

# Package ‘MCTest’

January 20, 2025

**Type** Package

**Title** Monte Carlo Hypothesis Tests with Sequential Stopping

**Version** 1.0-3

**Date** 2019-05-14

**Author** Michael P. Fay

**Maintainer** Michael P. Fay <mfay@niaid.nih.gov>

**Depends** stats

**Description** Performs Monte Carlo hypothesis tests, allowing a couple of different sequential stopping boundaries. For example, a truncated sequential probability ratio test boundary (Fay, Kim and Hachey, 2007 <[DOI:10.1198/106186007X257025](https://doi.org/10.1198/106186007X257025)>) and a boundary proposed by Besag and Clifford, 1991 <[DOI:10.1093/biomet/78.2.301](https://doi.org/10.1093/biomet/78.2.301)>. Gives valid p-values and confidence intervals on p-values.

**License** GPL

**URL** <https://www.niaid.nih.gov/about/brb-staff-fay>

**NeedsCompilation** no

**Repository** CRAN

**Date/Publication** 2019-05-16 13:10:03 UTC

## Contents

MCTest-package . . . . .	2
find.ab . . . . .	3
MCbound . . . . .	4
MCbound.precalc1 . . . . .	6
MCTest . . . . .	7
p1.given.p0 . . . . .	9
plot.MCbound . . . . .	10
rrisk . . . . .	11
tSPRT.to.Bvalue . . . . .	12

<b>Index</b>	<b>13</b>
--------------	-----------

---

MCTest-package

*Monte Carlo hypothesis tests allowing sequential stopping*

---

## Description

Performs Monte Carlo hypothesis tests. It allows a couple of different sequential stopping boundaries (a truncated sequential probability ratio test boundary and a boundary proposed by Besag and Clifford, 1991). Gives valid p-values and confidence intervals on p-values.

## Details

Package: MCTest  
Type: Package  
Version: 1.0-3  
Date: 2019-05-14  
License: GPL

Use [MCbound](#) to create sequential stopping boundaries. These may take considerable set-up time, but once the stopping boundary is calculated then it can be used in [MCTest](#) to save time in computation of Monte Carlo hypothesis tests. The idea of the truncated sequential probability ratio test boundary is that it takes many resamples if the true p-value (i.e., the one from an infinite resample size) is close to the significance level (e.g., 0.05), but takes much fewer if the true p-value is far from the significance level.

## Author(s)

Michael P. Fay

Maintainer: Michael P. Fay <mfay@niaid.nih.gov>

## References

Besag, J. and Clifford, P. (1991). Sequential Monte Carlo p-values. *Biometrika*. 78: 301-304.

Fay, M.P., Kim, H-J. and Hachey, M. (2007). Using truncated sequential probability ratio test boundaries for Monte Carlo implementation of hypothesis tests. *Journal of Computational and Graphical Statistics*. 16(4):946-967.

## See Also

Precalculated MCbound: [MCbound.precalc1](#)

**Examples**

```

## Create a stopping boundary
##### May take a long time if Nmax is large
B<-MCbound("tsprt",c(alpha0=.001,beta0=.01,Nmax=99,p0=.04,p1=.06))
## do Monte Carlo test
x<-data.frame(y=1:100,z=rnorm(100),group=c(rep(1,50),rep(2,50)))
stat<-function(x){ cor(x[,1],x[,2]) }
### nonparametric bootstrap test on correlation between y and z
### low p-value means that such a large correlation unlikely due to chance
resamp<-function(x){ n<-dim(x)[[1]] ; x[sample(1:n,replace=TRUE),] }
MCtest(x,stat,resamp,bound=B)
## Package comes with a large precalculated MC bound as the default
## the precalculated bound is good for testing at the 0.05 level
MCtest(x,stat,resamp)

```

find.ab

*Find beta parameters to approximate distribution of p-values.***Description**

Find parameters of a beta distribution to approximate distribution of a p-value derived from a normal test statistic with one-sided significance level=ALPHA and power=1-BETA.

**Usage**

```
find.ab(n = 1e+05, ALPHA = 0.05, BETA = 0.2, higha = 100)
```

**Arguments**

n	the number of divisions for the numeric integration used to estimate the mean of p-value distribution, H (see details).
ALPHA	one-sided significance level of normal test statistic
BETA	type II error for normal test stastic
higha	an upper bound on the beta parameter (see details).

**Details**

The cumulative distribution function of the p-value from a normally distributed test statistic with one-sided significance level=ALPHA and power=1-BETA is  $H(p) = 1 - \text{pnorm}(\text{qnorm}(1-p) - \text{qnorm}(1-\text{ALPHA}) + \text{qnorm}(\text{BETA}))$ . We approximate this distribution with a beta distribution, B, which has the same mean as H and has  $B(\text{ALPHA}) = 1 - \text{BETA}$ . If two beta distributions meet both those criteria, we select the one closest to H in terms of integrated square error of the cumulative distribution function. That error is estimated by the sample variance of the differences in the two CDFs evaluated at  $(0:n)/n$ . Note that the two beta distributions come from the two roots of the following function:  $1 - \text{BETA} - B(\text{ALPHA})$ . We search for those two roots as the beta parameter within the range  $(1/\text{higha}, \text{higha})$ .

**Value**

A list with two elements:

- a                    numeric value of one of the shape parameters of the beta distribution
- b                    numeric value of the other shape parameter of the beta distribution

**Author(s)**

M.P. Fay

**References**

Fay, M.P., and Follmann, D.A. (2002). "Designing Monte Carlo implementations of permutation or bootstrap hypothesis tests" *American Statistician*, 56: 63-70.

**Examples**

```
## See first line of Table 1, Fay and Follmann, 2002
find.ab(ALPHA=.05,BETA=.1)
```

---

MCbound

*Create Monte Carlo stopping boundary*

---

**Description**

Creates one of several different types of Monte Carlo stopping boundaries

**Usage**

```
MCbound(type, parms, conf.level = 0.99)
```

**Arguments**

- type                a character vector of type of boundary, possible values: "fixed", "tsprt", "Bvalue", and "BC"
- parms              a numeric vector of parameter values, different for each type (see details)
- conf.level        confidence level for intervals about Monte Carlo p-values

**Details**

Create Monte Carlo stopping boundaries for use with `MCtest`, where we keep resampling until hitting the stopping boundary. There are several possible types, each with a different length parameter vector.

```
type="fixed"        then names(parms)=c("Nmax")
type="tsprt"        then names(parms)=c("p0","p1","A","B","Nmax")
                    or names(parms)=c("p0","p1","alpha0","beta0","Nmax")
```

```

type="Bvalue"  then names(parms)=c("Nmax","alpha","e0","e1")
type="BC"      then names(parms)=c("Nmax","Smax")

```

The object `parms` should be a named vector, although unnamed vectors will work if the parameters are in the above order (for the `tsprt` it assumes the first parameterization). For `type="fixed"` we keep reampling until  $N=N_{\max}$  resamples. For `type="tsprt"` we keep resampling until stopping for a truncated sequential probability ratio test for a binary parameter. The parameterizations are the usual Wald notation, except  $\alpha_0=\alpha$  and  $\beta_0=\beta$ , where  $A=(1-\beta_0)/\alpha_0$  and  $B=\beta_0/(1-\alpha_0)$ . The `Bvalue` is a test that  $p=\alpha$  or not and we stop if the `B-value` at information time  $t$ ,  $B(t)$ , is  $B(t)\leq q_{\text{norm}}(e_0)$  or  $B(t)\geq q_{\text{norm}}(1-e_1)$ . Note that the `B-value` stopping boundary is just a reparameterization of the truncated sequential probability ratio test. For `type="BC"` we keep resampling until  $N=N_{\max}$  or  $S=S_{\max}$  following a design recommended by Besag and Clifford (1991). For each stopping boundary we calculate valid  $p$ -values at each stopping point ordering by  $S/N$ . For details see Fay, Kim and Hachey, 2006.

### Value

An object of class `MCbound`. A list with the following elements:

<code>S</code>	number of successes at points on the boundary
<code>N</code>	number of resamples at points on the boundary
<code>p.value</code>	valid $p$ -value at each point on boundary, calculated using ordering by $S/N$
<code>ci.lower</code>	lower confidence limit of $p$ -value at each boundary point
<code>ci.upper</code>	upper confidence limit of $p$ -value at each boundary point
<code>Kstar</code>	number of ways to reach each point, $(S,N)$ , on boundary times $\beta_0(S+1,N-S+1)$
<code>conf.level</code>	confidence level for intervals on $p$ -values
<code>type</code>	type of boundary: either "fixed", "tsprt", "Bvalue" or "BC"
<code>parms</code>	parameter vector that defines boundary (see details)

### Author(s)

Michael P. Fay

### References

- Besag, J. and Clifford, P. (1991). Sequential Monte Carlo  $p$ -values. *Biometrika*. 78: 301-304.
- Fay, M.P., Kim, H-J. and Hachey, M. (2007). Using truncated sequential probability ratio test boundaries for Monte Carlo implementation of hypothesis tests. *Journal of Computational and Graphical Statistics*. 16(4):946-967.

### Examples

```
MCbound("tsprt",c(alpha0=.001,beta0=.01,Nmax=99,p0=.06,p1=.04))
```

---

MCbound.precalc1	<i>Precalculated object of class MCbound</i>
------------------	--

---

### Description

Because the calculation of this truncated sequential probability ratio test stopping boundary takes a long time, it is calculated ahead of time and included in the package. It is created with the following code, `MCbound("tsprt", parms=c(p0=p0.given.p1(0.04), p1=0.04, alpha0=.0001, beta0=0.0001, Nmax=9999), conf`

### Usage

```
MCbound.precalc1
```

### Format

The format is: List of 10

- S : num [1:10000] 22 22 22 22 22 22 22 22 22 22 ...
- N : num [1:10000] 22 23 24 25 26 27 28 29 30 31 ...
- p.value : num [1:10000] 1.000 0.957 0.917 0.880 0.846 ...
- ci.lower : num [1:10000] 0.786 0.719 0.668 0.626 0.590 ...
- ci.upper : num [1:10000] 1.000 1.000 0.995 0.985 0.972 ...
- Kstar : num [1:10000] 0.0435 0.0399 0.0367 0.0338 0.0313 ...
- conf.level: num 0.99
- type : chr "tsprt"
- parms : Named num [1:5] 6.14e-02 4.00e-02 1.00e-04 1.00e-04 1.00e+04
- ..- attr(\*, "names")= chr [1:5] "p0" "p1" "alpha0" "beta0" ...
- check : num 1
- - attr(\*, "class")= chr "MCbound"

### Examples

```
plot(MCbound.precalc1)
```

---

 MCtest

---

*Perform Monte Carlo hypothesis tests.*


---

### Description

Performs Monte Carlo hypothesis test with either a fixed number of resamples or a sequential stopping boundary on the number of resamples. Outputs p-value and confidence interval for p-value. The program is very general and different bootstrap or permutation tests may be done by defining the statistic function and the resample function.

### Usage

```
MCtest(x, statistic, resample, bound, extreme = "geq", seed = 1234325)
MCtest.fixed(x, statistic, resample, Nmax, extreme = "geq",
  conf.level=.99, seed = 1234325)
```

### Arguments

x	data object
statistic	function that inputs data object, x, and outputs a scalar numeric
resample	function that inputs data object,x, and outputs a "resampling" of x, an object of the same type as x
Nmax	the number of resamples for the fixed boundary
bound	a object of class MCbound, can be created by <a href="#">MCbound</a> see that help
extreme	character value either "geq" or "leq". Defines which Monte Carlo outputs from statistic (T1,T2,...) are denoted extreme with respect to the original output (T0) ; extreme values are either all $T_i \geq T_0$ ("geq") or all $T_i \leq T_0$ ("leq").
conf.level	confidence level for interval about p-value
seed	a numeric value used in set.seed

### Details

Performs Monte Carlo hypothesis test. MCtest allows any types of Monte Carlo boundary created by [MCbound](#), while MCtest.fixed only performs Monte Carlo tests using fixed boundaries. The only advantage of MCtest.fixed is that one can do the test without first creating the fixed stopping boundary through MCbound. The default boundary is described in [MCbound.precalc1](#).

Let  $T_0 = \text{statistic}(x)$  and let  $T_1, T_2, \dots$  be  $\text{statistic}(\text{resample}(x))$ . Then in the simplest type of boundary, the "fixed" type, then the resulting p-value is  $p = (S+1)/(N+1)$ , where  $S = (\# T_i \geq T_0)$  (if extreme is "geq") or  $S = (\# T_i \leq T_0)$  (if extreme is "leq"). The confidence interval on the p-value is calculated by an exact method.

There are several different types of MC designs that may be used for the MCtest. These are described in the [MCbound](#) help.

**Value**

A LIST of class "MCtest",with elements:

Ti	an N vector of the outputs of the resampled statistic
type	type of boundary: "fixed", "tsprt", "Bvalue" or "BC"
parms	vector of parameters that define boundary specified by type
T0	the value of the test statistic applied to the original data
p.value	p.value
pvalue.ci	confidence interval about the p-value

**Author(s)**

M.P. Fay

**References**

Fay, M.P., Kim, H-J. and Hachey, M. (2007). Using truncated sequential probability ratio test boundaries for Monte Carlo implementation of hypothesis tests. *Journal of Computational and Graphical Statistics*. 16(4):946-967.

**See Also**

[MCbound](#),[MCbound.precalc1](#)

**Examples**

```
x<-data.frame(y=1:100,z=rnorm(100),group=c(rep(1,50),rep(2,50)))
stat<-function(x){ cor(x[,1],x[,2]) }
### nonparametric bootstrap test on correlation between y and z
### low p-value means that such a large correlation unlikely due to chance
resamp<-function(x){ n<-dim(x)[[1]] ; x[sample(1:n,replace=TRUE),] }
out<-MCtest(x,stat,resamp,extreme="geq")
out$p.value
out$p.value.ci
### permutation test, permuting y only within group
resamp<-function(x){
  ug<-unique(x[,"group"])
  y<- x[,"y"]
  for (i in 1:length(ug)){
    pick.strata<- x[,"group"]==ug[i]
    y[pick.strata]<-sample(y[pick.strata],replace=FALSE)
  }
  x[,1]<-y
  x
}

out<-MCtest.fixed(x,stat,resamp,N=199)
out$p.value
out$p.value.ci
```

---

p1.given.p0

*Find p0 (or p1) associated with p1 (or p0) that gives minimax tsprt*

---

### Description

Consider the SPRT for testing  $H_0:p=p_0$  vs  $H_1:p=p_1$  where  $p_1 < \text{Alpha} < p_0$ . For Monte Carlo tests, we want to reject and conclude that  $p < \text{Alpha}$ . In terms of the resampling risk at  $p$  (i.e., the probability of reaching a wrong decision at  $p$ ) the minimax SPRT has a particular relationship between  $p_0$  and  $p_1$ . Here we calculate  $p_1$  given  $p_0$  or vice versa to obtain that relationship.

### Usage

```
p1.given.p0(p0, Alpha = 0.05, TOL = 10^-9)
p0.given.p1(p1, Alpha = 0.05, TOL = 10^-9)
```

### Arguments

p0	null such that $p_0 > \text{Alpha}$
p1	alternative such that $p_1 < \text{Alpha}$
Alpha	want tsprt associated with testing $p = \text{Alpha}$ or not
TOL	tolerance level input into call to <a href="#">uniroot</a>

### Value

either  $p_0$  or  $p_1$

### Author(s)

Michael P. Fay

### References

Fay, M.P., Kim, H-J. and Hachey, M. (2007). Using truncated sequential probability ratio test boundaries for Monte Carlo implementation of hypothesis tests. *Journal of Computational and Graphical Statistics*. 16(4):946-967.

### Examples

```
p1.given.p0(.04)
```

---

`plot.MCbound`*Plot stopping boundary*

---

### Description

Creates two plots of an object of class `MCbound`, an stopping boundary for use with Monte Carlo hypothesis tests. First, it plots the stopping boundary as number of replications (i.e.,  $N$ ) vs. number of successes ( $S$ ). Second, it plots the estimated p-values vs. the confidence limits minus the estimated p-values (this nicely shows the width of the confidence intervals).

### Usage

```
## S3 method for class 'MCbound'  
plot(x, rdigit=4, plimit=500,...)
```

### Arguments

<code>x</code>	an object of class <code>MCbound</code>
<code>rdigit</code>	the rounding digit for the parms values in the title
<code>plimit</code>	if the number of points in the <code>MCbound</code> is $>$ <code>plimit</code> then plot lines, otherwise plot points
<code>...</code>	additional arguments to both plot functions

### Value

Does not return any values. Does two plots only.

### Author(s)

M.P. Fay

### See Also

[MCbound](#)

### Examples

```
plot(MCbound.precalc1)
```

---

 rrisk

---

*Calculate resampling risk and expected resampling size*


---

### Description

Calculates for a particular stopping boundary the resampling risk of making the wrong accept/reject decision. Can be calculated for different distributions of the p-value. If type="p" then assume point mass at pparms. If type="b" then assume a beta distribution with two shape parameters given by pparms.

### Usage

```
rrisk(bound, pparms, sig.level = 0.05, type = "b")
```

### Arguments

bound	an object of class MCbound, i.e., a stopping boundary. See <a href="#">MCbound</a> to create
pparms	either a vector of possible point mass p-value distributions (type="p"), or a vector (or matrix with two columns) representing two beta shape parameters (type="b")
sig.level	significance level for defining resampling risk
type	either "p" for point mass p-value distributions, or "b" for a beta distribution

### Details

The resampling risk (RR) is defined as the probability of making an accept/reject decision different from complete enumeration. In other words, for any Monte Carlo test the true p-value for any data is either below the sig.level (reject the null) or above the sig.level (accept the null), and the RR is the probability of either deciding  $p \leq \text{sig.level}$  when  $p > \text{sig.level}$  or vice versa. We also calculate the expected resampling size for the assumed distributions on the p-values. As a check of the MCbound, we sum the probability of stopping at any point in the boundary over the entire stopping boundary for each assumed distribution on the p-values; the output value check should give a vector of all ones if the MCbound is calculated correctly.

### Value

A list with the following elements:

check	Sum of the probabilities of the stopping boundary corresponding to the p-value distribution(s). Should be a vector with all values equal to 1.
rr	resampling risk corresponding to the p-value distribution(s)
EN	expected resampling size corresponding to the p-value distribution(s)

### Author(s)

Michael P. Fay

## References

Fay, M.P., Kim, H-J. and Hachey, M. (2007). Using truncated sequential probability ratio test boundaries for Monte Carlo implementation of hypothesis tests. *Journal of Computational and Graphical Statistics*. 16(4):946-967.

## Examples

```
### caculate resampling risk and E(N) under null, i.e., uniform distribution on p-values
rrisk(MCbound.precalc1,c(1,1))
```

---

<code>tSPRT.to.Bvalue</code>	<i>Convert between MCbound parameterizations</i>
------------------------------	--

---

## Description

Convert from the tSPRT to the Bvalue parametrization or vice versa.

## Usage

```
tSPRT.to.Bvalue(parms)
Bvalue.to.tSPRT(parms,p0,TOL=10^-8)
```

## Arguments

<code>parms</code>	named vector of parameters
<code>p0</code>	To pick a unique parameterization of the type tSPRT, you must specify p0
<code>TOL</code>	tolerance for solution of <a href="#">p1.given.p0</a>

## Value

Parameter vector of other parameterization.

## Note

`tsprt.to.Bvalue` called by [MCbound](#) when `type="tsprt"`.

## Author(s)

Michael P. Fay

## See Also

[MCbound](#)

## Examples

```
temp<-tSPRT.to.Bvalue(c(p0=.04,p1=p1.given.p0(.04),alpha0=.001,beta0=.001,Nmax=9999))
temp
Bvalue.to.tSPRT(temp,p0=.04)
```

# Index

- \* **hplot**
    - plot.MCbound, 10
  - \* **htest**
    - MCbound, 4
    - MChtest-package, 2
    - MCtest, 7
  - \* **misc**
    - find.ab, 3
    - p1.given.p0, 9
    - rrisk, 11
    - tSPRT.to.Bvalue, 12
  - \* **package**
    - MChtest-package, 2
  - \* **sysdata**
    - MCbound.precalc1, 6
- Bvalue.to.tSPRT (tSPRT.to.Bvalue), 12
- find.ab, 3
- MCbound, 2, 4, 7, 8, 10–12
- MCbound.precalc1, 2, 6, 7, 8
- MChtest (MChtest-package), 2
- MChtest-package, 2
- MCtest, 2, 4, 7
- p0.given.p1 (p1.given.p0), 9
- p1.given.p0, 9, 12
- plot.MCbound, 10
- rrisk, 11
- tSPRT.to.Bvalue, 12
- uniroot, 9