates. These are specified below, instantiated for the PPL domain `Pointset_Powerset_C_Polyhedron`. Note that predicates for other pointset powerset domains will follow similar patterns.

#### Predicates for pointset powerset iterators and disjuncts.

Iterators allow the user to examine and change individual elements (called here disjuncts) of a pointset powerset. Detailed descriptions for adding and removing disjuncts can be found in the main *PPL User Manual* in Section *Adding a Disjunct*. The following predicates support useful operations on these iterators and disjuncts via the usual handles.

```
ppl_new_Pointset_Powerset_C_Polyhedron_iterator_from_iterator(+Iterator_1, -Iterator_2)
```

*Builds a new iterator `it` from the iterator referenced by `Iterator_1`. `Iterator_2` is unified with the handle for `it`.*

```
ppl_Pointset_Powerset_C_Polyhedron_begin_iterator(+Handle, -Iterator)
```

*Unifies `Iterator` with a handle to an iterator "pointing" to the beginning of the sequence of disjuncts of the powerset referred to by `Handle`.*

```
ppl_Pointset_Powerset_C_Polyhedron_end_iterator(+Handle, -Iterator)
```

*Unifies `Iterator` with a handle to an iterator "pointing" to the end of the sequence of disjuncts of the powerset referred to by `Handle`.*

```
ppl_Pointset_Powerset_C_Polyhedron_iterator_equals_iterator(+Iterator_1, +Iterator_2)
```

*Succeeds if and only if the iterator referenced by `Iterator_1` is equal to the iterator referenced by `Iterator_2`.*

```
ppl_Pointset_Powerset_C_Polyhedron_iterator_increment(+Iterator)
```

*Increments the iterator referenced by `Iterator` so that it "points" to the next disjunct.*

```
ppl_Pointset_Powerset_C_Polyhedron_iterator_decrement(+Iterator)
```

*Decrements the iterator referenced by `Iterator` so that it "points" to the previous disjunct.*

```
ppl_Pointset_Powerset_C_Polyhedron_iterator_get_disjunct(+Iterator, -Handle)
```

*Unifies with `Handle` a reference to the disjunct referred to by `Iterator_1`.*

```
ppl_delete_Pointset_Powerset_C_Polyhedron_iterator(+Iterator)
```

*Invalidates the handle referenced by `Iterator`: this makes sure the corresponding resources will eventually be released.*

```
ppl_Pointset_Powerset_C_Polyhedron_add_disjunct(+Handle_1, +Handle_2)
```

*Adds to the pointset powerset referenced by `Handle_1` a disjunct referred to by `Handle_2`.*



```
ppl_Pointset_Powerset_C_Polyhedron_drop_disjunct(+Handle, +Iterator)
```

*If it is the iterator referred to by `Iterator`, drops from the pointset powerset referenced by `Handle` the disjunct pointed to by it and assigns to it an iterator to the next disjunct.*

```
ppl_Pointset_Powerset_C_Polyhedron_drop_disjuncts(+Handle, +Iterator_1,
+Iterator_2)
```

*If `it_1` and `it_2` are the iterators referred to by `Iterator_1` and `Iterator_2`, respectively, drops from the pointset powerset referenced by `Handle` all the disjuncts from `it_1` to `it_2` (excluded).*

#### Other Ad Hoc Predicates for the pointset powerset domains.

Collected here are some other predicates that are specific to pointset powersets of C polyhedra; these provide operations for simplifying the powerset, geometric comparisons and widening and extrapolation. Detailed descriptions of these can be found in the main *PPL User Manual* in Sections *Geometric Comparisons*, *Certificate-Based Widening*, *Powerset Extrapolation Operators*.

```
ppl_Pointset_Powerset_C_Polyhedron_pairwise_reduce(+Handle)
```

*Assigns the result of pairwise reduction on the pointset powerset referenced by `Handle`.*

```
ppl_Pointset_Powerset_C_Polyhedron_omega_reduce(+Handle)
```

*Assigns the result of omega reduction on the pointset powerset referenced by `Handle`.*

```
ppl_Pointset_Powerset_C_Polyhedron_geometrically_covers_Pointset_Powerset-
_C_Polyhedron(+Handle_1, +Handle_2)
```

*Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically covers the pointset powerset referenced by `Handle_1`; see Section *Geometric Comparisons* in the main *PPL User Manual*.*

```
ppl_Pointset_Powerset_C_Polyhedron_geometrically_equals_Pointset_Powerset-
_C_Polyhedron(+Handle_1, +Handle_2)
```

*Succeeds if and only if the pointset powerset referenced by `Handle_2` geometrically equals the pointset powerset referenced by `Handle_1`; see Section *Geometric Comparisons* in the main *PPL User Manual*.*

```
ppl_Pointset_Powerset_C_Polyhedron_BHZ03_BHRZ03_BHRZ03_widening_assign(+
Handle_1, +Handle_2)
```

*Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHRZ03-widening certified by the convergence certificate for BHRZ03.*

```
ppl_Pointset_Powerset_C_Polyhedron_BHZ03_H79_H79_widening_assign(+Handle-
_1, +Handle_2)
```

*Assigns to the pointset powerset `P_1` referenced by `Handle_1` the BHZ03-widening between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening certified by the convergence certificate for H79.*

```
ppl_Pointset_Powerset_C_Polyhedron_BGP99_BHRZ03_extrapolation_assign(+
Handle_1, +Handle_2, C_unsigned)
```

*Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the BHRZ03-widening and the cardinality threshold `C_unsigned`.*

```
ppl_Pointset_Powerset_C_Polyhedron_BGP99_H79_extrapolation_assign(+Handle-
_1, +Handle_2, C_unsigned)
```

*Assigns to the pointset powerset `P_1` referenced by `Handle_1` the result of applying the BGP99 extrapolation operator between `P_1` and the pointset powerset referenced by `Handle_2`, using the H79-widening*



and the cardinality threshold `C_unsigned`.

## 6 Compilation and Installation

When the Parma Polyhedra Library is configured, it tests for the existence of each supported Prolog system. If a supported Prolog system is correctly installed in a standard location, things are arranged so that the corresponding interface is built and installed.

The Prolog interface files are all installed in the directory `prefix/lib/ppl`. Since this includes shared and dynamically loaded libraries, you must make your dynamic linker/loader aware of this fact. If you use a GNU/Linux system, try the commands `man ld.so` and `man ldconfig` for more information.

As an option, the Prolog interface can track the creation and disposal of polyhedra. In fact, differently from native Prolog data, PPL polyhedra must be explicitly disposed and forgetting to do so is a very common mistake. To enable this option, configure the library adding `-DPROLOG_TRACK_ALLOCATION` to the options passed to the C++ compiler. Your configure command would then look like

```
path/to/configure --with-cxxflags="-DPROLOG_TRACK_ALLOCATION" ...
```

## 7 Prolog Interface System-Dependent Features

### CIAO Prolog

The Ciao Prolog interface to the PPL is available both as “PPL enhanced” Ciao Prolog interpreter and as a library that can be linked to Ciao Prolog programs. Only Ciao Prolog versions 1.10 ‘#5 and later are supported.’

So that it can be used with the Ciao Prolog PPL interface, the Ciao Prolog installation must be configured with the `--disable-regs` option.

#### The `ppl_ciao` Executable

If an appropriate version of Ciao Prolog is installed on the machine on which you compiled the library, the command `make install` will install the executable `ppl_ciao` in the directory `prefix/bin`. The `ppl_ciao` executable is simply the Ciao Prolog interpreter with the Parma Polyhedra Library linked in. The only thing you should do to use the library is to call `ppl_initialize/0` before any other PPL predicate and to call `ppl_finalize/0` when you are done with the library.

#### Linking the Library To Ciao Prolog Programs

In order to allow linking Ciao Prolog programs to the PPL, the following files are installed in the directory `prefix/lib/ppl`: `ppl_ciao.pl` contains the required foreign declarations; `libppl_ciao.*` contain the executable code for the Ciao Prolog interface in various formats (static library, shared library, libtool library). If your Ciao Prolog program is constituted by, say, `source1.pl` and `source2.pl` and you want to create the executable `myprog`, your compilation command may look like

```
ciaoc -o myprog prefix/lib/ppl/ppl_ciao.pl ciao_pl_check.pl \
-L '-Lprefix/lib/ppl -lppl_ciao -Lprefix/lib -lppl -lgmpxx -lgmp -lstdc++'
```

### GNU Prolog

The GNU Prolog interface to the PPL is available both as a “PPL enhanced” GNU Prolog interpreter and as a library that can be linked to GNU Prolog programs. The only GNU Prolog version that is known to work is a patched version of the “unstable version” tagged `20040608` (which unpacks to a directory



called `gprolog-1.2.18`). The patch is contained in the `interfaces/Prolog/GNU/README` file of the PPL's distribution.

So that it can be used with the GNU Prolog PPL interface (and, for that matter, with any foreign code), the GNU Prolog installation must be configured with the `--disable-regs` option.

### The `ppl_gprolog` Executable

If an appropriate version of GNU Prolog is installed on the machine on which you compiled the library, the command `make install` will install the executable `ppl_gprolog` in the directory `prefix/bin`. The `ppl_gprolog` executable is simply the GNU Prolog interpreter with the Parma Polyhedra Library linked in. The only thing you should do to use the library is to call `ppl_initialize/0` before any other PPL predicate and to call `ppl_finalize/0` when you are done with the library.

### Linking the Library To GNU Prolog Programs

In order to allow linking GNU Prolog programs to the PPL, the following files are installed in the directory `prefix/lib/ppl`: `ppl_gprolog.pl` contains the required foreign declarations; `libppl_gprolog.*` contain the executable code for the GNU Prolog interface in various formats (static library, shared library, libtool library). If your GNU Prolog program is constituted by, say, `source1.pl` and `source2.pl` and you want to create the executable `myprog`, your compilation command may look like

```
gplc -o myprog prefix/lib/ppl/ppl_gprolog.pl source1.pl source2.pl \
-L '-Lprefix/lib/ppl -lppl_gprolog -Lprefix/lib -lppl -lgmpxx -lgmp -lstdc++'
```

GNU Prolog uses several stacks to execute a Prolog program each with a pre-defined default size. If the size of a stack is too small for the application an overflow will occur. To change the default size of a stack, the user has to set the value of the relevant environment variable; in particular, to execute some of the tests, we found it necessary to increase the size of `GLOBALSZ`. Thus, for the above example, the compilation command would be

```
GLOBALSZ=32768 gplc -o myprog prefix/lib/ppl/ppl_gprolog.pl source1.pl source2.
pl \
-L '-Lprefix/lib/ppl -lppl_gprolog -Lprefix/lib -lppl -lgmpxx -lgmp -lstdc++'
```

More information on adjusting the size of the stacks can be found in Section 3.3 in the [GNU Prolog Manual](#)

### SICStus Prolog

The SICStus Prolog interface to the PPL is available both as a statically linked module or as a dynamically linked one. Only SICStus Prolog versions 3.9.0 and later are supported.

#### The Statically Linked `ppl_sicstus` Executable

If an appropriate version of SICStus Prolog is installed on the machine on which you compiled the library, the command `make install` will install the executable `ppl_sicstus` in the directory `prefix/bin`. The `ppl_sicstus` executable is simply the SICStus Prolog system with the Parma Polyhedra Library statically linked. The only thing you should do to use the library is to load `prefix/lib/ppl/ppl_sicstus.pl`.

#### Loading the SICStus Interface Dynamically

In order to dynamically load the library from SICStus Prolog you should simply load `prefix/lib/ppl/ppl_sicstus.pl`. Notice that, for dynamic linking to work, you should have configured the library with the `--enable-shared` option.



## SWI-Prolog

The SWI-Prolog interface to the PPL is available both as a statically linked module or as a dynamically linked one. Only SWI-Prolog version 5.6.0 and later versions are supported.

### The `ppl_pl` Executable

If an appropriate version of SWI-Prolog is installed on the machine on which you compiled the library, the command `make install` will install the executable `ppl_pl` in the directory `prefix/bin`. The `ppl_pl` executable is simply the SWI-Prolog shell with the Parma Polyhedra Library statically linked: from within `ppl_pl` all the services of the library are available without further action.

### Loading the SWI-Prolog Interface Dynamically

In order to dynamically load the library from SWI-Prolog you should simply load `prefix/lib/ppl/ppl-_swiprolog.pl`. This will invoke `ppl_initialize/0` and `ppl_finalize/0` automatically. Alternatively, you can load the library directly with

```
:- load_foreign_library('prefix/lib/ppl/libppl_swiprolog').
```

This will call `ppl_initialize/0` automatically. Analogously,

```
:- unload_foreign_library('prefix/lib/ppl/libppl_swiprolog').
```

will, as part of the unload process, invoke `ppl_finalize/0`.

Notice that, for dynamic linking to work, you should have configured the library with the `--enable-shared` option.

## XSB

The XSB Prolog interface to the PPL is available as a dynamically linked module. Only some CVS versions of XSB starting from 2 July 2005 are known to work. CVS versions starting from 11 November 2005 are known not to work.

In order to dynamically load the library from XSB you should load the `ppl_xsb` module and import the predicates you need. For things to work, you may have to copy the files `prefix/lib/ppl/ppl-_xsb.xwam` and `prefix/lib/ppl/ppl_xsb.so` in your current directory or in one of the XSB library directories.

## YAP

The YAP Prolog interface to the PPL is available as a dynamically linked module. Only YAP versions following 5.1.0 and CVS HEAD versions starting from 4 January 2006 are supported. Notice that support for unbounded integers in YAP is young and may have errors that could affect programs using the PPL (see, e.g., <http://www.cs.unipr.it/pipermail/ppl-devel/2006-January/007780.-html>).

In order to dynamically load the library from YAP you should simply load `prefix/lib/ppl/ppl-_yap.pl`. This will invoke `ppl_initialize/0` automatically; it is the programmer's responsibility to call `ppl_finalize/0` when the PPL library is no longer needed. Notice that, for dynamic linking to work, you should have configured the library with the `--enable-shared` option.

# 8 Module Index



## 8.1 Modules

Here is a list of all modules:

**Prolog Language Interface**

**42**

## 9 Module Documentation

### 9.1 Prolog Language Interface

The Parma Polyhedra Library comes equipped with an interface for the Prolog language.





## Index

Prolog Language Interface, [42](#)

