

# Package ‘QGameTheory’

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**Title** Quantum Game Theory Simulator

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**Imports** dplyr, RColorBrewer, R.utils

**Depends** R(>= 3.4)

**Description** General purpose toolbox for simulating quantum versions of game theoretic models (Flitney and Abbott 2002) <[arXiv:quant-ph/0208069](#)>. Quantum (Nielsen and Chuang 2010, ISBN:978-1-107-00217-3) versions of models that have been handled are: Penny Flip Game (David A. Meyer 1998) <[arXiv:quant-ph/9804010](#)>, Prisoner's Dilemma (J. Orlin Grabbe 2005) <[arXiv:quant-ph/0506219](#)>, Two Person Duel (Flitney and Abbott 2004) <[arXiv:quant-ph/0305058](#)>, Battle of the Sexes (Nawaz and Toor 2004) <[arXiv:quant-ph/0110096](#)>, Hawk and Dove Game (Nawaz and Toor 2010) <[arXiv:quant-ph/0108075](#)>, Newcomb's Paradox (Piotrowski and Sladkowski 2002) <[arXiv:quant-ph/0202074](#)> and Monty Hall Problem (Flitney and Abbott 2002) <[arXiv:quant-ph/0109035](#)>.

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---

Bell

*Bell States*

---

### Description

The function builds one of the four Bell states, according to the input qubits

### Usage

```
Bell(qubit1, qubit2)
```

### Arguments

qubit1	1st input qubit
qubit2	2nd input qubit

### Value

One of the Bell states as a vector depending on the input qubits.

### References

[https://en.wikipedia.org/wiki/Bell\\_state](https://en.wikipedia.org/wiki/Bell_state)  
[https://books.google.co.in/books?id=66TgFp2YqrAC&pg=PA25&redir\\_esc=y](https://books.google.co.in/books?id=66TgFp2YqrAC&pg=PA25&redir_esc=y)

### Examples

```
init()  
Bell(Q$Q0, Q$Q0)  
Bell(Q$Q0, Q$Q1)  
Bell(Q$Q1, Q$Q0)  
Bell(Q$Q1, Q$Q1)
```

---

CNOT

*CNOT gate*

---

### Description

This function operates the CNOT gate on a conformable input matrix/vector

### Usage

```
CNOT(n)
```

**Arguments**

n                    A vector/matrix

**Value**

A matrix or a vector after performing the CNOT gate operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>

**Examples**

```
init()  
CNOT(Q$I4)  
CNOT(Q$Q11)
```

---

col\_count

*Number of columns of a vector/matrix*

---

**Description**

This function counts the number of columns of a vector or a matrix

**Usage**

```
col_count(M)
```

**Arguments**

M                    A vector/matrix

**Value**

An integer that gives the number of columns in a vector or a matrix.

**Examples**

```
init()  
col_count(Q$Q11)  
col_count(Q$lambda4)  
col_count(Q$I2)
```

---

Fredkin	<i>Fredkin Gate</i>
---------	---------------------

---

**Description**

This function operates the Fredkin gate on a conformable input matrix/vector

**Usage**

```
Fredkin(n)
```

**Arguments**

n	A vector/matrix
---	-----------------

**Value**

A matrix or a vector after performing the Fredkin gate operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>

**Examples**

```
init()  
Fredkin(Q$I8)  
Fredkin(Q$Q110)
```

---

Hadamard	<i>Hadamard Gate</i>
----------	----------------------

---

**Description**

This function operates the Hadamard gate on a conformable input matrix/vector

**Usage**

```
Hadamard(n)
```

**Arguments**

n                    A vector/matrix

**Value**

A matrix or a vector after performing the Hadamard operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>

**Examples**

```
init()
Hadamard(Q$Q0)
Hadamard(Q$I2)
Hadamard(Hadamard(Q$Q1))
```

---

IDSDS

---

*Iterated Deletion of Strictly Dominated Strategies algorithm*


---

**Description**

This function applies the IDSDS algorithm to result in the equilibrium strategies based on the rationality of the players. The input parameters are equal dimensional payoff matrices for the first and the second players.

**Usage**

```
IDSDS(P1, P2)
```

**Arguments**

P1                    Payoff matrix to Alice  
P2                    Payoff matrix to Bob

**Value**

A list consisting of the equilibrium strategies based on the rationality of the players by application of the IDSDS algorithm on P1 and P2.

## References

<https://arxiv.org/abs/1512.06808>  
[https://en.wikipedia.org/wiki/Strategic\\_dominance](https://en.wikipedia.org/wiki/Strategic_dominance)

## Examples

```
init()  
Alice <- matrix(c(8, 0, 3, 3, 2, 4, 2, 1, 3), ncol=3, byrow=TRUE)  
Bob <- matrix(c(6, 9, 8, 2, 1, 3, 8, 5, 1), ncol=3, byrow=TRUE)  
IDSDS(Alice, Bob)
```

---

init	<i>Initialization</i>
------	-----------------------

---

## Description

Builds the parameters in the required environment after initialization

## Usage

```
init()
```

## Value

No return value, generates the required variables/parameters.

## References

<https://arxiv.org/pdf/Quant-ph/0512125.pdf>  
<https://arxiv.org/pdf/0910.4222.pdf>  
<https://arxiv.org/pdf/Quant-ph/9703032.pdf>  
[https://en.wikipedia.org/wiki/Quantum\\_computing](https://en.wikipedia.org/wiki/Quantum_computing)  
<https://en.wikipedia.org/wiki/Qubit>  
<https://en.wikipedia.org/wiki/Qutrit>  
[https://en.wikipedia.org/wiki/Clebsch%E2%80%93Gordan\\_coefficients\\_for\\_SU\(3\)](https://en.wikipedia.org/wiki/Clebsch%E2%80%93Gordan_coefficients_for_SU(3))

## Examples

```
init()  
Q$Q110  
Q$Qt12  
Q$Q_minus  
Q$lambda4
```

levi\_civita                      *Levi-Civita symbol*

---

**Description**

This function computes the Levi-Civita symbol depending on the permutations of the three inputs, lying in 0 to 2

**Usage**

```
levi_civita(i, j, k)
```

**Arguments**

i	an integer 0, 1 or 2
j	an integer 0, 1 or 2
k	an integer 0, 1 or 2

**Value**

0, 1 or -1 after computing the Levi-Civita symbol depending on the permutations of the three inputs 0, 1 and 2

**References**

[https://en.wikipedia.org/wiki/Levi-Civita\\_symbol](https://en.wikipedia.org/wiki/Levi-Civita_symbol)

**Examples**

```
init()  
levi_civita(0, 2, 1)  
levi_civita(1, 2, 0)  
levi_civita(1, 2, 1)
```

---

NASH                                      *Nash Equilibrium*

---

**Description**

This function finds out the Nash equilibria of the 2-D payoff matrix for the players. The input parameters are equal dimensional payoff matrices for the first and the second players.



**Usage**

```
NASH(P1, P2)
```

**Arguments**

P1	Payoff matrix to Alice
P2	Payoff matrix to Bob

**Value**

The cell positions of the Nash equilibrium/equilibria as a dataframe from the payoff matrices of the players.

**References**

<https://arxiv.org/abs/1512.06808>  
[https://en.wikipedia.org/wiki/Nash\\_equilibrium](https://en.wikipedia.org/wiki/Nash_equilibrium)

**Examples**

```
init()
Alice <- matrix(c(4, 3, 2, 4, 4, 2, 1, 0, 3, 5, 3, 5, 2, 3, 1, 3), ncol=4, byrow=TRUE)
Bob <- matrix(c(0, 2, 3, 8, 2, 1, 2, 2, 6, 5, 1, 0, 3, 2, 2, 3), ncol=4, byrow=TRUE)
NASH(Alice, Bob)
```

---

PayoffMatrix\_QBOS

*Quantum Battle of the Sexes game: Payoff Matrix*

---

**Description**

This function generates the payoff matrix for the Quantum Battle of Sexes game for all the four combinations of p and q. moves is a list of two possible strategies for each of the players and alpha, beta, gamma are the payoffs for the players corresponding to the choices available to them with the chain of inequalities,  $\alpha > \beta > \gamma$ .

**Usage**

```
PayoffMatrix_QBOS(moves, alpha, beta, gamma)
```

**Arguments**

moves	a list of matrices
alpha	a number
beta	a number
gamma	a number

**Value**

The payoff matrices for the two players as two elements of a list.

**References**

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/abs/quant-ph/0110096>

**Examples**

```
init()
moves <- list(Q$I2, sigmaX(Q$I2))
PayoffMatrix_QBOS(moves, 5, 3, 1)
```

---

PayoffMatrix\_QHawkDove

*Quantum Hawk and Dove game: Payoff Matrix*

---

**Description**

This function generates the payoff matrix for the Quantum Hawk and Dove game for all the four combinations of p and q. moves is a list of two possible strategies for each of the players and v, j, D are the value of resource, cost of injury and cost of displaying respectively.

**Usage**

```
PayoffMatrix_QHawkDove(moves, v, j, D)
```

**Arguments**

moves	a list of matrices
v	a number
j	a number
D	a number

**Value**

The payoff matrices for the two players as two elements of a list.

**References**

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/pdf/quant-ph/0108075.pdf>

**Examples**

```
init()
moves <- list(Q$I2, sigmaX(Q$I2))
PayoffMatrix_QHawkDove(moves, 50, -100, -10)
```

---

PayoffMatrix_QPD	<i>Quantum Prisoner's Dilemma game: Payoff Matrix</i>
------------------	---

---

**Description**

This function generates the payoff matrix for the Quantum Prisoner's Dilemma game . moves is a list of the possible strategies for each of the players and w, x, y, z are the payoffs for the players corresponding to the choices available to them with the chain of inequalities,  $z > w > x > y$ . This function also plots the probability distribution plots of the qubits for all the possible combinations of the strategies of the players.

**Usage**

```
PayoffMatrix_QPD(moves, w, x, y, z)
```

**Arguments**

moves	a list of matrices
w	a number
x	a number
y	a number
z	a number

**Value**

The payoff matrices for the two players as two elements of a list.

**References**

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/pdf/quant-ph/0004076.pdf>

**Examples**

```
init()
moves <- list(Q$I2, sigmaX(Q$I2), Hadamard(Q$I2), sigmaZ(Q$I2))
PayoffMatrix_QPD(moves, 3, 1, 0, 5)
```

---

 Phase

*Phase Gate*


---

**Description**

This function operates the Phase gate on a conformable input matrix/vector

**Usage**

Phase(n)

**Arguments**

n                    a vector/matrix

**Value**

A matrix or a vector after performing the Phase gate operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>

**Examples**

```
init()
Phase(Q$I2)
Phase(Q$Q_plus)
```

---

 PhaseDagger

*Hermitian Transpose of the Phase Gate*


---

**Description**

This function operates the hermitian transpose of the Phase gate on a conformable input matrix/vector

**Usage**

PhaseDagger(n)

**Arguments**

n                    a vector/matrix

**Value**

A matrix or a vector after performing the operation of the hermitian transpose of the Phase gate on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>

**Examples**

```
init()
Conj(t(Phase(Q$I2)))==PhaseDagger(Q$I2)
PhaseDagger(Q$Q_plus)
```

---

 QBOS

*Quantum Battle of the Sexes game*


---

**Description**

This function returns the expected payoffs to Alice and Bob with respect to the probabilities  $p$  and  $q$ .  $p+q$  should equal 1 and moves is a list of two possible strategies for each of the players and alpha, beta, gamma are the payoffs for the players corresponding to the choices available to them with the chain of inequalities,  $\alpha > \beta > \gamma$ .

**Usage**

```
QBOS(p, q, moves, alpha, beta, gamma)
```

**Arguments**

p                    a real number between 0 and 1 including the end points  
 q                    a real number between 0 and 1 including the end points  
 moves                alist of matrices  
 alpha                a number  
 beta                 a number  
 gamma                a number

**Value**

A vector consisting of the Payoffs to Alice and Bob as its two elements depending on the inputs.

**References**

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/abs/quant-ph/0110096>

**Examples**

```
init()
moves <- list(Q$I2, sigmaX(Q$I2))
QBOS(0, 1, moves, 5, 3, 1)
QBOS(1, 1, moves, 5, 3, 1)
QBOS(0.5, 0.5, moves, 5, 3, 1)
```

---

QDuelsPlot1

*Quantum Two Person Duel game*

---

**Description**

This function helps us to plot Alice's and Bob's expected payoffs as functions of  $\alpha_1$  and  $\alpha_2$ .  $\Psi$  is the initial state of the quantum game,  $n$  is the number of rounds,  $a$  is the probability of Alice missing the target,  $b$  is the probability of Bob missing the target, and  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$ ,  $\beta_2$  are arbitrary phase factors that lie in  $-\pi$  to  $\pi$  that control the outcome of a poorly performing player.

**Usage**

```
QDuelsPlot1(Psi, n, a, b, beta1, beta2)
```

**Arguments**

Psi	a vector representing the initial quantum state
n	an integer
a	a number
b	a number
beta1	a number
beta2	a number

**Value**

No return value, plots Alice's and Bob's expected payoffs as functions of  $\alpha_1$  and  $\alpha_2$ .

**References**

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/pdf/quant-ph/0305058.pdf>

**Examples**

```

init()
QDuelsPlot1(Q$Q10, 2, 0.66666, 0.5, 0.2, 0.8)

```

---

QDuelsPlot2

*Quantum Two Person Duel game*


---

**Description**

This function helps us to plot Alice's and Bob's expected payoffs as functions of the number of rounds  $n$  played in a repeated quantum duel.  $\Psi$  is the initial state of the quantum game,  $n$  is the number of rounds,  $a$  is the probability of Alice missing the target,  $b$  is the probability of Bob missing the target, and  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$ ,  $\beta_2$  are arbitrary phase factors that lie in  $-\pi$  to  $\pi$  that control the outcome of a poorly performing player.

**Usage**

```
QDuelsPlot2(Psi, n, a, b, alpha1, alpha2, beta1, beta2)
```

**Arguments**

$\Psi$	a vector representing the initial quantum state
$n$	an integer
$a$	a number
$b$	a number
$\alpha_1$	a number
$\alpha_2$	a number
$\beta_1$	a number
$\beta_2$	a number

**Value**

No return value, plots Alice's and Bob's expected payoffs as functions of the number of rounds  $n$  played in a repeated quantum duel.

**References**

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/pdf/quant-ph/0305058.pdf>

**Examples**

```
init()
QDuelsPlot2(Q$Q01, 10, 0.66666, 0.5, -pi/2, pi/4, 0.6, 0.4)
```

---

QDuelsPlot3

*Quantum Two Person Duel game*


---

**Description**

This function helps us to plot the improvement in Alice's expected payoff as a function of  $a$  and  $b$ , if Alice chooses to fire at the air in her second shot, in a two round game.  $\Psi$  is the initial state of the quantum game,  $n$  is the number of rounds,  $a$  is the probability of Alice missing the target,  $b$  is the probability of Bob missing the target, and  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$ ,  $\beta_2$  are arbitrary phase factors that lie in  $[-\pi, \pi]$  that control the outcome of a poorly performing player.

**Usage**

```
QDuelsPlot3(Psi, alpha1, alpha2)
```

**Arguments**

$\Psi$	a vector representing the initial quantum state
$\alpha_1$	a number
$\alpha_2$	a number

**Value**

No return value, plots the improvement in Alice's expected payoff as a function of  $a$  and  $b$ , if Alice chooses to fire at the air in her second shot, in a two round game.

**References**

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/pdf/quant-ph/0305058.pdf>



**Examples**

```

init()
Qs <- (Q$Q0+Q$Q1)/sqrt(2)
Psi <- kronecker(Q$Q1, Qs)
QDuelsPlot3(Psi, pi/3, pi/6)

```

---

QDuelsPlot4

*Quantum Two Person Duel game*


---

**Description**

This function helps us to plot the improvement in Bob's expected payoff as a function of  $a$  and  $b$ , if Bob chooses to fire at the air in her second shot, in a two round game.  $\Psi$  is the initial state of the quantum game,  $n$  is the number of rounds,  $a$  is the probability of Alice missing the target,  $b$  is the probability of Bob missing the target, and  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$ ,  $\beta_2$  are arbitrary phase factors that lie in  $-\pi$  to  $\pi$  that control the outcome of a poorly performing player.

**Usage**

```
QDuelsPlot4(Psi, alpha1, alpha2)
```

**Arguments**

Psi	a vector representing the initial quantum state
alpha1	a number
alpha2	a number

**Value**

No return value, plots the improvement in Bob's expected payoff as a function of  $a$  and  $b$ , if Bob chooses to fire at the air in her second shot, in a two round game.

**References**

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/pdf/quant-ph/0305058.pdf>

**Examples**

```

init()
Qs <- (Q$Q0+Q$Q1)/sqrt(2)
Psi <- kronecker(Q$Q1, Qs)
QDuelsPlot4(Psi, pi/3, pi/6)

```

---

QDuels\_Alice\_payoffs *Quantum Two Person Duel game*

---

### Description

This function returns the expected payoff to Alice for three possible cases for the Quantum Duel game:

1. The game is continued for  $n$  rounds and none of the players shoots at the air.
2. The game is continued for 2 rounds and Alice shoots at the air in her second round.
3. The game is continued for 2 rounds and Bob shoots at the air in her second round.

$\Psi$  is the initial state of the quantum game,  $n$  is the number of rounds,  $a$  is the probability of Alice missing the target,  $b$  is the probability of Bob missing the target, and  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$ ,  $\beta_2$  are arbitrary phase factors that lie in  $[-\pi, \pi]$  that control the outcome of a poorly performing player.

### Usage

`QDuels_Alice_payoffs( $\Psi$ ,  $n$ ,  $a$ ,  $b$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\beta_1$ ,  $\beta_2$ )`

### Arguments

$\Psi$	a vector representing the initial quantum state
$n$	an integer
$a$	a number
$b$	a number
$\alpha_1$	a number
$\alpha_2$	a number
$\beta_1$	a number
$\beta_2$	a number

### Value

A list consisting of the payoff value to Alice depending on three situations of the quantum duel game: 1) The game is continued for  $n$  rounds and none of the players shoots at the air, 2) The game is continued for 2 rounds and Alice shoots at the air in her second round and 3) The game is continued for 2 rounds and Bob shoots at the air in her second round.

### References

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/pdf/quant-ph/0305058.pdf>

**Examples**

```
init()
QDuels_Alice_payoffs(Q$Q11, 5, 0.666666, 0.5, 0, 0, 0.2, 0.7)
Qs <- (Q$Q0+Q$Q1)/sqrt(2)
Psi <- kronecker(Qs, Qs)
QDuels_Alice_payoffs(Psi, 5, 0.666666, 0.5, 0, 0, 0.2, 0.7)
```

QDuels\_Bob\_payoffs      *Quantum Two Person Duel game*

**Description**

This function returns the expected payoff to Bob for three possible cases for the Quantum Duel game:

1. The game is continued for n rounds and none of the players shoots at the air.
2. The game is continued for 2 rounds and Alice shoots at the air in her second round.
3. The game is continued for 2 rounds and Bob shoots at the air in her second round.

Psi is the initial state of the quantum game, n is the number of rounds, a is the probability of Alice missing the target, b is the probability of Bob missing the target, and alpha1, alpha2, beta1, beta2 are arbitrary phase factors that lie in -pi to pi that control the outcome of a poorly performing player.

**Usage**

```
QDuels_Bob_payoffs(Psi, n, a, b, alpha1, alpha2, beta1, beta2)
```

**Arguments**

Psi	a vector representing the initial quantum state
n	an integer
a	a number
b	a number
alpha1	a number
alpha2	a number
beta1	a number
beta2	a number

**Value**

A list consisting of the payoff value to Bob depending on three situations of the quantum duel game: 1) The game is continued for n rounds and none of the players shoots at the air, 2) The game is continued for 2 rounds and Alice shoots at the air in her second round and 3) The game is continued for 2 rounds and Bob shoots at the air in her second round.

## References

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/pdf/quant-ph/0305058.pdf>

## Examples

```
init()
QDuels_Bob_payoffs(Q$Q11, 5, 0.666666, 0.5, 0, 0, 0.2, 0.7)
Qs <- (Q$Q0+Q$Q1)/sqrt(2)
Psi <- kronecker(Qs, Qs)
QDuels_Bob_payoffs(Psi, 5, 0.666666, 0.5, 0, 0, 0.2, 0.7)
```

---

QFT

*Quantum Fourier Transform*

---

## Description

This function performs Quantum Fourier Transform for a given state  $|y\rangle$  from the computational basis to the Fourier basis.

## Usage

```
QFT(y)
```

## Arguments

`y` an integer

## Value

A vector representing the Quantum Fourier transformation of the state  $|y\rangle$  from the computational basis to the Fourier basis.

## References

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
[https://books.google.co.in/books?id=66TgFp2YqrAC&pg=PA25&redir\\_esc=y](https://books.google.co.in/books?id=66TgFp2YqrAC&pg=PA25&redir_esc=y)  
[https://en.wikipedia.org/wiki/Quantum\\_Fourier\\_transform](https://en.wikipedia.org/wiki/Quantum_Fourier_transform)

## Examples

```
init()
QFT(5)
```

**Description**

This function returns the expected payoffs to Alice and Bob with respect to the probabilities  $p$  and  $q$ .  $p+q$  should equal 1 and moves is a list of two possible strategies for each of the players and  $v$ ,  $j$ ,  $D$  are the value of resource, cost of injury and cost of displaying respectively.

**Usage**

```
QHawkDove(p, q, moves, v, j, D)
```

**Arguments**

$p$	a real number between 0 and 1 including the end points
$q$	a real number between 0 and 1 including the end points
moves	a list of matrices
$v$	a number
$j$	a number
$D$	a number

**Value**

A vector consisting of the expected payoffs to Alice and Bob as its elements calculated according to the probabilities  $p$  and  $q$  provided as inputs.

**References**

<https://arxiv.org/pdf/quant-ph/0506219.pdf>

<https://arxiv.org/pdf/quant-ph/0208069.pdf>

<https://arxiv.org/pdf/quant-ph/0108075.pdf>

**Examples**

```
init()
moves <- list(Q$I2, sigmaX(Q$I2))
QHawkDove(0, 1, moves, 50, -100, -10)
QHawkDove(0, 0, moves, 50, -100, -10)
```

---

 QMeasure

*Measurement*


---

**Description**

This function performs a projective measurement of a quantum state  $n$ , in the computational basis and plots the corresponding probability distributions of the qubits.

**Usage**

```
QMeasure(n)
```

**Arguments**

$n$  a vector representing a quantum state

**Value**

No return value, plots the probability distributions of the qubits after performing a projective measurement of a quantum state  $n$ .

**References**

[https://books.google.co.in/books?id=66TgFp2YqrAC&pg=PA25&redir\\_esc=y](https://books.google.co.in/books?id=66TgFp2YqrAC&pg=PA25&redir_esc=y)  
[https://en.wikipedia.org/wiki/Measurement\\_in\\_quantum\\_mechanics](https://en.wikipedia.org/wiki/Measurement_in_quantum_mechanics)

**Examples**

```
init()  
QMeasure(Q$Q10110)
```

---

 QMontyHall

*Quantum Monty Hall Problem*


---

**Description**

This function simulates the quantum version of the Monty Hall problem, by taking in  $\Psi_{in}$  as the initial quantum state of the game,  $\gamma$  lying in 0 to  $\pi/2$ ,  $A_{hat}$  and  $B_{hat}$  as the choice operators in  $SU(3)$  for Alice and Bob respectively as the inputs. It returns the expected payoffs to Alice and Bob after the end of the game.

**Usage**

```
QMontyHall( $\Psi_{in}$ ,  $\gamma$ ,  $A_{hat}$ ,  $B_{hat}$ )
```

**Arguments**

Psi_in	a vector representing the initial quantum state
gamma	a number between 0 and $\pi/2$ including the end points
Ahat	a matrix lying in SU(3)
Bhat	a matrix lying in SU(3)

**Value**

A vector consisting of the expected payoffs to Alice and Bob as its elements depending on the input parameters.

**References**

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/pdf/quant-ph/0109035.pdf>

**Examples**

```
init()
Psi_in <- kronecker(Q$Qt0, (Q$Qt00+Q$Qt11+Q$Qt22)/sqrt(3))
QMontyHall(Psi_in, pi/4, Q$Identity3, Q$Hhat)
```

---

QNewcomb

*Quantum Newcomb's Paradox*


---

**Description**

This function simulates the quantum version of the Newcomb's Paradox by taking in the choice of the qubit  $|0\rangle$  or  $|1\rangle$  by the supercomputer Omega and the probability 'probability' with which Alice plays the spin flip operator. It returns the final state of the quantum game along with plotting the probability densities of the qubits of the final state after measurement.

**Usage**

```
QNewcomb(Omega, probability)
```

**Arguments**

Omega	$ 0\rangle$ or $ 1\rangle$
probability	a real number between 0 and 1 including the end points

**Value**

The final state of the quantum game as a vector along with plotting the probability densities of the qubits of the final state after measurement.

## References

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/pdf/quant-ph/0202074.pdf>

## Examples

```

init()
QNewcomb(Q$Q0, 0)
QNewcomb(Q$Q1, 0)
QNewcomb(Q$Q1, 0.7)

```

---

QPD

*Quantum Prisoner's Dilemma game*

---

## Description

This function returns the expected payoffs to Alice and Bob, with the strategy moves by Alice and Bob as two of the inputs.  $w$ ,  $x$ ,  $y$ ,  $z$  are the payoffs to the players corresponding to the choices available to them with the chain of inequalities,  $z > w > x > y$ . This function also plots the probability distribution plots of the qubits for one of all the combinations of the strategies of the players.

## Usage

```
QPD(U_Alice, U_Bob, w, x, y, z)
```

## Arguments

U_Alice	a matrix lying in SU(2)
U_Bob	a matrix lying in SU(2)
w	a number
x	a number
y	a number
z	a number

## Value

A vector consisting of the expected payoffs to Alice and Bob as its elements according to the strategies played by Alice and Bob and also the payoff values.

## References

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/pdf/quant-ph/0004076.pdf>



**Examples**

```
init()
QPD(Hadamard(Q$I2), sigmaZ(Q$I2), 3, 1, 0, 5)
```

---

QPennyFlip

*Quantum Penny Flip game*


---

**Description**

This function simulates the Quantum Penny Flip game by taking in the initial state of the game that is set by Alice and the strategies available to Alice and Bob. It returns the final state of the game along with the plot of the probability distribution of the qubits after measurement of the final state.

**Usage**

```
QPennyFlip(initial_state, strategies_Alice, strategies_Bob)
```

**Arguments**

```
initial_state  a vector representing the initial quantum state
strategies_Alice
                a matrix lying in SU(2)
strategies_Bob a matrix lying in SU(2)
```

**Value**

The final state of the game along with the plot of the probability distribution of the qubits after measurement of the final state by taking in the initial state of the game that is set by Alice and the strategies available to Alice and Bob as the inputs.

**References**

<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
<https://arxiv.org/pdf/quant-ph/0208069.pdf>  
<https://arxiv.org/pdf/quant-ph/9804010.pdf>

**Examples**

```
init()
psi <- (u+d)/sqrt(2)
S1 <- sigmaX(Q$I2)
S2 <- Q$I2
H <- Hadamard(Q$I2)
SA <- list(S1, S2)
SB <- list(H)
QPennyFlip(psi, SA, SB)
```

---

row_count	<i>Number of rows of a vector/matrix</i>
-----------	--

---

**Description**

This function counts the number of rows of a vector or a matrix

**Usage**

```
row_count(M)
```

**Arguments**

M	A vector/matrix
---	-----------------

**Value**

An integer that gives the number of rows in a vector or a matrix.

**Examples**

```
init()
row_count(Q$Q01)
row_count(Q$lambda5)
row_count(Q$Qt12)
```

---

Rx	<i>Rotation operation about x-axis of the Bloch sphere</i>
----	--

---

**Description**

This function operates the Rotation gate about the x-axis of the Bloch sphere by an angle theta on a conformable input matrix n.

**Usage**

```
Rx(n, theta)
```

**Arguments**

n	a vector/matrix
theta	an angle

**Value**

A vector or a matrix after operating the Rotation gate about the x-axis of the Bloch sphere, by an angle theta, on a conformable input matrix or a vector n

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>  
<http://www.physics.udel.edu/~msafrono/650/Lecture%204%20-%205.pdf>

**Examples**

```
init()
Rx(Q$Q0, pi/6)
```

---

Ry

*Rotation operation about y-axis of the Bloch sphere*


---

**Description**

This function operates the Rotation gate about the y-axis of the Bloch sphere by an angle theta on a conformable input matrix n.

**Usage**

```
Ry(n, theta)
```

**Arguments**

n	a vector/matrix
theta	an angle

**Value**

A vector or a matrix after operating the Rotation gate about the y-axis of the Bloch sphere, by an angle theta, on a conformable input matrix or a vector n.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>  
<http://www.physics.udel.edu/~msafrono/650/Lecture%204%20-%205.pdf>

**Examples**

```
init()  
Ry(Q$Q1, pi/3)
```

---

**Rz***Rotation operation about z-axis of the Bloch sphere*

---

**Description**

This function operates the Rotation gate about the z-axis of the Bloch sphere by an angle theta on a conformable input matrix n.

**Usage**

```
Rz(n, theta)
```

**Arguments**

n	a vector/matrix
theta	an angle

**Value**

A vector or a matrix after operating the Rotation gate about the z-axis of the Bloch sphere, by an angle theta, on a conformable input matrix or a vector n.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>  
<http://www.physics.udel.edu/~msafrono/650/Lecture%204%20-%205.pdf>

**Examples**

```
init()  
Rz(Q$Q1, pi)
```

---

sigmaX	<i>Pauli-X gate</i>
--------	---------------------

---

**Description**

This function operates the Pauli-X gate on a conformable input matrix or a vector.

**Usage**

```
sigmaX(n)
```

**Arguments**

n                    a vector/matrix

**Value**

A matrix or a vector after performing the Pauli-X gate operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>  
<http://www.physics.udel.edu/~msafrono/650/Lecture%204%20-%205.pdf>

**Examples**

```
init()  
sigmaX(Q$I2)  
sigmaX(Hadamard(Q$I2))  
sigmaX(Q$Q1)
```

---

sigmaY	<i>Pauli-Y gate</i>
--------	---------------------

---

**Description**

This function operates the Pauli-Y gate on a conformable input matrix or a vector.

**Usage**

```
sigmaY(n)
```

**Arguments**

n                    a vector/matrix

**Value**

A matrix or a vector after performing the Pauli-Y gate operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>  
<http://www.physics.udel.edu/~msafrono/650/Lecture%204%20-%205.pdf>

**Examples**

```
init()  
sigmaY(Q$I2)  
sigmaY(Hadamard(Q$I2))  
sigmaY(Q$Q0)
```

---

sigmaZ

*Pauli-Z gate*

---

**Description**

This function operates the Pauli-Z gate on a conformable input matrix or a vector.

**Usage**

```
sigmaZ(n)
```

**Arguments**

n                    a vector/matrix

**Value**

A matrix or a vector after performing the Pauli-Z gate operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>  
<http://www.physics.udel.edu/~msafrono/650/Lecture%204%20-%205.pdf>

**Examples**

```

init()
sigmaZ(Q$I2)
sigmaZ(Hadamard(Q$I2))
sigmaZ(Q$Q0)

```

---

SWAP

*SWAP gate*


---

**Description**

This function operates the SWAP gate on a conformable input matrix or a vector.

**Usage**

```
SWAP(n)
```

**Arguments**

n                    a vector/matrix

**Value**

A matrix or a vector after performing the SWAP gate operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>

**Examples**

```

init()
SWAP(Q$I4)
SWAP(Q$Q10)

```

T *T gate*

---

**Description**

This function operates the T gate on a conformable input matrix or a vector.

**Usage**

T(n)

**Arguments**

n a vector/matrix

**Value**

A matrix or a vector after performing the T gate operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>

**Examples**

```
init()  
T(Q$I2)  
T(Q$Q_minus)
```

---

TDagger *Hermitian Transpose of the T gate*

---

**Description**

This function operates the hermitian transpose of the T gate on a conformable input matrix or a vector.

**Usage**

TDagger(n)



**Arguments**

n                    a vector/matrix

**Value**

A matrix or a vector after performing the operation of the hermitian transpose of the T gate on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>

**Examples**

```
init()
TDagger(Q$I2)
TDagger(Q$Q_plus)
```

---

Toffoli	<i>Toffoli gate</i>
---------	---------------------

---

**Description**

This function operates the Toffoli gate on a conformable input matrix or a vector.

**Usage**

```
Toffoli(n)
```

**Arguments**

n                    a vector/matrix

**Value**

A matrix or a vector after performing the Toffoli gate operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>

**Examples**

```
init()  
Toffoli(Q$I8)  
Toffoli(Q$Q010)
```

---

Walsh

*Walsh-Hadamard gate*

---

**Description**

This function operates the Walsh-Hadamard gate on a conformable input matrix or a vector.

**Usage**

```
Walsh(n)
```

**Arguments**

n                    a vector/matrix

**Value**

A matrix or a vector after performing the Walsh-Hadamard gate operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>  
<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
[https://en.wikipedia.org/wiki/Hadamard\\_transform](https://en.wikipedia.org/wiki/Hadamard_transform)

**Examples**

```
init()  
Walsh(Q$I2)  
Walsh(Q$Q0)
```

---

Walsh16

*Walsh-Hadamard gate*

---

### Description

This function operates the Walsh-16 gate on a conformable input matrix or a vector.

### Usage

Walsh16(n)

### Arguments

n                    a vector/matrix

### Value

A matrix or a vector after performing the Walsh-16 gate operation on a conformable input matrix or a vector.

### References

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>  
<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
[https://en.wikipedia.org/wiki/Hadamard\\_transform](https://en.wikipedia.org/wiki/Hadamard_transform)

### Examples

```
init()  
Walsh16(Q$I16)  
Walsh16(Q$Q1001)
```

---

Walsh32

*Walsh-Hadamard gate*

---

### Description

This function operates the Walsh-32 gate on a conformable input matrix or a vector.

### Usage

Walsh32(n)

**Arguments**

n                    a vector/matrix

**Value**

A matrix or a vector after performing the Walsh-32 gate operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>  
<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
[https://en.wikipedia.org/wiki/Hadamard\\_transform](https://en.wikipedia.org/wiki/Hadamard_transform)

**Examples**

```
init()
Walsh32(Q$I32)
Walsh32(Q$Q10011)
```

---

Walsh4

*Walsh-Hadamard gate*

---

**Description**

This function operates the Walsh-4 gate on a conformable input matrix or a vector.

**Usage**

```
Walsh4(n)
```

**Arguments**

n                    a vector/matrix

**Value**

A matrix or a vector after performing the Walsh-4 gate operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>  
<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
[https://en.wikipedia.org/wiki/Hadamard\\_transform](https://en.wikipedia.org/wiki/Hadamard_transform)

**Examples**

```

init()
Walsh4(Q$I4)
Walsh4(Q$Q10)

```

---

Walsh8

*Walsh-Hadamard gate*


---

**Description**

This function operates the Walsh-8 gate on a conformable input matrix or vector.

**Usage**

```
Walsh8(n)
```

**Arguments**

n                    a vector/matrix

**Value**

A matrix or a vector after performing the Walsh-8 gate operation on a conformable input matrix or a vector.

**References**

[https://en.wikipedia.org/wiki/Quantum\\_logic\\_gate](https://en.wikipedia.org/wiki/Quantum_logic_gate)  
<http://www2.optics.rochester.edu/~stroud/presentations/muthukrishnan991/LogicGates.pdf>  
<https://arxiv.org/pdf/quant-ph/0506219.pdf>  
[https://en.wikipedia.org/wiki/Hadamard\\_transform](https://en.wikipedia.org/wiki/Hadamard_transform)

**Examples**

```
init()  
Walsh8(Q$I8)  
Walsh8(Q$Q000)
```

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