

# Package ‘Lmoments’

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**Title** L-Moments and Quantile Mixtures

**Maintainer** Juha Karvanen <juha.karvanen@iki.fi>

**Description** Contains functions to estimate

L-moments and trimmed L-moments from the data. Also contains functions to estimate the parameters of the normal polynomial quantile mixture and the Cauchy polynomial quantile mixture from L-moments and trimmed L-moments.

**Imports** stats, Rcpp

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**License** GPL-2

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**Author** Juha Karvanen [cre, aut],  
Santeri Karppinen [aut]

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cauchypoly

*Cauchy-polynomial quantile mixture***Description**

Density, distribution function, quantile function and random generation for the Cauchy-polynomial quantile mixture.

**Usage**

```
dcauchypoly(x,param)
pcauchypoly(x,param)
qcauchypoly(cp,param)
rcauchypoly(n,param)
cauchypoly_pdf(x,param)
cauchypoly_cdf(x,param)
cauchypoly_inv(cp,param)
cauchypoly_rnd(n,param)
```

**Arguments**

x	vector of quantiles
cp	vector of probabilities
n	number of observations
param	vector of parameters

**Details**

The length the parameter vector specifies the order of the polynomial in the quantile mixture. If  $k < \text{length}(\text{param})$  then  $\text{param}[1:(k-1)]$  contains the mixture coefficients of polynomials starting from the constant and  $\text{param}[k]$  is the mixture coefficient for Cauchy distribution. (Functions `cauchypoly_pdf`, `cauchypoly_cdf`, `cauchypoly_inv` and `cauchypoly_rnd` are aliases for compatibility with older versions of this package.)

**Value**

`'dcauchypoly'` gives the density, `'pcauchypoly'` gives the cumulative distribution function, `'qcauchypoly'` gives the quantile function, and `'rcauchypoly'` generates random deviates.

**Author(s)**

Juha Karvanen <[juha.karvanen@iki.fi](mailto:juha.karvanen@iki.fi)>

**References**

Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [http://www.bsp.brain.riken.jp/publications/2006/karvanen\\_quantile\\_mixtures.pdf](http://www.bsp.brain.riken.jp/publications/2006/karvanen_quantile_mixtures.pdf).

**See Also**

[data2cauchypoly4](#) for the parameter estimation and [dnormpoly](#) for the normal-polynomial quantile mixture.

**Examples**

```
#Generates 500 random variables from the Cauchy-polynomial quantile mixture,
#calculates the trimmed L-moments,
#estimates parameters via trimmed L-moments and
#plots the true pdf and the estimated pdf together with the histogram of the data.
true_params<-t1lmom2cauchypoly4(c(0,1,0.075,0.343));
x<-rcauchypoly(500,true_params);
t1lmom<-t1lmoments(x);
estim_params<-t1lmom2cauchypoly4(t1lmom);
plotpoints<-seq(-10,10,by=0.01);
histpoints<-c(seq(min(x)-1,-20,length.out=50),seq(-10,10,by=0.5),seq(20,max(x)+1,length.out=50));
hist(x,breaks=histpoints,freq=FALSE,xlim=c(-10,10));
lines(plotpoints,dcauchypoly(plotpoints,estim_params),col='red');
lines(plotpoints,dcauchypoly(plotpoints,true_params),col='blue');
```

covnormpoly4

*Covariance matrix of the parameters of the normal-polynomial quantile mixture*

**Description**

Estimates covariance matrix of the four parameters of normal-polynomial quantile mixture

**Usage**

```
covnormpoly4(data)
```

**Arguments**

data	vector of observations
------	------------------------

**Value**

covariance matrix of the four parameters of normal-polynomial quantile mixture

**Author(s)**

Juha Karvanen <juha.karvanen@iki.fi>

**References**

Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [http://www.bsp.brain.riken.jp/publications/2006/karvanen\\_quantile\\_mixtures.pdf](http://www.bsp.brain.riken.jp/publications/2006/karvanen_quantile_mixtures.pdf).

**See Also**

[Lmomcov](#) for covariance matrix of L-moments, [dnormpoly](#) for the normal-polynomial quantile mixture and [data2normpoly4](#) for the estimation of the normal-polynomial quantile mixture.

**data2cauchypoly***Estimation of the Cauchy-polynomial quantile mixture***Description**

Estimates the parameters of the Cauchy-polynomial quantile mixture from data or from trimmed L-moments

**Usage**

```
data2cauchypoly4(data)
t11mom2cauchypoly4(t11mom)
```

**Arguments**

data	vector
t11mom	vector of trimmed L-moments

**Value**

vector containing the four parameters of the Cauchy-polynomial quantile mixture

**Author(s)**

Juha Karvanen <[juha.karvanen@iki.fi](mailto:juha.karvanen@iki.fi)>

**References**

Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [http://www.bsp.brain.riken.jp/publications/2006/karvanen\\_quantile\\_mixtures.pdf](http://www.bsp.brain.riken.jp/publications/2006/karvanen_quantile_mixtures.pdf).

**See Also**

[t11moments](#) for trimmed L-moments, [dcauchypoly](#) for the Cauchy-polynomial quantile mixture and [data2normpoly4](#) for the estimation of the normal-polynomial quantile mixture.

## Examples

```
#Generates 500 random variables from the Cauchy-polynomial quantile mixture,
#calculates the trimmed L-moments,
#estimates parameters via trimmed L-moments and
#plots the true pdf and the estimated pdf together with the histogram of the data.
true_params<-t1lmom2cauchypoly4(c(0,1,0.075,0.343));
x<-rcauchypoly(500,true_params);
t1lmom<-t1lmoments(x);
estim_params<-t1lmom2cauchypoly4(t1lmom);
plotpoints<-seq(-10,10,by=0.01);
histpoints<-c(seq(min(x)-1,-20,length.out=50),seq(-10,10,by=0.5),seq(20,max(x)+1,length.out=50));
hist(x,breaks=histpoints,freq=FALSE,xlim=c(-10,10));
lines(plotpoints,dcauchypoly(plotpoints,estim_params),col='red');
lines(plotpoints,dcauchypoly(plotpoints,true_params),col='blue');
```

data2normpoly

*Estimation of normal-polynomial quantile mixture*

## Description

Estimates the parameters of normal-polynomial quantile mixture from data or from L-moments

## Usage

```
data2normpoly4(data)
lmom2normpoly4(lmom)
data2normpoly6(data)
lmom2normpoly6(lmom)
```

## Arguments

data	matrix or data frame
lmom	vector or matrix of L-moments

## Value

vector or matrix containing the four or six parameters of normal-polynomial quantile mixture

## Author(s)

Juha Karvanen <juha.karvanen@iki.fi>

## References

Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [http://www.bsp.brain.riken.jp/publications/2006/karvanen\\_quantile\\_mixtures.pdf](http://www.bsp.brain.riken.jp/publications/2006/karvanen_quantile_mixtures.pdf).

**See Also**

[dnormpoly](#) for L-moments, [dnormpoly](#) for the normal-polynomial quantile mixture and [data2cauchypoly4](#) for the estimation of Cauchy-polynomial quantile mixture.

**Examples**

```
#Generates a sample 500 observations from the normal-polynomial quantile mixture,
#calculates L-moments and their covariance matrix,
#estimates parameters via L-moments and
#plots the true pdf and the estimated pdf together with the histogram of the data.
true_params<-lmom2normpoly4(c(0,1,0.2,0.05));
x<-rnormpoly(500,true_params);
lmoments<-Lmoments(x);
lmomcov<-Lmomcov(x);
estim_params<-lmom2normpoly4(lmoments);
hist(x,30,freq=FALSE);
plotpoints<-seq(min(x)-1,max(x)+1,by=0.01);
lines(plotpoints,dnormpoly(plotpoints,estim_params),col='red');
lines(plotpoints,dnormpoly(plotpoints,true_params),col='blue');
```

**Description**

Calculates sample L-moments, L-coefficients and covariance matrix of L-moments.

**Usage**

```
Lmoments(data, rmax = 4, na.rm = FALSE, returnobject = FALSE, trim = c(0, 0))
Lcoefs(data, rmax = 4, na.rm = FALSE, trim = c(0, 0))
Lmomcov(data, rmax = 4, na.rm = FALSE)
Lmoments_calc(data, rmax = 4)
Lmomcov_calc(data, rmax = 4)
shiftedlegendre(rmax)
```

**Arguments**

<code>data</code>	matrix or data frame.
<code>rmax</code>	maximum order of L-moments.
<code>na.rm</code>	a logical value indicating whether 'NA' values should be removed before the computation proceeds.
<code>returnobject</code>	a logical value indicating whether a list object should be returned instead of an array of L-moments.
<code>trim</code>	<code>c(0, 0)</code> for ordinary L-moments and <code>c(1, 1)</code> for trimmed ( $t = 1$ ) L-moments

## Value

`Lmoments` returns an array of L-moments containing a row for each variable in data, or if `returnobject=TRUE`, a list containing

<code>lambdas</code>	an array of L-moments
<code>ratios</code>	an array of mean, L-scale and L-moment ratios
<code>trim</code>	the value of the parameter 'trim'
<code>source</code>	a string with value "Lmoments" or "t1lmoments".

`Lcoefs` returns an array of L-coefficients (mean, L-scale, L-skewness, L-kurtosis, ...) containing a row for each variable in data.

`Lmomcov` returns the covariance matrix of L-moments or a list of covariance matrices if the input has multiple columns. The numerical accuracy of the results decreases with increasing `rmax`. With `rmax > 5`, a warning is thrown, as the numerical accuracy of the results is likely less than `sqrt(.Machine$double.eps)`.

`shiftedlegendre` returns a matrix of the coefficients of the shifted Legendre polynomials up to a given order.

## Note

Functions `Lmoments` and `Lcoefs` calculate trimmed L-moments if you specify `trim = c(1, 1)`. `Lmoments_calc` and `Lmomcov_calc` are internal C++ functions called by `Lmoments` and `Lmomcov`. The direct use of these functions is not recommended.

## Author(s)

Juha Karvanen <[juha.karvanen@iki.fi](mailto:juha.karvanen@iki.fi)>, Santeri Karppinen

## References

- Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [http://www.bsp.brain.riken.jp/publications/2006/karvanen\\_quantile\\_mixtures.pdf](http://www.bsp.brain.riken.jp/publications/2006/karvanen_quantile_mixtures.pdf).
- Elamir, E. A., Seheult, A. H. 2004. Exact variance structure of sample L-moments, *Journal of Statistical Planning and Inference* **124** (2) 337–359.
- Hosking, J. 1990. L-moments: Analysis and estimation distributions using linear combinations of order statistics, *Journal of Royal Statistical Society B* **52**, 105–124.

## See Also

`t1lmoments` for trimmed L-moments, `dnormpoly`, `lmom2normpoly4` and `covnormpoly4` for the normal-polynomial quantile mixture and package `lmomco` for additional L-moment functions

## Examples

```
#Generates a sample 500 observations from the normal-polynomial quantile mixture,
#calculates the L-moments and their covariance matrix,
#estimates parameters via L-moments and
#plots the true pdf and the estimated pdf together with the histogram of the data.
true_params<-lmom2normpoly4(c(0.1,0.2,0.05));
x<-rnormpoly(500,true_params);
lmoments<-Lmoments(x);
lmomcov<-Lmomcov(x);
estim_params<-lmom2normpoly4(lmoments);
hist(x,30,freq=FALSE)
plotpoints<-seq(min(x)-1,max(x)+1,by=0.01);
lines(plotpoints,dnormpoly(plotpoints,estim_params),col='red');
lines(plotpoints,dnormpoly(plotpoints,true_params),col='blue');
```

normpoly

*Normal-polynomial quantile mixture*

## Description

Density, distribution function, quantile function and random generation for the normal-polynomial quantile mixture.

## Usage

```
dnormpoly(x,param)
pnormpoly(x,param)
qnormpoly(cp,param)
rnormpoly(n,param)
normpoly_pdf(x,param)
normpoly_cdf(x,param)
normpoly_inv(cp,param)
normpoly_rnd(n,param)
```

## Arguments

x	vector of quantiles
cp	vector of probabilities
n	number of observations
param	vector of parameters

## Details

The length the parameter vector specifies the order of the polynomial in the quantile mixture. If  $k < \text{length}(\text{param})$  then  $\text{param}[1:(k-1)]$  contains the mixture coefficients of polynomials starting from the constant and  $\text{param}[k]$  is the mixture coefficient for normal distribution. (Functions `normpoly_pdf`, `normpoly_cdf`, `normpoly_inv` and `normpoly_rnd` are aliases for compatibility with older versions of this package.)

**Value**

'dnormpoly' gives the density, 'pnormpoly' gives the cumulative distribution function, 'qnormpoly' gives the quantile function, and 'rnormpoly' generates random deviates.

**Author(s)**

Juha Karvanen <juha.karvanen@iki.fi>

**References**

Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [http://www.bsp.brain.riken.jp/publications/2006/karvanen\\_quantile\\_mixtures.pdf](http://www.bsp.brain.riken.jp/publications/2006/karvanen_quantile_mixtures.pdf).

**See Also**

[data2normpoly4](#) for the parameter estimation and [dcauchypoly](#) for the Cauchy-polynomial quantile mixture.

**Examples**

```
#Generates a sample 500 observations from the normal-polynomial quantile mixture,
#calculates L-moments and their covariance matrix,
#estimates parameters via L-moments and
#plots the true pdf and the estimated pdf together with the histogram of the data.
true_params<-lmom2normpoly4(c(0,1,0.2,0.05));
x<-rnormpoly(500,true_params);
lmoments<-Lmoments(x);
lmomcov<-Lmomcov(x);
estim_params<-lmom2normpoly4(lmoments);
hist(x,30,freq=FALSE)
plotpoints<-seq(min(x)-1,max(x)+1,by=0.01);
lines(plotpoints,dnormpoly(plotpoints,estim_params),col='red');
lines(plotpoints,dnormpoly(plotpoints,true_params),col='blue');
```

**Description**

Calculates sample trimmed L-moments with trimming parameter 1.

**Usage**

```
t11moments(data, rmax = 4)
t11moments_calc(data, rmax = 4)
```

## Arguments

<code>data</code>	matrix or data frame.
<code>rmax</code>	maximum order of trimmed L-moments.

## Value

array of trimmed L-moments (trimming parameter = 1) up to order 4 containing a row for each variable in `data`.

## Note

Functions `link{Lmoments}` and `link{Lcoefs}` calculate trimmed L-moments if you specify `trim = c(1, 1)`. `t11moments_calc` is an internal C++ function called by `t11moments`. The direct use of this function is not recommended.

## Author(s)

Juha Karvanen <[juha.karvanen@iki.fi](mailto:juha.karvanen@iki.fi)>, Santeri Karppinen

## References

- Karvanen, J. 2006. Estimation of quantile mixtures via L-moments and trimmed L-moments, *Computational Statistics & Data Analysis* **51**, (2), 947–959. [http://www.bsp.brain.riken.jp/publications/2006/karvanen\\_quantile\\_mixtures.pdf](http://www.bsp.brain.riken.jp/publications/2006/karvanen_quantile_mixtures.pdf).
- Elamir, E. A., Seheult, A. H. 2003. Trimmed L-moments, *Computational Statistics & Data Analysis* **43**, 299–314.

## See Also

`Lmoments` for L-moments, and `dcauchypoly` and `t11mom2cauchypoly4` for the Cauchy-polynomial quantile mixture

## Examples

```
#Generates 500 random variables from the Cauchy-polynomial quantile mixture,
#calculates the trimmed L-moments,
#estimates parameters via trimmed L-moments and
#plots the true pdf and the estimated pdf together with the histogram of the data.
true_params<-t11mom2cauchypoly4(c(0,1,0.075,0.343));
x<-rcauchypoly(500,true_params);
t11mom<-t11moments(x);
estim_params<-t11mom2cauchypoly4(t11mom);
plotpoints<-seq(-10,10,by=0.01);
histpoints<-c(seq(min(x)-1,-20,length.out=50),seq(-10,10,by=0.5),seq(20,max(x)+1,length.out=50));
hist(x,breaks=histpoints,freq=FALSE,xlim=c(-10,10));
lines(plotpoints,dcauchypoly(plotpoints,estim_params,col='red'));
lines(plotpoints,dcauchypoly(plotpoints,true_params),col='blue');
```

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