Package ‘HDRFA’

January 20, 2025

Type Package

Title High-Dimensional Robust Factor Analysis

Version 0.1.5

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Description
Factor models have been widely applied in areas such as economics and finance, and the well-known heavy-tailedness of macroeconomic/financial data should be taken into account when conducting factor analysis. We propose two algorithms to do robust factor analysis by considering the Huber loss. One is based on minimizing the Huber loss of the idiosyncratic error's L2 norm, which turns out to do Principal Component Analysis (PCA) on the weighted sample covariance matrix and thereby named as Huber PCA. The other one is based on minimizing the element-wise Huber loss, which can be solved by an iterative Huber regression algorithm. In this package we also provide the code for traditional PCA, the Robust Two Step (RTS) method by He et al. (2022) and the Quantile Factor Analysis (QFA) method by Chen et al. (2021) and He et al. (2023).

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Encoding UTF-8

Imports quantreg, pracma

Depends R (>= 3.5.0)

NeedsCompilation no

Repository CRAN

Date/Publication 2024-07-22 04:50:02 UTC

Contents

HPCA *Huber Principal Component Analysis for Large-Dimensional Factor Models*

Description

This function is to fit the factor models via the Huber Principal Component Analysis (HPCA) method. One is based on minimizing the Huber loss of the idiosyncratic error's ℓ_2 norm, which turns out to do Principal Component Analysis (PCA) on the weighted sample covariance matrix and thereby named as Huber PCA. The other one is based on minimizing the elementwise Huber loss, which can be solved by an iterative Huber regression algorithm.

Usage

```
HPCA(X, r, Method = "E", tau = NULL, scale_est="MAD", L_init = NULL,
     F_init = NULL, maxiter_HPCA = 100, maxiter_HLM = 100, eps = 0.001)
```

Arguments

X Input matrix, of dimension $T \times N$. Each row is an observation with N features at time point t.

r A positive integer indicating the factor numbers.

Method `Method="P"` indicates minimizing the Huber loss of the idiosyncratic error's ℓ_2 norm while `Method="E"` indicates minimizing the elementwise Huber loss. The default is the elementwise Huber loss.

tau Optional user-supplied parameter for Huber loss; default is NULL, and τ is provided by default.

scale_est A parameter for the elementwise Huber loss. `scale_est="MAD"` indicates robust variance estimation in each iteration, while `scale_est="const"` indicates fixing user-supplied τ. The default is `scale_est="MAD"`.

L_init User-supplied inital value of loadings; default is the PCA estimator.

F_init User-supplied inital value of factors; default is the PCA estimator.

maxiter_HPCA The maximum number of iterations in the HPCA. The default is 100.

maxiter_HLM The maximum number of iterations in the iterative Huber regression algorithm. The default is 100.

eps The stopping critetion parameter in the HPCA. The default is 1e-3.

Details

See He et al. (2023) for details.

Value

The return value is a list. In this list, it contains the following:

`Fhat` The estimated factor matrix of dimension $T \times r$.

`Lhat` The estimated loading matrix of dimension $N \times r$.

`m` The number of iterations.

Author(s)

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

References

He Y, Li L, Liu D, Zhou W., 2023 Huber Principal Component Analysis for Large-dimensional Factor Models.

Examples

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E

fit=HPCA(X,r,Method = "E")
fit$Fhat;fit$Lhat

fit=HPCA(X,r,Method = "P")
fit$Fhat;fit$Lhat
```

HPCA_FN *Estimating Factor Numbers via Rank Minimization Corresponding to Huber PCA*

Description

This function is to estimate factor numbers via rank minimization corresponding to Huber Principal Component Analysis (HPCA).

Usage

```
HPCA_FN(X, rmax, Method = "E", tau = NULL, scale_est="MAD", threshold = NULL,
        L_init = NULL, F_init = NULL, maxiter_HPCA = 100, maxiter_HLM = 100,
        eps = 0.001)
```

Arguments

`X` Input matrix, of dimension $T \times N$. Each row is an observation with N features at time point t.

`rmax` The user-supplied maximum factor numbers.

`Method` `Method="P"` indicates minimizing the Huber loss of the idiosyncratic error's ℓ_2 norm while `Method="E"` indicates minimizing the elementwise Huber loss. The default is the elementwise Huber loss.

`tau` Optional user-supplied parameter for Huber loss; default is NULL, and τ is provided by default.

`scale_est` A parameter for the elementwise Huber loss. `scale_est="MAD"` indicates robust variance estimation in each iteration, while `scale_est="const"` indicates fixing user-supplied τ. The default is `scale_est="MAD"`.

`threshold` The threshold of rank minimization; default is NULL.

`L_init` User-supplied inital value of loadings in the HPCA; default is the PCA estimator.

`F_init` User-supplied inital value of factors in the HPCA; default is the PCA estimator.

`maxiter_HPCA` The maximum number of iterations in the HPCA. The default is 100.

`maxiter_HLM` The maximum number of iterations in the iterative Huber regression algorithm. The default is 100.

`eps` The stopping critetion parameter in the HPCA. The default is 1e-3.

Details

See He et al. (2023) for details.

Value

`rhat` The estimated factor number.

Author(s)

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

References

He Y, Li L, Liu D, Zhou W., 2023 Huber Principal Component Analysis for Large-dimensional Factor Models.

Examples

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E


HPCA_FN(X,8,Method="E")
HPCA_FN(X,8,Method="P")
```

IQR *Iterative Quantile Regression Methods for Quantile Factor Models*

Description

This function is to fit the quantile factor model via the Iterative Quantile Regression (IQR) algorithm.

Usage

```
IQR(X, r, tau, L_init = NULL, F_init = NULL, max_iter = 100, eps = 0.001)
```

Arguments

`X` Input matrix, of dimension $T \times N$. Each row is an observation with N features at time point t.

`r` A positive integer indicating the factor numbers.

`tau` The user-supplied quantile level.

`L_init` User-supplied inital value of loadings; default is the PCA estimator.

`F_init` User-supplied inital value of factors; default is the PCA estimator.

`max_iter` The maximum number of iterations. The default is 100.

`eps` The stopping critetion parameter. The default is 1e-06.

Details

See Chen et al. (2021) and He et al. (2023) for details.

Value

The return value is a list. In this list, it contains the following:

`Fhat` The estimated factor matrix of dimension $T \times r$.

`Lhat` The estimated loading matrix of dimension $N \times r$.

`t` The number of iterations.

Author(s)

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

References

Chen, L., Dolado, J.J., Gonzalo, J., 2021. Quantile factor models. Econometrica 89, 875–910.

He Y, Kong X, Yu L, Zhao P., 2023 Quantile factor analysis for large-dimensional time series with statistical guarantee <arXiv:2006.08214>.

Examples

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E

tau=0.5
fit=IQR(X,r,tau)
fit$Fhat;fit$Lhat
```

IQR_FN *Estimating Factor Numbers via Rank Minimization Corresponding to IQR*

Description

This function is to estimate factor numbers via rank minimization corresponding to Iterative Quantile Regression (IQR).

Usage

```
IQR_FN(X, rmax, tau, threshold = NULL, L_init = NULL, F_init = NULL,
       max_iter = 100, eps = 10^(-6))
```

Arguments

X	Input matrix, of dimension $T \times N$. Each row is an observation with N features at time point t.
rmax	The user-supplied maximum factor numbers.
tau	The user-supplied quantile level.
threshold	The threshold of rank minimization; default is NULL.
L_init	User-supplied inital value of loadings in the IQR; default is the PCA estimator.
F_init	User-supplied inital value of factors in the IQR; default is the PCA estimator.
max_iter	The maximum number of iterations. The default is 100.
eps	The stopping critetion parameter of the IQR method. The default is 1e-06.

Details

See Chen et al. (2021) for more details.

Value

rhat	The estimated factor number.

Author(s)

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

References

Chen, L., Dolado, J.J., Gonzalo, J., 2021. Quantile factor models. Econometrica 89, 875–910.

Examples

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E

tau=0.5
IQR_FN(X,8,tau)
```

PCA *Principal Component Analysis for Large-Dimensional Factor Models*

Description

This function is to fit the factor models via Principal Component Analysis (PCA) methods.

Usage

```
PCA(X, r, constraint = "L")
```

Arguments

`X`	Input matrix, of dimension $T \times N$. Each row is an observation with N features at time point t.
`r`	A positive integer indicating the factor numbers.
`constraint`	The type of identification condition. If `constraint="L"`, the columns of the estimated loading matrix are orthogonal and `constraint="F"` indicates the columns of the estimated factor matrix are orthogonal.

Details

See Bai (2003) for details.

Value

The return value is a list. In this list, it contains the following:

`Fhat`	The estimated factor matrix of dimension $T \times r$.
`Lhat`	The estimated loading matrix of dimension $N \times r$.

Author(s)

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

References

Bai, J., 2003. Inferential theory for factor models of large dimensions. Econometrica 71, 135–171.

Examples

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E

fit=PCA(X,3,"L")
t(fit$Lhat)%*%fit$Lhat/N

fit=PCA(X,3,"F")
t(fit$Fhat)%*%fit$Fhat/T
```

PCA_FN *Estimating Factor Numbers via Eigenvalue Ratios Corresponding to PCA*

Description

This function is to estimate factor numbers via eigenvalue ratios corresponding to Principal Component Analysis (PCA).

Usage

```
PCA_FN(X, rmax)
```

Arguments

`X` Input matrix, of dimension $T \times N$. Each row is an observation with N features at time point t.

`rmax` The user-supplied maximum factor numbers.

Details

See Ahn and Horenstein (2013) for details.

Value

`rhat` The estimated factor numbers.

Author(s)

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

References

Ahn, S.C., Horenstein, A.R., 2013. Eigenvalue ratio test for the number of factors. Econometrica 81, 1203–1227.

Examples

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E

PCA_FN(X,8)
```

`RTS` *Robust Two Step Algorithm for Large-Dimensional Elliptical Factor Models*

Description

This function is to fit the large-dimensional elliptical factor models via the Robust Two Step (RTS) algorithm.

Usage

```
RTS(X, r)
```

Arguments

`X` Input matrix, of dimension $T \times N$. Each row is an observation with N features at time point t.

`r` A positive integer indicating the factor numbers.

Details

See He et al. (2022) for details.

Value

The return value is a list. In this list, it contains the following:

`Fhat` The estimated factor matrix of dimension $T \times r$.

`Lhat` The estimated loading matrix of dimension $N \times r$.

Author(s)

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

References

He, Y., Kong, X., Yu, L., Zhang, X., 2022. Large-dimensional factor analysis without moment constraints. Journal of Business & Economic Statistics 40, 302–312.

Examples

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E

fit=RTS(X,3)
fit$Fhat;fit$Lhat
```

RTS_FN *Estimating Factor Numbers Robustly via Multivariate Kendall's Tau Eigenvalue Ratios*

Description

This function is to estimate factor numbers robustly via multivariate Kendall's tau eigenvalue ratios.

Usage

```
RTS_FN(X, rmax)
```

Arguments

X	Input matrix, of dimension $T \times N$. Each row is an observation with N features at time point t.
rmax	The user-supplied maximum factor numbers.

Details

See Yu et al. (2019) for details.

Value

rhat	The estimated factor number.

Author(s)

Yong He, Lingxiao Li, Dong Liu, Wenxin Zhou.

References

Yu, L., He, Y., Zhang, X., 2019. Robust factor number specification for large-dimensional elliptical factor model. Journal of Multivariate analysis 174, 104543.

Examples

```
set.seed(1)
T=50;N=50;r=3
L=matrix(rnorm(N*r,0,1),N,r);F=matrix(rnorm(T*r,0,1),T,r)
E=matrix(rnorm(T*N,0,1),T,N)
X=F%*%t(L)+E

RTS_FN(X,8)
```

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