

# Package ‘freegroup’

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**Type** Package

**Title** The Free Group

**Version** 1.1-9

**Maintainer** Robin K. S. Hankin <hankin.robin@gmail.com>

**Description** The free group in R; juxtaposition is represented by a plus. Includes inversion, multiplication by a scalar, group-theoretic power operation, and Tietze forms. To cite the package in publications please use Hankin (2022) <[doi:10.48550/ARXIV.2212.05883](https://doi.org/10.48550/ARXIV.2212.05883)>.

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**Suggests** knitr, rmarkdown, permutations, testthat, covr

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**Imports** freealg (>= 1.0-4), magic (>= 1.5-9), magrittr

**VignetteBuilder** knitr

**License** GPL-2

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## Contents

freegroup-package . . . . .	2
abelianize . . . . .	4
abs.free . . . . .	5
alpha . . . . .	6

backwards . . . . .	7
c . . . . .	8
char_to_free . . . . .	9
cumsum . . . . .	10
cycled . . . . .	11
donames . . . . .	12
dot-class . . . . .	13
Extract . . . . .	14
free . . . . .	15
getlet . . . . .	16
identity . . . . .	17
keep . . . . .	18
nielsen . . . . .	19
Ops.free . . . . .	20
primitive . . . . .	22
print . . . . .	23
reduce . . . . .	24
rfree . . . . .	26
shift . . . . .	27
size . . . . .	28
subs . . . . .	29
sum . . . . .	30
tietze . . . . .	31

<b>Index</b>	<b>33</b>
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freegroup-package	<i>The Free Group</i>
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## Description

The free group in R; juxtaposition is represented by a plus. Includes inversion, multiplication by a scalar, group-theoretic power operation, and Tietze forms. To cite the package in publications please use Hankin (2022) <doi:10.48550/ARXIV.2212.05883>.

## Details

The DESCRIPTION file:

```

Package:      freegroup
Type:         Package
Title:        The Free Group
Version:      1.1-9
Authors@R:   c(person(c("Robin", "K. S. "), "Hankin", role=c("aut","cre"), email="hankin.robin@gmail.com", comment=
Maintainer:   Robin K. S. Hankin <hankin.robin@gmail.com>
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Suggests:    knitr, rmarkdown, permutations, testthat, covr

```

LazyData: yes  
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 Author: Robin K. S. Hankin [aut, cre] (<<https://orcid.org/0000-0001-5982-0415>>)

#### Index of help topics:

Extract.free	Extract or replace parts of a free group object
Ops.free	Arithmetic Ops methods for the free group
abelianize	Abelianization of free group elements
abs.free	Absolute value of a 'free' object
alpha	Alphabetical free group elements
backwards	Write free objects backwards
c	Concatenation of free objects
char_to_free	Convert character vectors to free objects
cumsum	Cumulative sum
cycled	Cyclic reductions of a word
donames	Names attributes of free group elements
dot-class	Class "dot"
free	Objects of class 'free'
freegroup-package	The Free Group
getlet	Get letters of a freegroup object
identity	The identity element
keep	Keep or drop symbols
nielsen	Outer automorphisms of the free group
primitive	Primitive elements of the free algebra
print.free	Print free objects
reduce	Reduction of a word to reduced form
rfree	Random free objects
shift	Permute elements of a vector in a cycle
size	Bignesses of a free object
subs	Substitute and invert symbols
sum	Repeated summation by concatenation
tietze	Tietze form for free group objects

#### Author(s)

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#### Examples

```
p <- rfree(10,6,3)
x <- as.free('x')
```

```
p+x  
p^x  
sum(p)  
abelianize(p)  
subsu(p, "ab", "z")  
subs(p, a='z')  
  
discard(p+x, 'a')
```

---

abelianize

*Abelianization of free group elements*

---

### Description

Function `abelianize()` returns a word that is equivalent to its argument under assumption of Abelianness. The symbols are placed in alphabetical order.

### Usage

```
abelianize(x)  
is.abelian(x)
```

### Arguments

`x`                    An object of class `free`

### Details

Abelianizing a free group element means that the symbols can commute past one another. Abelianization is vectorized.

Function `is.abelian()` is trivial: it just checks to see whether argument `x` has its symbols in alphabetical order. It might have been better to call this `abelianized()`.

Package **frab** presents extensive R-centric functionality for dealing with the free Abelian group. It is much more efficient than this package for Abelian operations, and contains bespoke methods for working with a range of applications such as tables of counts.

### Author(s)

Robin K. S. Hankin

**Examples**

```
x <- as.free("aabAA")
x
abelianize(x)

x <- rfree(10,10,2)
x
abelianize(x)

abelianize(.[rfree(),rfree()])

p <- free(rbind(rep(1:5,4),rep(1:4,5)))
p
abelianize(p)
```

---

abs.free	<i>Absolute value of a free object</i>
----------	--

---

**Description**

Replaces every term's power with its absolute value

**Usage**

```
## S3 method for class 'free'
abs(x)
```

**Arguments**

x                    Object of class free

**Details**

Replaces every term's power with its absolute value

**Note**

The function's name is motivated by the inequality in the examples section.

**Author(s)**

Robin K. S. Hankin

**See Also**

[subs](#)

**Examples**

```

abs(abc(-5:5))

a <- rfree(10,4,7)
b <- rfree(10,4,7)

a
abs(a)

## following should all be TRUE:
all(size(abs(a+b)) <= size(abs(a) + abs(b)))
all(total(abs(a+b)) <= total(abs(a) + abs(b)))
all(number(abs(a+b)) <= number(abs(a) + abs(b)))

all(size(a+b) <= size(abs(a) + abs(b)))
all(total(a+b) <= total(abs(a) + abs(b)))
all(number(a+b) <= number(abs(a) + abs(b)))

```

---

alpha

*Alphabetical free group elements*


---

**Description**

Produces simple vectors of free group elements based on the alphabet

**Usage**

```

alpha(v)
abc(v)

```

**Arguments**

v                      Vector of integers

**Details**

Function `alpha()` takes an integer `i` and returns the letter `i` of the alphabet. Thus `alpha(3)` returns `c`. The function is vectorised: `alpha(1:3)` returns `a b c`.

Function `abc()` takes an integer `i` and returns letters 1 to `i` of the alphabet. Thus `abc(4)` returns `a.b.c.d`. The function is vectorised.

Remember that “letters of the alphabet” is just a phrase: above it refers to the default print method which can be changed, see the examples.

**Author(s)**

Robin K. S. Hankin

**Examples**

```

alpha(5) # just the single letter 'e'
abc(5)   # product of a,b,c,d,e

alpha(1:26) # the whole alphabet; c

all(alpha(1:26) == as.free(letters)) # should be TRUE

z <- alpha(26) # variable 'z' is symbol 26, aka 'z'.
abc(1:10) ^ z

abc(-5:5)
alpha(-5:5)
sum(abc(-5:5))

## bear in mind that the symbols used are purely for the print method:
jj <- LETTERS[1:10]
options(freegroup_symbols = apply(expand.grid(jj,jj),1,paste,collapse=""))
alpha(c(66,67,68,69)) # sensible output
options(freegroup_symbols=NULL) # restore to symbols to default letters
alpha(c(66,67,68,69)) # print method not very helpful now

```

---

backwards

*Write free objects backwards*


---

**Description**

Write free objects in reverse order

**Usage**

```
backwards(x)
```

**Arguments**

x                    Object of class free

**Note**

For each element of a free object, function backwards() writes the symbols in reverse order. It is distinct from rev(), see examples.

Function backwards is an involution: it is its own inverse.

**Author(s)**

Robin K. S. Hankin

**Examples**

```

abc(1:5)
backwards(abc(1:5))
rev(abc(1:5))

x <- rfree(10,5)
backwards(backwards(x)) == x # involution
all(abelianize(x) == abelianize(backwards(x))) # should be TRUE

```

c

*Concatenation of free objects***Description**

Concatenate free objects together

**Usage**

```

## S3 method for class 'free'
c(...)
## S3 method for class 'free'
rep(x, ...)

```

**Arguments**

...	In the method for <code>c()</code> , objects to be concatenated. Should all be of the same type
x	In the method for <code>rep()</code> , a free object

**Author(s)**

Robin K. S. Hankin

**Examples**

```

(x <- abc(1:3))
(y <- alpha(22:25))

c(x,y,x,x)

## NB: compare
rep(x,2)
x*2

```



---

`char_to_free`*Convert character vectors to free objects*

---

## Description

Convert character vectors to free objects

## Usage

```
char_to_matrix(x)
```

## Arguments

`x`                    A character vector

## Details

Function `char_to_matrix()` gives very basic conversion between character vectors and free objects. Current functionality is limited to strings like “aaabaacd”, which would give  $a^3ba^2cd$ . It would be nice to take a string like “ $a^3b^{-3}$ ” but this is not yet implemented.

Function `char_to_free()` is a vectorized version that coerces output to free.

## Note

The function is not particularly robust; for example, passing anything other than letters a-z or A-Z will give possibly undesirable behaviour.

Upper-case letters A-Z are interpreted by `char_to_matrix()` as the inverse of their corresponding lower-case equivalents. This behaviour is inherited by `char_to_free()` and `as.free()`, so that `as.free("A") == inverse(as.free("a"))`.

Function `char_to_free()` is consistent with the default print options (which are that the symbols are the lowercase letters a-z). If you change the symbols’ names, for example `options(freegroup_symbols=sample(letters, 26))`, then things can get confusing. The print method does not change the internal representation of a free object, which is a list of integer matrices.

## Author(s)

Robin K. S. Hankin

## See Also

[print.free](#)

**Examples**

```
char_to_matrix("aaabcABC")

rfree(10,3) + as.free('xxxxxxxxxxx')

as.free(letters)*7

all(is.id(as.free(letters) + as.free(LETTERS)))

as.free('') # identity element
```

---

cumsum

*Cumulative sum*

---

**Description**

Cumulative sum of free vectors

**Usage**

```
## S3 method for class 'free'
cumsum(x)
```

**Arguments**

x                      Vector of class free

**Author(s)**

Robin K. S. Hankin

**See Also**

[sum](#)

**Examples**

```
abc(1:6)
cumsum(abc(1:6))

x <- rfree(10,2)
cumsum(c(x, -rev(x)))
```

cycled

*Cyclic reductions of a word***Description**

Functionality to cyclically reduce words and detect conjugacy

**Usage**

```
is.cyclically_reduced(a)
as.cyclically_reduced(a)
cyclically_reduce(a)
cyclically_reduce_tietze(p)
is.conjugate_single(u,v)
x %% y
## S3 method for class 'free'
is.conjugate(x,y)
allconj(x)
```

**Arguments**

a, x, y	An object of class free
p, u, v	Integer vector corresponding to Tietze form of a word

**Details**

A free object is *cyclically reduced* iff every cyclic permutation of the word is reduced. A reduced word is cyclically reduced iff the first letter is not the inverse of the last one. A reduced word is cyclically reduced if the first and last symbol differ (irrespective of power) or, if identical, have powers of opposite sign. For example, abac and abca are cyclically reduced but  $abca^{-1}$  is not. Function `is.cyclically_reduced()` tests for this.

Function `as.cyclically_reduced()` takes a vector of free objects and returns the elementwise cyclically reduced equivalents. Function `cyclically_reduce()` is a synonym with better (English) grammar.

The identity is cyclically reduced: it cannot be shortened by a combination of cyclic permutation followed by reduction. This ensures that `is.cyclically_reduced(as.cyclically_reduced(x))` is always TRUE. Also, it is clear that the identity should be conjugate to itself.

Two words  $a, b$  are *conjugate* if there exists a  $x$  such that  $ax = xb$  (or equivalently  $a = x^{-1}bx$ ). This is detected by function `is.conjugate()`. Functions `is_conjugate_single()` and `cyclically_reduce_tietze()` are lower-level helper functions.

Function `allconj()` returns all cyclically reduced words conjugate to its argument.

**Author(s)**

Robin K. S. Hankin

**See Also**[reduce](#)**Examples**

```
(x <- abc(1:9) - abc(9:1))
as.cyclically_reduced(x)
```

```
a <- rfree(1000,3)
all(size(as.cyclically_reduced(a)) <= size(a))
all(total(as.cyclically_reduced(a)) <= total(a))
all(number(as.cyclically_reduced(a)) <= number(a))
```

```
x <- rfree(1000,2)
y <- as.free('ab')
table(conjugate = (x%~%y), equal = (x==y)) # note zero at top right
```

```
allconj(as.free('aaaaab'))
allconj(sum(abc(seq_len(3))))
```

```
x <- rfree(1,10,8,8)
all(is.id(allconj(x) + allconj(-x)[shift(rev(seq_len(total(x))))]))
```

---

 donames

*Names attributes of free group elements*


---

**Description**

Get and set names of free group elements and arithmetic operations

**Usage**

```
donames(f, e1, e2)
```

**Arguments**

f                    A vector, typically of class free  
 e1, e2              Objects of class free, possibly with names

**Details**

Function `donames()` is a low-level helper function that ensures that the result of arithmetic operations such as `+` and `^` have the correct names attributes. The behaviour is inherited from that of `base::`+``.

**Author(s)**

Robin K. S. Hankin

**See Also**

[Ops.free](#)

**Examples**

```
x <- rfree(9,4)
x
names(x) <- letters[1:9]
x

z <- as.free('z')
x + x
x^z
z^x

n <- 1:9
names(n) <- LETTERS[1:9]

x*n
n*x # note different names
```

---

dot-class

*Class "dot"*

---

**Description**

The dot object is defined in the **freealg** package, and imported here, so that idiom like `.[x,y]` returns the commutator, that is,  $x^{-1}y^{-1}xy$ .

**Arguments**

<code>x</code>	Object of any class
<code>i, j</code>	elements to commute
<code>...</code>	Further arguments to <code>dot_error()</code> , currently ignored

**Value**

Always returns an object of the same class as `xy`.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
.[as.free("x"), as.free("y")]

.[abc(1:6), "z"]

x <- rfree()
y <- rfree()
z <- rfree()

.[x,y] == -x-y+x+y # should be TRUE

abelianize(.[x,y])

## Jacobi identity _not_ satisfied with this definition:
is.id(.[x,.[y,z]] + .[y,.[z,x]] + .[z,.[x,y]])

## But the Hall-Witt identity is:
all(is.id(.[.[x,-y],z]^y + .[.[y,-z],x]^z + .[.[z,-x],y]^x))
```

---

Extract

*Extract or replace parts of a free group object*

---

**Description**

Extract or replace subsets of free objects

**Arguments**

<code>x</code>	Object of class <code>free</code>
<code>index</code>	elements to extract or replace
<code>value</code>	replacement value

**Details**

These methods (should) work as expected: an object of class `free` is a list but standard extraction techniques should work.

**Examples**

```
(x <- rfree(20,8,8))

x[5:6]
x[1:2] <- -x[11:12]

x[1:5] <- keep(x[1:5],1:3)
```

---

free

*Objects of class free*


---

**Description**

Generate, and test for, objects of class free

**Usage**

```
free(x)
as.free(x)
is.free(x)
list_to_free(x)
```

**Arguments**

`x`                      Function `free()` needs either a two-row matrix, or a list of two-row matrices; function `as.free()` attempts to coerce different types of argument before passing to `free()` (possibly via `list_to_free()`)

**Details**

The basic structure of an element of the free group is a two-row matrix. The top row is the symbols (1=a, 2=b, 3=c, etc) and the bottom row is the corresponding power. Thus  $a^2ba^{-1}c^9$  would be

```
> rbind(c(1,2,1,3),c(2,1,-1,9))
      [,1] [,2] [,3] [,4]
[1,]   1   2   1   3
[2,]   2   1  -1   9
>
```

Function `free()` needs either a two-row matrix or a list of two-row matrices. It is the only place in the package that sets the class of an object to `free`. Function `as.free()` is a bit more user-friendly and tries a bit harder to do the Right Thing.

The package uses `setOldClass("free")` for the dot methods.

**Author(s)**

Robin K. S. Hankin

**See Also**[char\\_to\\_free](#)**Examples**

```
free(rbind(1:5,5:1))

x <- rfree(10,3)
x
x+x
x-x
x[1:5]*(1:5)

as.free(c(4,3,2,2,2))
as.free("aaaabccccaaaa")
as.free(c("a","A","abAAA"))
```

---

**getlet***Get letters of a freegroup object*

---

**Description**

Get the symbols in a freegroup object

**Usage**

```
getlet(x)
```

**Arguments**

x                    Object of class free

**Note**

By default, return a list with elements corresponding to the elements of x. But, if object x is of length 1, a vector is returned. The result is sorted for convenience.

**Author(s)**

Robin K. S. Hankin



**Examples**

```
(x <- rfree(6,7,3))  
  
getlet(x)  
  
as.free(getlet(x))  
  
identical(as.free(getlet(abc(1:26))), abc(1:26))
```

---

identity

*The identity element*

---

**Description**

Create and test for the identity element

**Usage**

```
is.id(x)  
id(n)  
## S3 method for class 'free'  
is.id(x)
```

**Arguments**

x	Object of class free
n	Strictly positive integer

**Details**

Function `id()` returns a vector of  $n$  free objects, all of which are the identity element. Do not ask what happens if  $n = 0$ .

Function `is.id()` returns a Boolean indicating whether an element is the identity or not. The identity can also be generated using `as.free(0)`.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
id()  
as.free(0) # convenient R idiom for creating the identity  
  
x <- rfree(10,3)  
stopifnot(all(x == x + as.free(0)))  
stopifnot(all(is.id(x-x)))
```

---

keep	<i>Keep or drop symbols</i>
------	-----------------------------

---

**Description**

Keep or drop symbols

**Usage**

```
keep(a, yes)
discard(a, no)
```

**Arguments**

a	Object of class free
yes, no	Specification of symbols to either keep (yes) or discard (no), coerced to a free object

**Note**

Function `keep()` needs an explicit `return()` to prevent it from returning invisibly.

The functions are vectorised in the first argument but not the second.

The second argument—the symbols to keep or discard—is formally a vector of nonnegative integers, but the functions coerce it to a free object. The symbols kept or dropped are the union of the symbols in the elements of the vector. Function `discard()` was formerly known as `drop()` but this conflicted with `base::drop()`.

These functions have nothing in common with APL's `take()` and `drop()`.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
(x <- rfree(20,5,8))

keep(x,abc(4))          # keep only symbols a,b,c,d
discard(x,as.free('cde')) # drop symbols c,d,e

keep(x,alpha(3)) # keep only abc
```

**Description**

Vectorized functionality to implement outer automorphisms of the free group

**Usage**

```
permsymb_single_X(X,f)
permsymb_single_f(X,f)
permsymb_vec(X,f)
permsymb(X,f)
autosub_lowlevel(M,e,S)
autosub(X,e,S,automorphism_warning=TRUE)
```

**Arguments**

X, S	Object of class free
f	Permutation function
M	Single free group element, in two-row matrix form
e	Single element to substitute
automorphism_warning	Boolean, with default TRUE meaning to give a warning if the requested substitution is not an automorphism and FALSE meaning not to give the warning

**Details**

In 1924, Nielsen showed that the automorphism group of the free group with basis  $[x_1, \dots, x_n]$  is generated by the following four elementary Nielsen transformations:

1. switch  $x_1$  and  $x_2$
2. Cyclically permute  $x_1, x_2, \dots, x_n$  to  $x_2, \dots, x_n, x_1$
3. Replace  $x_1$  with  $x_1^{-1}$
4. Replace  $x_1$  with  $x_1x_2$ .

The functions documented here give vectorized methods to effect such outer automorphisms, using the **permutations** package.

Operations 1 and 2 above generate the symmetric group  $S_n$  and such automorphisms are effected by function `permsymb()`. Operation 3 is carried out by `flip()` and operation 4 by `subsymb()`.

Functions `permsymb_single_X()`, `permsymb_single_f()`, `permsymb_vec()` and `subsymb_lowlevel()` are low-level helper functions that are not really suited for the end user; use `permsymb()`, `flip()` and `subsymb()` instead.

**Note**

Function `permsymb()` is intended to work nicely with the **permutations** package; see `inst/outer.Rmd` for some illustrations. The function is not perfect.

**Author(s)**

Robin K. S. Hankin

**References**

Wikipedia contributors. (2018, October 29). "Automorphism group of a free group". In *Wikipedia, The Free Encyclopedia*. Retrieved 19:58, January 10, 2019, from [https://en.wikipedia.org/w/index.php?title=Automorphism\\_group\\_of\\_a\\_free\\_group&oldid=866270661](https://en.wikipedia.org/w/index.php?title=Automorphism_group_of_a_free_group&oldid=866270661)

**See Also**

[flip](#)

**Examples**

```
P <- as.free(c("abc", "aba", "cc", "ca"))
autosub(P, "c", as.free("xyz"))

flip(P, "c")
flip(P, "ac")
```

---

Ops.free

*Arithmetic Ops methods for the free group*

---

**Description**

Allows arithmetic operators to be used for manipulation of free group elements such as addition, multiplication, powers, etc

**Usage**

```
## S3 method for class 'free'
Ops(e1, e2)
free_equal(e1, e2)
free_power(e1, e2)
free_repeat(e1, n)
juxtapose(e1, e2)
## S3 method for class 'free'
inverse(e1)
## S3 method for class 'matrix'
inverse(e1)
```

**Arguments**

e1, e2	Objects of class free
n	An integer, possibly non-positive

**Details**

The function `Ops.free()` passes binary arithmetic operators (“+”, “-”, “\*”, “^”, and “==”) to the appropriate specialist function.

There are two non-trivial basic operations: juxtaposition, denoted “a+b”, and inversion, denoted “-a”. Note that juxtaposition is noncommutative and a+b will not, in general, be equal to b+a.

All operations return a reduced word.

The caret, as in  $a^b$ , denotes group-theoretic exponentiation ( $-b+a+b$ ); the notation is motivated by the identities  $x^{(yz)}=(x^y)^z$  and  $(xy)^z=x^z*y^z$ , as in the `permutations` package.

Multiplication between a free object  $a$  and an integer  $n$  (as in  $a*n$  or  $n*a$ ) is defined as juxtaposing  $n$  copies of  $a$  and reducing. Zero and negative values of  $n$  work as expected.

Comparing a free object with a numeric does not make sense and idiom such as `rfree() > 4` will return an error. Comparing a free object with another free object might be desirable [specifically, lexicographic ordering], but is not currently implemented.

**Note**

The package uses additive notation but multiplicative notation might have been better.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
x <- as.free(c("a", "ab", "aaab", "abacc"))
y <- as.free(c("aa", "BA", "Bab", "aaaaa"))
x
y

x + x
x + y
x + as.free("xyz")

x+y == y+x    # not equal in general

x*5 == x+x+x+x+x    # always true

x + alpha(26)

x^y
```

---

 primitive

*Primitive elements of the free algebra*


---

### Description

A *primitive word* is one that is not of the form  $a^m$  for any  $m > 1$ . The concept is used in Lyndon and Schutzenberger 1962.

### Usage

```
is.primitive(x)
is.power(d,n)
```

### Arguments

x	Freegroup object, coerced to Tietze form
d	Numeric vector
n	Integer

### Details

Function `is.primitive()` returns a boolean vector indicating whether the elements of its argument are primitive.

Function `is.power()` is a lower-level helper function. `is.power(d,n)` determines whether `d` is an `n`-th power (that is, `d` may be written as `n` copies of some numeric vector).

Thus `is_power(c(4,5,7,4,5,7,4,5,7,4,5,7),4)` returns TRUE because its primary argument is indeed a fourth power (of `c(4,5,7)`).

### Value

Returns a boolean.

### Author(s)

Robin K. S. Hankin. The code for finding the factors of an integer was (somewhat more than) inspired by the **numbers** package.

### References

R. C. Lyndon and M. P. Schutzenberger 1962. “The equation  $a^M = b^N c^P$  in a free group”. Michigan Mathematical Journal, 9(4): 289–298.

**Examples**

```
is.primitive(as.free(c("a","aaaa", "aaaab", "abaab", "aabcaabcaabcaabc")))

is.power(c(7,8,4,7,8,4,7,8,4,7,8,4),4)

table(is.primitive(rfree(100)))

## primitive with >1 symbols is rare:
x <- rfree(1000)
x[!is.primitive(x) & number(x)>1]
## try x <- rfree(10000) [but this takes a long time]
```

---

print

---

*Print free objects*


---

**Description**

Print methods for free objects

**Usage**

```
## S3 method for class 'free'
print(x,...)
as.character_free(m,latex=getOption("latex"))
```

**Arguments**

x	Object of class free in the print method
m	A two-row matrix in function as.character_free()
latex	Boolean, with TRUE meaning to print latex-friendly output including curly braces, and default NULL option meaning to give a nicer-looking output that latex would typeset incorrectly
...	Further arguments, currently ignored

**Note**

The print method does not change the internal representation of a free object, which is a list of integer matrices.

The default print method uses multiplicative notation (powers) which is inconsistent with the juxtaposition method “+”.

The print method has special dispensation for length-zero free objects but these are not handled entirely consistently.

The default print method uses lowercase letters a-z, but it is possible to override this using `options("freegroup_symbols" = foo)`, where `foo` is a character vector. This is desirable if you have more than 26 symbols, because unallocated symbols appear as NA.

The package will allow the user to set `options("freegroup_symbols")` to unhelpful things like `rep("a", 20)` without complaining (but don't actually do it, you crazy fool).

### Author(s)

Robin K. S. Hankin

### See Also

[char\\_to\\_free](#)

### Examples

```
## default symbols:

abc(26)
rfree(1,10)

# if we need more than 26:
options(freegroup_symbols=state.name)
rfree(10,4)

# or even:
jj <- letters[1:10]
options(freegroup_symbols=apply(expand.grid(jj, jj), 1, paste, collapse=""))
rfree(10,10,100,4)

options(freegroup_symbols=NULL) # NULL is interpreted as letters a-z
rfree(10,4) # back to normal
```

---

reduce

*Reduction of a word to reduced form*

---

### Description

Given a word, remove redundant zero-power terms, and consolidate adjacent like terms into a single power

### Usage

```
reduce(a)
is_reduced(a)
remove_zero_powers(a)
consolidate(a)
is_proper(a)
```



**Arguments**

a                    An object of class free

**Details**

A word is *reduced* if no symbol appears next to its own inverse and no symbol has zero power. The essence of the package is to reduce a word into a reduced form. Thus  $a^2b^{-1}ba$  will be transformed into  $a^3$ .

In the package, reduction happens automatically at creation, in function `free()`.

Apart from `is_proper()`, the functions all take a free object, but the meat of the function operates on a single two-row matrix.

Reduction is carried out by repeatedly consolidating adjacent terms of identical symbol (function `consolidate()`), and removing zero power terms (function `remove_zero_power()`) until the word is in reduced form (function `is_reduced()`).

Function `is_proper()` checks to see whether a matrix is suitably formed for passing to `reduce()`.

A free object is *cyclically reduced* iff every cyclic permutation of the word is reduced. A reduced word is cyclically reduced iff the first letter is not the inverse of the last one. A reduced word is cyclically reduced if the first and last symbol differ (irrespective of power) or, if identical, have powers of opposite sign. For example, `abac` and `abca` are cyclically reduced but `abca^{-1}` is not. Function `is.cyclically.reduced()` tests for this, documented at `cycled.Rd`.

**Author(s)**

Robin K. S. Hankin

**See Also**

[cycled](#)

**Examples**

```
## create a matrix:
(M <- rbind(c(1,2,3,3,2,3,2,1),c(1,2,3,-3,5,0,7,0)))

## call the print method (note non-reduced form):
as.character_free(M)

## show the effect of reduce():
as.character_free(reduce(M))

## free() calls reduce() automatically:
free(M)
```

---

`rfree`*Random free objects*

---

**Description**

Creates a vector of random free objects. Intended as a quick “get you going” example of free group objects

**Usage**

```
rfree(n = 7, size = 4, number = size, powers = seq(from = -size, to = size))
rfreee(n = 30, size = 8, number = size, powers = seq(from = -size, to = size))
rfreeee(n = 40, size = 25, number = size, powers = seq(from = -size, to = size))
```

**Arguments**

<code>n</code>	Length of random vector to generate
<code>size</code>	Maximum length of each element
<code>number</code>	How many distinct letters to sample from
<code>powers</code>	Powers in resulting polynomial. An integer <code>n</code> is interpreted (via <code>sample()</code> ) as <code>seq_len(n)</code>

**Details**

The auxiliary arguments specify the general complexity of the returned object with small meaning simpler.

Functions `rfreee()` and `rfreeee()` give, by default, successively more complicated expressions.

**Author(s)**

Robin K. S. Hankin

**See Also**

[size](#)

**Examples**

```
rfree()

abelianize(rfree())

rfree(10,2)
rfree(10,30,26)

rfree(powers=5)
rfree(powers=5:6)
```

```
rfree(20,2)^alpha(26)
```

---

shift *Permute elements of a vector in a cycle*

---

### Description

Given a vector, permute the elements with a cyclic permutation

### Usage

```
shift(x, i=1)
```

### Arguments

x	Vector
i	Integer, number of places to permute. Negative values mean to count from the end

### Details

This function is that of the **magic** package, where it is motivated and discussed.

### Value

Returns a vector

### Author(s)

Robin K. S. Hankin

### Examples

```
shift(1:9)
shift(1:9,-1)
shift(1:9,2)
```

---

size

*Bignesses of a free object*

---

### Description

Various metrics to describe how “big” a free object is

### Usage

```
size(a)
total(a)
number(a)
bigness(a)
```

### Arguments

a                    Vector of free group objects

### Details

- The *size* of an object is the number of pure powers in it (this is the number of columns of the matrix representation of the word)
- The *total* of an object is the sum of the absolute values of its powers
- The *number* of an object is the number of distinct symbols in it

Thus  $\text{size}(a^2ba)=3$ ,  $\text{total}(a^2ba)=4$ , and  $\text{number}(a^2ba)=2$ .

Function `bigness()` is a convenience wrapper that returns all three bigness measures.

### Value

These functions return an integer vector.

### Note

I would like to thank Murray Jorgensen for his insightful comments which inspired this functionality.

### Author(s)

Robin K. S. Hankin

### See Also

[abs](#)

**Examples**

```
(a <- rfree(20,6,4))
size(a)
total(a)
number(a)

a <- rfree(20,6,4)
b <- rfree(20,6,4)

## Following should all be TRUE
size(a+b) <= size(a) + size(b)
total(a+b) <= total(a) + total(b)
number(a+b) <= number(a)+ number(b)

bigness(rfree(10,3,3))
bigness(allconj(rfree(1,6,1)))
```

---

subs

*Substitute and invert symbols*


---

**Description**

Substitute and invert specific symbols in a free object

**Usage**

```
subsu(X, from, to)
subs(X, ...)
flip(X, turn)
```

**Arguments**

X	Object of class free
from, to, turn	Objects coerced to class free specifying symbols to alter. These arguments are coerced to symbols using <code>getlet(as.free())</code>
...	Named arguments for substitution

**Details**

Function `subsu(X, from, to)` takes object `X` and transforms every symbol present in `from` into the symbol specified in `to`.

Function `flip(X, turn)` takes object `X` and replaces every symbol present in `turn` with its inverse.

Function `discard()`, documented at `keep.Rd`, effectively substitutes a symbol with the identity element (thereby discarding it).

Experimental function `subs()` is modelled on similar functionality in the **freealg** package and makes idiom such as `subs(X, a='z')` work as expected (viz, taking each instance of symbol `a` and replacing it with `x`).

**Note**

Functions `subs()` and `subsu()` substitute for particular symbols, not free group elements. In particular, be careful with uppercase (inverse) symbols; because the power is discarded, substituting with `x` is the same as substituting for `X`. This behaviour might change in the future.

**Author(s)**

Robin K. S. Hankin

**See Also**

[abs](#), [discard](#)

**Examples**

```
subsu(abc(1:10), abc(5), 'z')
flip(abc(1:10), abc(5))

o <- rfree(30, 5, 10)

# Following tests should all be TRUE:
size(flip(o, 'a')) == size(o)
number(flip(o, 'a')) == number(o)
total(flip(o, 'a')) == total(o)

size(subsu(o, 'a', 'b')) <= size(o)
number(subsu(o, 'a', 'b')) <= number(o)
total(subsu(o, 'a', 'b')) <= total(o)

frog <- rfree()
subs(frog, a='x')
```

---

sum

*Repeated summation by concatenation*

---

**Description**

Concatenates its arguments to give a single free object

**Usage**

```
## S3 method for class 'free'
sum(..., na.rm = FALSE)
```

**Arguments**

...                    Objects of class `free`, to be summed  
`na.rm`                Boolean, indicating whether to ignore NA entries (currently ignored)

**Details**

Concatenates its arguments and gives a single element of the free group. It works nicely with `rev()`, see the examples.

**Note**

The package uses additive notation, but it is easy to forget this and wonder why idiom like `prod(rfree())` does not work as desired. Of course, the package using additive notation means that one probably wants `sum(rfree())`.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
(x <- rfree(10,3))
sum(x)
abelianize(sum(x))

(y <- rfree(10,6))

sum(x,y)
sum(x,y) == sum(sum(x),sum(y))
x+y # not the same!

sum(x,-x)
sum(x,rev(-x))

z <- alpha(26)
stopifnot(sum(x^z) == sum(x)^z)
```

---

tietze

*Tietze form for free group objects*


---

**Description**

Translate an object of class `free` to and from Tietze form

**Usage**

```
## S3 method for class 'free'  
tietze(x)  
## S3 method for class 'matrix'  
tietze(x)  
vec_to_matrix(x)
```

**Arguments**

x                    Object to be converted

**Details**

The Tietze form for a word is a list of integers corresponding to the symbols of the word; typically  $a = 1, b = 2, c = 3, d = 4$ , etc. Negative integers represent the inverses of the symbols. Thus  $c^4.d^{-2}.a.c$  becomes 3 3 3 3 -4 -4 1 3.

Function `vec_to_matrix()` is a low-level helper function that returns a two-row integer matrix. If given  $\emptyset$  or NULL, it returns a two-row, zero-column matrix.

**Author(s)**

Robin K. S. Hankin

**Examples**

```
(x <- rfree(10,3))  
tietze(x)  
  
vec_to_matrix(c(1,3,-1,-1,-1,2))  
  
as.free(list(c(1,1,8),c(2,-4,-4)))  
  
all(as.free(tietze(abc(1:30))) == abc(1:30))
```



# Index

- \* **package**
  - freegroup-package, 2
  - . (dot-class), 13
  - [,dot,ANY,ANY-method (dot-class), 13
  - [,dot,ANY,missing-method (dot-class), 13
  - [,dot,free,ANY,ANY-method (dot-class), 13
  - [,dot,free,ANY-method (dot-class), 13
  - [,dot,matrix,matrix-method (dot-class), 13
  - [,dot,missing,ANY-method (dot-class), 13
  - [,dot,missing,missing-method (dot-class), 13
  - [,dot-method (dot-class), 13
  - [.dot (dot-class), 13
  - [.free (Extract), 14
  - [<-.free (Extract), 14
  - %-% (cycled), 11
- abc (alpha), 6
- abelianize, 4
- abelianized (abelianize), 4
- abs, 28, 30
- abs (abs.free), 5
- abs.free, 5
- allconj (cycled), 11
- alpha, 6
- alphabet (alpha), 6
- as.character\_free (print), 23
- as.cyclically\_reduced (cycled), 11
- as.free (free), 15
- automorphism (nielsen), 19
- automorphisms (nielsen), 19
- autosub (nielsen), 19
- autosub\_lowlevel (nielsen), 19
- backwards, 7
- bigness (size), 28
- c, 8
- char\_to\_free, 9, 16, 24
- char\_to\_matrix (char\_to\_free), 9
- commutator (dot-class), 13
- conjugate (cycled), 11
- consolidate (reduce), 24
- cumsum, 10
- cyclic (cycled), 11
- cyclic\_reduction (cycled), 11
- cyclically (cycled), 11
- cyclically\_reduce (cycled), 11
- cyclically\_reduce\_tietze (cycled), 11
- cyclically\_reduced (cycled), 11
- cycled, 11, 25
- discard, 30
- discard (keep), 18
- donames, 12
- dot (dot-class), 13
- dot-class, 13
- dot\_error (dot-class), 13
- drop (keep), 18
- Extract, 14
- extract (dot-class), 13
- flip, 20
- flip (subs), 29
- free, 15
- free-class (free), 15
- free\_equal (Ops.free), 20
- free\_power (Ops.free), 20
- free\_repeat (Ops.free), 20
- freegroup (freegroup-package), 2
- freegroup-package, 2
- freeprod (sum), 30
- getlet, 16
- id (identity), 17
- identity, 17
- inverse (Ops.free), 20

is.abelian (abelianize), 4  
 is.conjugate (cycled), 11  
 is.conjugate\_single (cycled), 11  
 is.cyclically\_reduced (cycled), 11  
 is.cyclically\_reduced (cycled), 11  
 is.free (free), 15  
 is.id (identity), 17  
 is.identity (identity), 17  
 is.power (primitive), 22  
 is.primitive (primitive), 22  
 is\_proper (reduce), 24  
 is\_reduced (reduce), 24  
  
 jacobi (dot-class), 13  
 juxtapose (Ops.free), 20  
  
 keep, 18  
  
 list\_to\_free (free), 15  
  
 names (donames), 12  
 neutral (identity), 17  
 nielsen, 19  
 number (size), 28  
  
 ops (Ops.free), 20  
 Ops.free, 13, 20  
 outer (nielsen), 19  
  
 permsymb (nielsen), 19  
 permsymb\_single\_f (nielsen), 19  
 permsymb\_single\_X (nielsen), 19  
 permsymb\_vec (nielsen), 19  
 primitive, 22  
 print, 23  
 print.free, 9  
 print.free (print), 23  
 print.freegroup (print), 23  
 prodfree (sum), 30  
  
 reduce, 12, 24  
 remove\_zero\_powers (reduce), 24  
 rep.free (c), 8  
 retain (keep), 18  
 rfree, 26  
 rfreee (rfree), 26  
 rfreeee (rfree), 26  
  
 shift, 27  
 size, 26, 28  
  
 subs, 5, 29  
 subsu (subs), 29  
 sum, 10, 30  
  
 Tietze (tietze), 31  
 tietze, 31  
 total (size), 28  
  
 vec\_to\_matrix (tietze), 31