# Package 'rqPen'

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Type Package

Title Penalized Quantile Regression

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LinkingTo Rcpp, RcppArmadillo

RdMacros Rdpack

Suggests splines, knitr

Maintainer Ben Sherwood <ben.sherwood@ku.edu>

#### **Description**

Performs penalized quantile regression with LASSO, elastic net, SCAD and MCP penalty functions including group penalties. In addition, offers a group penalty that provides consistent variable selection across quantiles. Provides a function that automatically generates lambdas and evaluates different models with cross validation or BIC, including a large p version of BIC. Below URL provides a link to a work in progress vignette.

ByteCompile TRUE

**Encoding UTF-8** 

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URL https://github.com/bssherwood/rqpen/blob/master/ignore/rqPenArticle.pdf

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**NeedsCompilation** yes

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bytau.plot

# **Contents**

Index		36
	rqPen	34
	rq.pen.cv	
	rq.pen	
	rq.group.pen.cv	
	rq.group.pen	
	rq.gq.pen.cv	
	rq.gq.pen	18
	qic.select.rq.pen.seq.cv	17
	qic.select.rq.pen.seq	15
	qic.select	
	print.rq.pen.seq.cv	
	print.rq.pen.seq	
	print.qic.select	
	predict.rq.pen.seq.cv	
	predict.rq.pen.seq	
	predict.qic.select	
	plot.rq.pen.seq.cv	_
	plot.rq.pen.seq	_
	coef.rq.pen.seq.cv	
	coef.rq.pen.seq	5
	bytau.plot.rq.pen.seq.cv	
	bytau.plot.rq.pen.seq	
	bytau.plot	- 2

bytau.plot

Plot of how coefficients change with tau

# Description

Plot of how coefficients change with tau

# Usage

```
bytau.plot(x, ...)
```

# **Arguments**

x A rq.pen.seq or rq.pen.seq.cv object.

... Additional arguments see bytau.plot.rq.pen.seq() or bytau.plot.rq.pen.seq.cv() for more information.

# Value

Returns the plot of how coefficients change with tau.

bytau.plot.rq.pen.seq 3

#### Author(s)

Ben Sherwood, <ben.sherwood@ku.edu>

bytau.plot.rq.pen.seq Plot of how coefficients change with tau.

# Description

Plot of how coefficients change with tau.

# Usage

```
## S3 method for class 'rq.pen.seq'
bytau.plot(x, a = NULL, lambda = NULL, lambdaIndex = NULL, vars = NULL, ...)
```

# **Arguments**

X	An rq.pen.seq object
а	The tuning parameter a of interest
lambda	The lambda value of interest.
lambdaIndex	The lambda index of interest. Only specify lambdaIndex or lambda, not both.
vars	Index of the variables to plot with 1 being the intercept, 2 being the first predictor, etc. Default is to include all variables.

Additional parameters sent to coef()

# Value

A plot of coefficient values by tau.

# Author(s)

Ben Sherwood, <ben.sherwood@ku.edu>

# **Examples**

```
set.seed(1)
x <- matrix(rnorm(800),nrow=100)
y <- 1 + x[,1] - 3*x[,5] + rnorm(100)
lassoModels <- rq.pen(x,y,tau=seq(.1,.9,.1))
bytau.plot(lassoModels,lambda=lassoModels$lambda[5])</pre>
```

```
bytau.plot.rq.pen.seq.cv
```

Plot of coefficients varying by quantiles for rq.pen.seq.cv object

# **Description**

Produces plots of how coefficient estimates vary by quantile for models selected by using cross validation.

#### Usage

```
## S3 method for class 'rq.pen.seq.cv'
bytau.plot(
    x,
    septau = ifelse(x$fit$penalty != "gq", TRUE, FALSE),
    cvmin = TRUE,
    useDefaults = TRUE,
    vars = NULL,
    ...
)
```

# **Arguments**

X	An rq.pen.seq.cv object
septau	Whether optimal tuning parameters are estimated separately for each quantile.
cvmin	Whether the minimum cv error should be used or the one standard error rule.
useDefaults	Set to FALSE if you want to use something besides minimum cv or 1se.
vars	Index of the variables to plot with 1 being the intercept, 2 being the first predictor, etc. Default is to include all variables.
	Additional parameters sent to coef()

#### Value

Returns plots of coefficient estimates varying by quantile.

# Author(s)

Ben Sherwood, <ben.sherwood@ku.edu>

# **Examples**

```
set.seed(1)
x <- matrix(runif(800),nrow=100)
y <- 1 + x[,1] - 3*x[,5] + (1+x[,4])*rnorm(100)
lmcv <- rq.pen.cv(x,y,tau=seq(.1,.9,.1))
bytau.plot(lmcv)</pre>
```

coef.rq.pen.seq 5

coef.rq.pen.seq

Returns coefficients of a rq.pen.seq object

# Description

Returns coefficients of a rq.pen.seq object

# Usage

```
## $3 method for class 'rq.pen.seq'
coef(
  object,
  tau = NULL,
  a = NULL,
  lambda = NULL,
  modelsIndex = NULL,
  lambdaIndex = NULL,
  ...
)
```

# Arguments

object	rq.pen.seq object
tau	Quantile of interest. Default is NULL, which will return all quantiles. Should not be specified if modelsIndex is used.
a	Tuning parameter of a. Default is NULL, which returns coefficients for all values of a. Should not be specified if modelsIndex is used.
lambda	Tuning parameter of $\lambda$ . Default is NULL, which returns coefficients for all values of $\lambda$ .
modelsIndex	Index of the models for which coefficients should be returned. Does not need to be specified if tau or a are specified.
lambdaIndex	Index of the lambda values for which coefficients should be returned. Does not need to be specified if lambda is specified.
	Additional parameters.

# Value

A list of a matrix of coefficients for each tau and a combination

# Author(s)

Ben Sherwood, <ben. sherwood@ku.edu>

6 coef.rq.pen.seq.cv

#### **Examples**

coef.rq.pen.seq.cv

Returns coefficients from a rq.pen.seq.cv object.

#### **Description**

Returns coefficients from a rq.pen.seq.cv object.

### Usage

```
## $3 method for class 'rq.pen.seq.cv'
coef(
  object,
  septau = ifelse(object$fit$penalty != "gq", TRUE, FALSE),
  cvmin = TRUE,
  useDefaults = TRUE,
  tau = NULL,
  ...
)
```

#### **Arguments**

object An rq.pen.seq.cv object.

septau Whether tuning parameter should be optimized separately for each quantile.

cvmin If TRUE then minimum error is used, if FALSE then one standard error rule is

used.

useDefaults Whether the default results are used. Set to FALSE if you you want to specify

specific models and lambda values.

tau Quantiles of interest.

... Additional parameters sent to coef.rq.pen.seq()

#### Value

Returns coefficients

### Author(s)

Ben Sherwood, <ben.sherwood@ku.edu>

plot.rq.pen.seq 7

#### **Examples**

```
## Not run:
set.seed(1)
x <- matrix(rnorm(800),nrow=100)
y <- 1 + x[,1] - 3*x[,5] + rnorm(100)
lassoModels <- rq.pen.cv(x,y,tau=seq(.1,.9,.1))
coefficients(lassoModels,septau=FALSE)
coefficients(lassoModels,cvmin=FALSE)
## End(Not run)</pre>
```

plot.rq.pen.seq

Plot of coefficients of rq.pen.seq object as a function of lambda

# **Description**

Plot of coefficients of rq.pen.seq object as a function of lambda

# Usage

```
## $3 method for class 'rq.pen.seq'
plot(
    x,
    vars = NULL,
    logLambda = TRUE,
    tau = NULL,
    a = NULL,
    lambda = NULL,
    modelsIndex = NULL,
    lambdaIndex = NULL,
    main = NULL,
    ...
)
```

#### **Arguments**

x rq.pen.seq objectvars Variables of interest

logLambda Whether lambda should be reported on the log scale

tau Quantiles of interest

a Tuning parameter a values of interest.

lambda Values of lambda of interest.

modelsIndex Specific models of interest.

lambdaIndex Specific lambda values of interest.

main Title of the plots. Can be a vector of multiple titles if multiple plots are created.

... Additional arguments sent to plot

8 plot.rq.pen.seq.cv

#### Value

Returns plot(s) of coefficients as they change with lambda.

#### Author(s)

Ben Sherwood, <ben. sherwood@ku.edu>

# **Examples**

```
set.seed(1)
x <- matrix(rnorm(100*8,sd=10),ncol=8)
y <- 1 + x[,1] + 3*x[,3] - x[,8] + rt(100,3)
m1 <- rq.pen(x,y,tau=c(.1,.5,.7),penalty="SCAD",a=c(3,4))
plot(m1,a=3,tau=.7)
plot(m1)
mlist <- list()
for(i in 1:6){
mlist[[i]] <- paste("Plot",i)
}
plot(m1,main=mlist)</pre>
```

plot.rq.pen.seq.cv

Plots cross validation results from a rq.pen.seq.cv object

# Description

Provides plots of cross-validation results by lambda. If septau is set to TRUE then plots the cross-validation results for each quantile. If septau is set to FALSE then provides one plot for cross-validation results across all quantiles.

### Usage

```
## S3 method for class 'rq.pen.seq.cv'
plot(
    x,
    septau = ifelse(x$fit$penalty != "gq", TRUE, FALSE),
    tau = NULL,
    logLambda = TRUE,
    main = NULL,
    ...
)
```

# **Arguments**

x The rq.pen.seq.cv object

septau

If set to true then optimal tuning parameters are selected seperately for each quantile and there will be a different plot for each quantile.

predict.qic.select 9

tau Quantiles of interest.

logLambda Whether log(lambda) is used for the x-axis

main Title to the plot

... Additional parameters sent to the plot function.

#### Value

Plots of the cross validation results by lambda.

#### Author(s)

Ben Sherwood, <ben. sherwood@ku.edu>

# **Examples**

```
set.seed(1)
x <- matrix(rnorm(100*8,sd=1),ncol=8)
y <- 1 + x[,1] + 3*x[,3] - x[,8] + rt(100,3)
m1 <- rq.pen.cv(x,y,tau=c(.1,.3,.7))
plot(m1)
plot(m1,septau=FALSE)</pre>
```

predict.qic.select

Predictions from a qic.select object

### **Description**

Predictions from a qic.select object

# Usage

```
## S3 method for class 'qic.select'
predict(object, newx, sort = FALSE, ...)
```

# Arguments

object qic.select object

newx Data matrix to make predictions from.

sort If there are crossing quantiles the predictions will be sorted to avoid this issue.

No additional parameters are used but this is needed for how R handles predict

functions.

### Value

A matrix of predicted values.

10 predict.rq.pen.seq

# Author(s)

Ben Sherwood, <ben.sherwood@ku.edu>

# **Examples**

```
x <- matrix(runif(800),ncol=8)
y <- 1 + x[,1] + x[,8] + (1+.5*x[,3])*rnorm(100)
m1 <- rq.pen(x,y,tau=c(.25,.75))
q1 <- qic.select(m1)
newx <- matrix(runif(80),ncol=8)
preds <- predict(q1,newx)</pre>
```

predict.rq.pen.seq

Predictions from rq.pen.seq object

# Description

Predictions from rq.pen.seq object

# Usage

```
## S3 method for class 'rq.pen.seq'
predict(
   object,
   newx,
   tau = NULL,
   a = NULL,
   lambda = NULL,
   modelsIndex = NULL,
   lambdaIndex = NULL,
   sort = FALSE,
   ...
)
```

# **Arguments**

object	rq.pen.seq object
newx	Matrix of predictors
tau	Quantile of interest. Default is NULL, which will return all quantiles. Should not be specified if modelsIndex is used.
a	Tuning parameter of a. Default is NULL, which returns coefficients for all values of a. Should not be specified if modelsIndex is used.
lambda	Tuning parameter of $\lambda$ . Default is NULL, which returns coefficients for all values of $\lambda$ .
modelsIndex	Index of the models for which coefficients should be returned. Does not need to be specified if tau or a are specified.

predict.rq.pen.seq.cv 11

lambdaIndex	Index of the lambda values for which coefficients should be returned. Does not need to be specified if lambda is specified.
sort	If there are crossing quantiles the predictions will be sorted to avoid this issue.
	Additional parameters passed to coef.rq.pen.seq()

#### Value

A matrix of predictions for each tau and a combination

### Author(s)

Ben Sherwood, <ben. sherwood@ku.edu>

# **Examples**

```
x <- matrix(runif(800),ncol=8)
y <- 1 + x[,1] + x[,8] + (1+.5*x[,3])*rnorm(100)
m1 <- rq.pen(x,y,penalty="ENet",a=c(0,.5,1),tau=c(.25,.75),lambda=c(.1,.05,.01))
newx <- matrix(runif(80),ncol=8)
allCoefs <- predict(m1,newx)
targetCoefs <- predict(m1,newx,tau=.25,a=.5,lambda=.1)
idxApproach <- predict(m1,newx,modelsIndex=2)
bothIdxApproach <- predict(m1,newx,modelsIndex=2,lambdaIndex=1)</pre>
```

 $\verb"predict.rq.pen.seq.cv" \textit{ Predictions from rq.pen.seq.cv object}$ 

# **Description**

Predictions from rq.pen.seq.cv object

#### Usage

```
## S3 method for class 'rq.pen.seq.cv'
predict(
   object,
   newx,
   tau = NULL,
   septau = ifelse(object$fit$penalty != "gq", TRUE, FALSE),
   cvmin = TRUE,
   useDefaults = TRUE,
   sort = FALSE,
   lambda = NULL,
   lambdaIndex = NULL,
   ...
)
```

12 print.qic.select

### **Arguments**

object	rq.pen.seq.cv object
newx	Matrix of predictors
tau	Quantile of interest. Default is NULL, which will return all quantiles. Should not be specified if modelsIndex is used.
septau	Whether tuning parameter should be optimized separately for each quantile.
cvmin	If TRUE then minimum error is used, if FALSE then one standard error rule is used.
useDefaults	Whether the default results are used. Set to FALSE if you you want to specify specific models and lambda values.
sort	If there are crossing quantiles the predictions will be sorted to avoid this issue.
lambda	The value of lambda for which predictions are wanted. Ignored unless useDefaults is set to false.
lambdaIndex	The indices for lambda for which predictions are wanted. Ignored unless useDefaults is set to false.
	Additional parameters sent to coef.rq.pen.seq.cv().

#### Value

A matrix of predictions for each tau and a combination

# Author(s)

Ben Sherwood, <ben.sherwood@ku.edu>

# **Examples**

```
x <- matrix(runif(1600),ncol=8)</pre>
y \leftarrow 1 + x[,1] + x[,8] + (1+.5*x[,3])*rnorm(200)
m1 \leftarrow rq.pen.cv(x,y,penalty="ENet",a=c(0,.5,1),tau=c(.25,.75),lambda=c(.1,.05,.01))
newx <- matrix(runif(80),ncol=8)</pre>
cvpreds <- predict(m1,newx)</pre>
```

print.qic.select

Print a qic.select object

# Description

Print a qic.select object

#### Usage

```
## S3 method for class 'qic.select'
print(x, ...)
```

print.rq.pen.seq 13

# **Arguments**

x qic.select object

... optional arguments

# Value

Prints the coefficients of the qic.select object

# Author(s)

Ben Sherwood, <ben. sherwood@ku.edu>

print.rq.pen.seq

Print a rq.pen.seq object

# Description

Print a rq.pen.seq object

# Usage

```
## S3 method for class 'rq.pen.seq'
print(x, ...)
```

# **Arguments**

x rq.pen.seq object

... optional arguments

### Value

If only one model, prints a data.frame of the number of nonzero coefficients and lambda. Otherwise prints information about the quantiles being modeled and choices for a.

#### Author(s)

Ben Sherwood, <ben. sherwood@ku.edu>

14 qic.select

print.rq.pen.seq.cv

Prints a rq.pen.seq.cv object

# **Description**

Prints a rq.pen.seq.cv object

### Usage

```
## S3 method for class 'rq.pen.seq.cv'
print(x, ...)
```

# **Arguments**

x A req.pen.seq.cv object.

... Additional arguments.

#### Value

Print of btr and gtr from a rq.pen.seq.cv object. If only one quantile is modeled then only btr is returned.

qic.select

Select tuning parameters using IC

#### **Description**

Selects tuning parameter  $\lambda$  and a according to information criterion of choice. For a given  $\hat{\beta}$  the information criterion is calculated as

$$\log(\sum_{i=1}^{n} w_{i} \rho_{\tau}(y_{i} - x_{i}^{\top} \hat{\beta})) + d * b/(2n),$$

where d is the number of nonzero coefficients and b depends on the method used. For AIC b=2, for BIC b=log(n) and for PBIC d=log(n)\*log(p) where p is the dimension of  $\hat{\beta}$ . If septau set to FALSE then calculations are made across the quantiles. Let  $\hat{\beta}^q$  be the coefficient vector for the qth quantile of Q quantiles. In addition let  $d_q$  and  $b_q$  be d and b values from the qth quantile model. Note, for all of these we are assuming eqn and a are the same. Then the summary across all quantiles is

$$\sum_{q=1}^{Q} w_q [\log(\sum_{i=1}^{n} m_i \rho_{\tau} (y_i - x_i^{\top} \hat{\beta}^q)) + d_q * b_q / (2n)],$$

where  $w_q$  is the weight assigned for the qth quantile model.

qic.select.rq.pen.seq 15

#### Usage

```
qic.select(obj, ...)
```

### **Arguments**

obj A rq.pen.seq or rq.pen.seq.cv object.

. . . Additional arguments see qic.select.rq.pen.seq() or qic.select.rq.pen.seq.cv() for more information.

#### Value

Returns a qic.select object.

### Author(s)

Ben Sherwood, <ben. sherwood@ku.edu>

#### References

Lee ER, Noh H, Park BU (2014). "Model Selection via Bayesian Information Criterion for Quantile Regression Models." *Journal of the American Statistical Association*, **109**(505), 216–229. ISSN 01621459.

#### **Examples**

```
set.seed(1)
x <- matrix(runif(800),ncol=8)
y <- 1 + x[,1] + x[,8] + (1+.5*x[,3])*rnorm(100)
m1 <- rq.pen(x,y,penalty="ENet",a=c(0,.5,1),tau=c(.25,.75))
qic.select(m1)</pre>
```

qic.select.rq.pen.seq Select tuning parameters using IC

# Description

Selects tuning parameter  $\lambda$  and a according to information criterion of choice. For a given  $\hat{\beta}$  the information criterion is calculated as

$$\log(\sum_{i=1}^{n} w_{i} \rho_{\tau}(y_{i} - x_{i}^{\top} \hat{\beta})) + d * b/(2n),$$

where d is the number of nonzero coefficients and b depends on the method used. For AIC b=2, for BIC b=log(n) and for PBIC d=log(n)\*log(p) where p is the dimension of  $\hat{\beta}$ . If septau set to FALSE then calculations are made across the quantiles. Let  $\hat{\beta}^q$  be the coefficient vector for the qth quantile of Q quantiles. In addition let  $d_q$  and  $b_q$  be d and b values from the qth quantile

16 qic.select.rq.pen.seq

model. Note, for all of these we are assuming eqn and a are the same. Then the summary across all quantiles is

$$\sum_{q=1}^{Q} w_q [\log(\sum_{i=1}^{n} m_i \rho_{\tau} (y_i - x_i^{\top} \hat{\beta}^q)) + d_q * b_q / (2n)],$$

where  $w_q$  is the weight assigned for the qth quantile model.

# Usage

```
## S3 method for class 'rq.pen.seq'
qic.select(
  obj,
  method = c("BIC", "AIC", "PBIC"),
  septau = ifelse(obj$penalty != "gq", TRUE, FALSE),
  tauWeights = NULL,
  ...
)
```

# Arguments

obj A rq.pen.seq or rq.pen.seq.cv object.

method Choice of BIC, AIC or PBIC, a large p BIC.

septau If optimal values of  $\lambda$  and a can vary with  $\tau$ . Default is TRUE.

tauWeights Weights for each quantile. Useful if you set septau to FALSE but want different

weights for the different quantiles. If not specified default is to have  $w_q = 1$  for

all quantiles.

... Additional arguments.

# Value

coefficients Coefficients of the selected models.

ic Information criterion values for all considered models.

**modelsInfo** Model info for the selected models related to the original object obj.

gic Information criterion summarized across all quantiles. Only returned if septau set to FALSE

### Author(s)

Ben Sherwood, <ben.sherwood@ku.edu>

# References

Lee ER, Noh H, Park BU (2014). "Model Selection via Bayesian Information Criterion for Quantile Regression Models." *Journal of the American Statistical Association*, **109**(505), 216–229. ISSN 01621459.

qic.select.rq.pen.seq.cv 17

### **Examples**

```
set.seed(1)
x <- matrix(runif(800),ncol=8)
y <- 1 + x[,1] + x[,8] + (1+.5*x[,3])*rnorm(100)
m1 <- rq.pen(x,y,penalty="ENet",a=c(0,.5,1),tau=c(.25,.75))
qic.select(m1)</pre>
```

```
qic.select.rq.pen.seq.cv
```

Select tuning parameters using IC

# **Description**

Selects tuning parameter  $\lambda$  and a according to information criterion of choice. For a given  $\hat{\beta}$  the information criterion is calculated as

$$\log(\sum_{i=1}^{n} w_{i} \rho_{\tau}(y_{i} - x_{i}^{\top} \hat{\beta})) + d * b/(2n),$$

where d is the number of nonzero coefficients and b depends on the method used. For AIC b=2, for BIC b=log(n) and for PBIC d=log(n)\*log(p) where p is the dimension of  $\hat{\beta}$ . If septau set to FALSE then calculations are made across the quantiles. Let  $\hat{\beta}^q$  be the coefficient vector for the qth quantile of Q quantiles. In addition let  $d_q$  and  $b_q$  be d and b values from the qth quantile model. Note, for all of these we are assuming eqn and a are the same. Then the summary across all quantiles is

$$\sum_{q=1}^{Q} w_q [\log(\sum_{i=1}^{n} \rho_{\tau}(y_i - x_i^{\top} \hat{\beta}^q)) + d_q * b_q/(2n)],$$

where  $w_q$  is the weight assigned for the qth quantile model.

#### Usage

```
## S3 method for class 'rq.pen.seq.cv'
qic.select(
  obj,
  method = c("BIC", "AIC", "PBIC"),
  septau = ifelse(obj$fit$penalty != "gq", TRUE, FALSE),
  weights = NULL,
   ...
)
```

### **Arguments**

obj A rq.pen.seq.cv object.

method Choice of BIC, AIC or PBIC, a large p BIC.

septau If optimal values of  $\lambda$  and a can vary with  $\tau$ . Default is TRUE.

rq.gq.pen

weights Weights for each quantile. Useful if you set septau to FALSE but want different

weights for the different quantiles. If not specified default is to have  $w_q=1$  for

all quantiles.

. . . Additional arguments.

### Value

coefficients Coefficients of the selected models.

ic Information criterion values for all considered models.

modelsInfo Model info for the selected models related to the original object obj.

gic Information criterion summarized across all quantiles. Only returned if septau set to FALSE

#### Author(s)

Ben Sherwood, <ben. sherwood@ku.edu>

#### References

Lee ER, Noh H, Park BU (2014). "Model Selection via Bayesian Information Criterion for Quantile Regression Models." *Journal of the American Statistical Association*, **109**(505), 216–229. ISSN 01621459.

### **Examples**

```
set.seed(1)
x <- matrix(runif(800),ncol=8)
y <- 1 + x[,1] + x[,8] + (1+.5*x[,3])*rnorm(100)
m1 <- rq.pen.cv(x,y,penalty="ENet",a=c(0,.5,1),tau=c(.25,.75))
qic.select(m1)</pre>
```

rq.gq.pen

Title Quantile regression estimation and consistent variable selection across multiple quantiles

# Description

Uses the group lasso penalty across the quantiles to provide consistent selection across all, K, modeled quantiles. Let  $\beta^q$  be the coefficients for the kth quantiles,  $\beta_j$  be the Q-dimensional vector of the jth coefficient for each quantile, and  $\rho_{\tau}(u)$  is the quantile loss function. The method minimizes

$$\sum_{q=1}^{Q} \frac{1}{n} \sum_{i=1}^{n} m_i \rho_{\tau} (y_i - x_i^{\top} \beta^q) + \lambda \sum_{j=1}^{p} ||\beta_j||_{2,w}.$$

Uses a Huber approximation in the fitting of model, as presented in Sherwood and Li (2022). Where,

$$||\beta_j||_{2,w} = \sqrt{\sum_{k=1}^K w_k v_j \beta_{kj}^2},$$

rq.gq.pen

where  $w_k$  is a quantile weight that can be specified by tau.penalty.factor,  $v_j$  is a predictor weight that can be assigned by penalty.factor, and  $m_i$  is an observation weight that can be set by weights.

#### Usage

```
rq.gq.pen(
  Х,
  у,
  tau,
  lambda = NULL,
  nlambda = 100,
  eps = ifelse(nrow(x) < ncol(x), 0.01, 0.001),
 weights = NULL,
  penalty.factor = NULL,
  scalex = TRUE,
  tau.penalty.factor = NULL,
  gmma = 0.2,
 max.iter = 200,
  lambda.discard = TRUE,
  converge.eps = 1e-04,
  beta0 = NULL
)
```

### **Arguments**

x covariate matrix

y a univariate response variable

tau a sequence of quantiles to be modeled, must be of at least length 3.

lambda shrinkage parameter. Default is NULL, and the algorithm provides a solution

path.

nlambda Number of lambda values to be considered.

eps If not pre-specified the lambda vector will be from lambda\_max to lambda\_max

times eps

weights observation weights. Default is NULL, which means equal weights.

penalty.factor weights for the shrinkage parameter for each covariate. Default is equal weight.

scalex Whether x should be scaled before fitting the model. Coefficients are returned

on the original scale.

tau.penalty.factor

weights for different quantiles. Default is equal weight.

gmma tuning parameter for the Huber loss

max.iter maximum number of iteration. Default is 200.

lambda.discard Default is TRUE, meaning that the solution path stops if the relative deviance

changes sufficiently small. It usually happens near the end of solution path. However, the program returns at least 70 models along the solution path.

20 rq.gq.pen

converge.eps The epsilon level convergence. Default is 1e-4.

beta0 Initial estimates. Default is NULL, and the algorithm starts with the intercepts

being the quantiles of response variable and other coefficients being zeros.

#### Value

An rq.pen.seq object.

models: A list of each model fit for each tau and a combination.

n: Sample size.

**p:** Number of predictors.

alg: Algorithm used. Options are "huber" or any method implemented in rq(), such as "br".

tau: Quantiles modeled.

a: Tuning parameters a used.

modelsInfo: Information about the quantile and a value for each model.

**lambda:** Lambda values used for all models. If a model has fewer coefficients than lambda, say k. Then it used the first k values of lambda. Setting lambda.discard to TRUE will gurantee all values use the same lambdas, but may increase computational time noticeably and for little gain.

penalty: Penalty used.

call: Original call.

Each model in the models list has the following values.

coefficients: Coefficients for each value of lambda.

rho: The unpenalized objective function for each value of lambda.

**PenRho:** The penalized objective function for each value of lambda.

nzero: The number of nonzero coefficients for each value of lambda.

tau: Quantile of the model.

**a:** Value of a for the penalized loss function.

#### Author(s)

Shaobo Li and Ben Sherwood, <ben. sherwood@ku.edu>

#### References

Wang M, Kang X, Liang J, Wang K, Wu Y (2024). "Heteroscedasticity identification and variable selection via multiple quantile regression." *Journal of Statistical Computation and Simulation*, **94**(2), 297-314.

Sherwood B, Li S (2022). "Quantile regression feature selection and estimation with grouped variables using Huber approximation." *Statistics and Computing*, **32**(5), 75.

rq.gq.pen.cv 21

#### **Examples**

```
## Not run:
n<- 200
p<- 10
X<- matrix(rnorm(n*p),n,p)
y<- -2+X[,1]+0.5*X[,2]-X[,3]-0.5*X[,7]+X[,8]-0.2*X[,9]+rt(n,2)
taus <- seq(0.1, 0.9, 0.2)
fit<- rq.gq.pen(X, y, taus)
#use IC to select best model, see rq.gq.pen.cv() for a cross-validation approach
qfit <- qic.select(fit)
## End(Not run)</pre>
```

rq.gq.pen.cv

Title Cross validation for consistent variable selection across multiple quantiles.

### Description

Title Cross validation for consistent variable selection across multiple quantiles.

# Usage

```
rq.gq.pen.cv(
  x = NULL,
  y = NULL,
  tau = NULL,
  lambda = NULL,
  nfolds = 10,
  cvFunc = c("rq", "se"),
  tauWeights = NULL,
  foldid = NULL,
  printProgress = FALSE,
  weights = NULL,
  ...
)
```

#### **Arguments**

x covariate matrix. Not needed if model\_obj is supplied. y univariate response. Not needed if model\_obj is supplied. tau a sequence of tau to be modeled, must be at least of length 3. lambda Values of  $\lambda$ . Default will automatically select the  $\lambda$  values.

nfolds number of folds

cvFunc loss function to be evaluated for cross-validation. Supported loss functions in-

clude quantile ("rq") and squared loss("se"). Default is the quantile loss.

22 rq.gq.pen.cv

tauWeights weights for different quantiles in calculating the cv error. Default is equal

weight.

foldid indices of pre-split testing obervations

printProgress If set to TRUE prints which partition is being worked on.

weights Weights for the quantile loss objective function.

... other arguments for rq.gq.pen.cv sent to rq.gq.pen

#### **Details**

Let  $y_{b,i}$  and  $x_{b,i}$  index the observations in fold b. Let  $\hat{\beta}_{\tau,a,\lambda}^{-b}$  be the estimator for a given quantile and tuning parameters that did not use the bth fold. Let  $n_b$  be the number of observations in fold b. Then the cross validation error for fold b is

$$CV(b,\tau) = \sum_{q=1}^{Q} \frac{1}{n_b} \sum_{i=1}^{n_b} m_{b,i} v_q \rho_{\tau} (y_{b,i} - x_{b,i}^{\top} \hat{\beta}_{\tau_q,a,\lambda}^{-b}).$$

Where,  $m_{b,i}$  is the weight for the ith observation in fold b and  $v_q$  is a quantile specific weight. Note that  $\rho_{\tau}()$  can be replaced squared error loss. Provides results about how the average of the cross-validation error changes with  $\lambda$ . Uses a Huber approximation in the fitting of model, as presented in Sherwood and Li (2022).

#### Value

An rq.pen.seq.cv object.

**cverr:** Matrix of cvSummary function, default is average, cross-validation error for each model, tau and a combination, and lambda.

cvse: Matrix of the standard error of cverr foreach model, tau and a combination, and lambda.

fit: The rq.pen.seq object fit to the full data.

**btr:** Let blank, unlike rq.pen.seq.cv() or rq.group.pen.cv(), because optmizes the quantiles individually does not make sense with this penalty.

**gtr:** A data.table for the combination of a and lambda that minimize the cross validation error across all tau.

geve: Group, across all quantiles, cross-validation error results for each value of a and lambda.

call: Original call to the function.

### Author(s)

Shaobo Li and Ben Sherwood, <ben.sherwood@ku.edu>

#### References

Wang M, Kang X, Liang J, Wang K, Wu Y (2024). "Heteroscedasticity identification and variable selection via multiple quantile regression." *Journal of Statistical Computation and Simulation*, **94**(2), 297-314.

Sherwood B, Li S (2022). "Quantile regression feature selection and estimation with grouped variables using Huber approximation." *Statistics and Computing*, **32**(5), 75.

rq.group.pen 23

#### **Examples**

```
## Not run:
n<- 200
p<- 10
X<- matrix(rnorm(n*p),n,p)
y<- -2+X[,1]+0.5*X[,2]-X[,3]-0.5*X[,7]+X[,8]-0.2*X[,9]+rt(n,2)
taus <- seq(0.1, 0.9, 0.2)
cvfit<- rq.gq.pen.cv(x=X, y=y, tau=taus)
cvCoefs <- coefficients(cvfit)
## End(Not run)</pre>
```

rq.group.pen

Fits quantile regression models using a group penalized objective function.

### Description

Let the predictors be divided into G groups with G corresponding vectors of coefficients,  $\beta_1, \ldots, \beta_G$ . Let  $\rho_{\tau}(a) = a[\tau - I(a < 0)]$ . Fits quantile regression models for Q quantiles by minimizing the penalized objective function of

$$\sum_{q=1}^{Q} \frac{1}{n} \sum_{i=1}^{n} m_i \rho_{\tau}(y_i - x_i^{\top} \beta^q) + \sum_{q=1}^{Q} \sum_{g=1}^{G} P(||\beta_g^q||_k, w_q * v_j * \lambda, a).$$

Where  $w_q$  and  $v_j$  are designated by penalty.factor and tau.penalty.factor respectively and  $m_i$  can be set by weights. The value of k is chosen by norm. Value of P() depends on the penalty. Briefly, but see references or vignette for more details,

Group LASSO (gLASSO)  $P(||\beta||_k, \lambda, a) = \lambda ||\beta||_k$ 

**Group SCAD**  $P(||\beta||_k, \lambda, a) = SCAD(||\beta||_k, \lambda, a)$ 

**Group MCP**  $P(||\beta||_k, \lambda, a) = MCP(||\beta||_k, \lambda, a)$ 

Group Adaptive LASSO  $P(||\beta||_k, \lambda, a) = \frac{\lambda ||\beta||_k}{|\beta_0|^a}$ 

Note if k=1 and the group lasso penalty is used then this is identical to the regular lasso and thus function will stop and suggest that you use rq.pen() instead. For Adaptive LASSO the values of  $\beta_0$  come from a Ridge solution with the same value of  $\lambda$ . If the Huber algorithm is used than  $\rho_{\tau}(y_i-x_i^{\top}\beta)$  is replaced by a Huber-type approximation. Specifically, it is replaced by  $h_{\gamma}^{\tau}(y_i-x_i^{\top}\beta)/2$  where

$$h_{\gamma}^{\tau}(a) = a^2/(2\gamma)I(|a| \le \gamma) + (|a| - \gamma/2)I(|a| > \gamma) + (2\tau - 1)a.$$

Where if  $\tau = .5$ , we get the usual Huber loss function.

24 rq.group.pen

# Usage

```
rq.group.pen(
 х,
 у,
 tau = 0.5,
 groups = 1:ncol(x),
 penalty = c("gLASSO", "gAdLASSO", "gSCAD", "gMCP"),
 lambda = NULL,
 nlambda = 100,
 eps = ifelse(nrow(x) < ncol(x), 0.05, 0.01),
 alg = c("huber", "br"),
  a = NULL,
 norm = 2,
 group.pen.factor = NULL,
  tau.penalty.factor = rep(1, length(tau)),
  scalex = TRUE,
 coef.cutoff = 1e-08,
 max.iter = 5000,
 converge.eps = 1e-04,
 gamma = IQR(y)/10,
 lambda.discard = TRUE,
 weights = NULL,
)
```

# Arguments

X	Matrix of predictors.	
у	Vector of responses.	
tau	Vector of quantiles.	
groups	Vector of group assignments for predictors.	
penalty	Penalty used, choices are group lasso ("gLASSO"), group adaptive lasso ("gAd-LASSO"), group SCAD ("gSCAD") and group MCP ("gMCP")	
lambda	Vector of lambda tuning parameters. Will be autmoatically generated if it is not set.	
nlambda	The number of lambda tuning parameters.	
eps	The value to be multiplied by the largest lambda value to determine the smallest lambda value.	
alg	Algorithm used. Choices are Huber approximation ("huber") or linear programming ("lp").	
a	The additional tuning parameter for adaptive lasso, SCAD and MCP.	
norm	Whether a L1 or L2 norm is used for the grouped coefficients.	
group.pen.factor		
	Penalty factor for each group. Default is 1 for all groups if norm-1 and square	

Penalty factor for each group. Default is 1 for all groups if norm=1 and square root of group size if norm=2.

rq.group.pen 25

tau.penalty.factor

Penalty factor for each quantile.

scalex Whether X should be centered and scaled so that the columns have mean zero

and standard deviation of one. If set to TRUE, the coefficients will be returned

to the original scale of the data.

coef.cutoff Coefficient cutoff where any value below this number is set to zero. Useful

for the lp algorithm, which are prone to finding almost, but not quite, sparse

solutions.

max.iter The maximum number of iterations for the algorithm.

converge.eps The convergence criteria for the algorithms. gamma The tuning parameter for the Huber loss.

lambda.discard Whether lambdas should be discarded if for small values of lambda there is very

little change in the solutions.

weights Weights used in the quantile loss objective function.

... Additional parameters

#### Value

An rq.pen.seq object.

models A list of each model fit for each tau and a combination.

n Sample size.

**p** Number of predictors.

alg Algorithm used.

tau Quantiles modeled.

penalty Penalty used.

a Tuning parameters a used.

**lambda** Lambda values used for all models. If a model has fewer coefficients than lambda, say k. Then it used the first k values of lambda. Setting lambda.discard to TRUE will gurantee all values use the same lambdas, but may increase computational time noticeably and for little gain.

modelsInfo Information about the quantile and a value for each model.

call Original call.

Each model in the models list has the following values.

coefficients Coefficients for each value of lambda.

**rho** The unpenalized objective function for each value of lambda.

**PenRho** The penalized objective function for each value of lambda. If scalex=TRUE then this is the value for the scaled version of x.

**nzero** The number of nonzero coefficients for each value of lambda.

tau Quantile of the model.

a Value of a for the penalized loss function.

26 rq.group.pen.cv

#### Author(s)

Ben Sherwood, <ben.sherwood@ku.edu>, Shaobo Li <shaobo.li@ku.edu> and Adam Maidman

#### References

Peng B, Wang L (2015). "An iterative coordinate descent algorithm for high-dimensional nonconvex penalized quantile regression." *J. Comput. Graph. Statist.*, **24**(3), 676-694.

# **Examples**

```
## Not run:
set.seed(1)
x <- matrix(rnorm(200*8,sd=1),ncol=8)
y <- 1 + x[,1] + 3*x[,3] - x[,8] + rt(200,3)
g <- c(1,1,1,2,2,2,3,3)
tvals <- c(.25,.75)
r1 <- rq.group.pen(x,y,groups=g)
r5 <- rq.group.pen(x,y,groups=g,tau=tvals)
#Linear programming approach with group SCAD penalty and L1-norm
m2 <- rq.group.pen(x,y,groups=g,alg="br",penalty="gSCAD",norm=1,a=seq(3,4))
# No penalty for the first group
m3 <- rq.group.pen(x,y,groups=g,group.pen.factor=c(0,rep(1,2)))
# Smaller penalty for the median
m4 <- rq.group.pen(x,y,groups=g,tau=c(.25,.5,.75),tau.penalty.factor=c(1,.25,1))
## End(Not run)</pre>
```

rq.group.pen.cv

Performs cross validation for a group penalty.

# Description

Performs cross validation for a group penalty.

#### Usage

```
rq.group.pen.cv(
    x,
    y,
    tau = 0.5,
    groups = 1:ncol(x),
    lambda = NULL,
    a = NULL,
    cvFunc = NULL,
    nfolds = 10,
    foldid = NULL,
    groupError = TRUE,
    cvSummary = mean,
```

rq.group.pen.cv 27

```
tauWeights = rep(1, length(tau)),
printProgress = FALSE,
weights = NULL,
...
)
```

#### **Arguments**

x Matrix of predictors.y Vector of responses.tau Vector of quantiles.

groups Vector of group assignments for the predictors.

lambda Vector of lambda values, if set to NULL they will be generated automatically.

a Vector of the other tuning parameter values.

cvFunc Function used for cross-validation error, default is quantile loss.

nfolds Number of folds used for cross validation.

foldid Fold assignments, if not set this will be randomly created.

groupError If errors are to be reported as a group or as the average for each fold.

cvSummary The

tauWeights Weights for the tau penalty only used in group tau results (gtr). printProgress If set to TRUE will print which fold the process is working on.

weights Weights for the quantile loss function. Used in both model fitting and cross-

validation.

... Additional parameters that will be sent to rq.group.pen().

# **Details**

Two cross validation results are returned. One that considers the best combination of a and lambda for each quantile. The second considers the best combination of the tuning parameters for all quantiles. Let  $y_{b,i}$ ,  $x_{b,i}$ , and  $m_{b,i}$  index the response, predictors, and weights of observations in fold b. Let  $\hat{\beta}_{\tau,a,\lambda}^{-b}$  be the estimator for a given quantile and tuning parameters that did not use the bth fold. Let  $n_b$  be the number of observations in fold b. Then the cross validation error for fold b is

$$\text{CV}(b,\tau) = \frac{1}{n_b} \sum_{i=1}^{n_b} m_{b,i} \rho_{\tau} (y_{b,i} - x_{b,i}^{\top} \hat{\beta}_{\tau,a,\lambda}^{-b}).$$

Note that  $\rho_{\tau}()$  can be replaced by a different function by setting the cvFunc parameter. The function returns two different cross-validation summaries. The first is btr, by tau results. It provides the values of lambda and a that minimize the average, or whatever function is used for cvSummary, of CV(b). In addition it provides the sparsest solution that is within one standard error of the minimum results.

The other approach is the group tau results, gtr. Consider the case of estimating Q quantiles of  $\tau_1, \ldots, \tau_Q$  with quantile (tauWeights) of  $v_q$ . The gtr returns the values of lambda and a that minimizes the average, or again whatever function is used for cvSummary, of

$$\sum_{q=1}^{Q} v_q \text{CV}(b, \tau_q).$$

If only one quantile is modeled then the gtr results can be ignored as they provide the same minimum solution as btr.

#### Value

An rq.pen.seq.cv object.

**cverr** Matrix of cvSummary function, default is average, cross-validation error for each model, tau and a combination, and lambda.

cvse Matrix of the standard error of everr foreach model, tau and a combination, and lambda.

fit The rq.pen.seq object fit to the full data.

**btr** A data.table of the values of a and lambda that are best as determined by the minimum cross validation error and the one standard error rule, which fixes a. In btr the values of lambda and a are selected seperately for each quantile.

**gtr** A data.table for the combination of a and lambda that minimize the cross validation error across all tau.

geve Group, across all quantiles, cross-validation error results for each value of a and lambda.

call Original call to the function.

#### Author(s)

Ben Sherwood, <ben. sherwood@ku.edu> and Shaobo Li <shaobo.li@ku.edu>

# Examples

```
set.seed(1)
x <- matrix(rnorm(100*8,sd=1),ncol=8)
y <- 1 + x[,1] + 3*x[,3] - x[,8] + rt(100,3)
g <- c(1,1,1,1,2,2,3,3)
tvals <- c(.25,.75)
## Not run:
m1 <- rq.group.pen.cv(x,y,tau=c(.1,.3,.7),groups=g)
m2 <- rq.group.pen.cv(x,y,penalty="gAdLASSO",tau=c(.1,.3,.7),groups=g)
m3 <- rq.group.pen.cv(x,y,penalty="gSCAD",tau=c(.1,.3,.7),a=c(3,4,5),groups=g)
m4 <- rq.group.pen.cv(x,y,penalty="gMCP",tau=c(.1,.3,.7),a=c(3,4,5),groups=g)
## End(Not run)</pre>
```

#### **Description**

Let q index the Q quantiles of interest. Let  $\rho_{\tau}(a) = a[\tau - I(a < 0)]$ . Fits quantile regression models by minimizing the penalized objective function of

$$\frac{1}{n} \sum_{q=1}^{Q} \sum_{i=1}^{n} m_i \rho_{\tau}(y_i - x_i^{\top} \beta^q) + \sum_{q=1}^{Q} \sum_{j=1}^{p} P(\beta_p^q, w_q * v_j * \lambda, a).$$

Where  $w_q$  and  $v_j$  are designated by penalty.factor and tau.penalty.factor respectively, and  $m_i$  is designated by weights. Value of P() depends on the penalty. See references or vignette for more details,

**LASSO:**  $P(\beta, \lambda, a) = \lambda |\beta|$ 

**SCAD:**  $P(\beta, \lambda, a) = SCAD(\beta, \lambda, a)$ 

**MCP:**  $P(\beta, \lambda, a) = MCP(\beta, \lambda, a)$ 

**Ridge:**  $P(\beta, \lambda, a) = \lambda \beta^2$ 

**Elastic Net:**  $P(\beta, \lambda, a) = a * \lambda |\beta| + (1 - a) * \lambda * \beta^2$ 

**Adaptive LASSO:**  $P(\beta, \lambda, a) = \frac{\lambda |\beta|}{|\beta_0|^a}$ 

For Adaptive LASSO the values of  $\beta_0$  come from a Ridge solution with the same value of  $\lambda$ . Three different algorithms are implemented

huber: Uses a Huber approximation of the quantile loss function. See Yi and Huang 2017 for more details.

**br:** Solution is found by re-formulating the problem so it can be solved with the rq() function from quantreg with the br algorithm.

The huber algorithm offers substantial speed advantages without much, if any, loss in performance. However, it should be noted that it solves an approximation of the quantile loss function.

#### Usage

```
rq.pen(
 х,
 у,
  tau = 0.5,
 lambda = NULL,
 penalty = c("LASSO", "Ridge", "ENet", "aLASSO", "SCAD", "MCP"),
 a = NULL,
 nlambda = 100,
  eps = ifelse(nrow(x) < ncol(x), 0.05, 0.01),
 penalty.factor = rep(1, ncol(x)),
  alg = c("huber", "br", "fn"),
  scalex = TRUE,
  tau.penalty.factor = rep(1, length(tau)),
  coef.cutoff = 1e-08,
 max.iter = 5000,
 converge.eps = 1e-04,
```

```
lambda.discard = TRUE,
weights = NULL,
...
)
```

#### **Arguments**

x matrix of predictorsy vector of responsestau vector of quantiles

lambda vector of lambda, if not set will be generated automatically

penalty choice of penalty

Additional tuning parameter, not used for lasso or ridge penalties. However,

will be set to the elastic net values of 1 and 0 respectively. Defaults are ENet(0),

aLASSO(1), SCAD(3.7) and MCP(3).

nlambda number of lambda, ignored if lambda is set

eps If not pre-specified the lambda vector will be from lambda\_max to lambda\_max

times eps

penalty.factor penalty factor for the predictors

alg Algorithm used.

scalex Whether x should be scaled before fitting the model. Coefficients are returned

on the original scale.

tau.penalty.factor

A penalty factor for each quantile.

coef.cutoff Some of the linear programs will provide very small, but not sparse solutions.

Estimates below this number will be set to zero. This is ignored if a non-linear

programming algorithm is used.

max.iter Maximum number of iterations of non-linear programming algorithms.

converge.eps Convergence threshold for non-linear programming algorithms.

lambda.discard Algorithm may stop for small values of lambda if the coefficient estimates are

not changing drastically. One example of this is it is possible for the LLA weights of the non-convex functions to all become zero and smaller values of

lambda are extremely likely to produce the same zero weights.

weights Weights for the quantile objective function.

... Extra parameters.

# Value

An rq.pen.seq object.

models: A list of each model fit for each tau and a combination.

n: Sample size.

**p:** Number of predictors.

alg: Algorithm used. Options are "huber" or any method implemented in rq(), such as "br".

tau: Quantiles modeled.

a: Tuning parameters a used.

modelsInfo: Information about the quantile and a value for each model.

**lambda:** Lambda values used for all models. If a model has fewer coefficients than lambda, say k. Then it used the first k values of lambda. Setting lambda.discard to TRUE will gurantee all values use the same lambdas, but may increase computational time noticeably and for little gain.

penalty: Penalty used.

call: Original call.

Each model in the models list has the following values.

coefficients: Coefficients for each value of lambda.

**rho:** The unpenalized objective function for each value of lambda.

**PenRho:** The penalized objective function for each value of lambda. If scalex=TRUE then this is the value for the scaled version of x.

**nzero:** The number of nonzero coefficients for each value of lambda.

tau: Quantile of the model.

a: Value of a for the penalized loss function.

If the Huber algorithm is used than  $\rho_{\tau}(y_i - x_i^{\top}\beta)$  is replaced by a Huber-type approximation. Specifically, it is replaced by  $h_{\gamma}^{\tau}(y_i - x_i^{\top}\beta)/2$  where

$$h_{\gamma}^{\tau}(a) = a^2/(2\gamma)I(|a| \le \gamma) + (|a| - \gamma/2)I(|a| > \gamma) + (2\tau - 1)a.$$

Where if  $\tau = .5$ , we get the usual Huber loss function. The Huber implementation calls the package hqreg which implements the methods of Yi and Huang (2017) for Huber loss with elastic net penalties. For non-elastic net penalties the LLA algorithm of Zou and Li (2008) is used to approximate those loss functions with a lasso penalty with different weights for each predictor.

#### Author(s)

Ben Sherwood, <ben. sherwood@ku.edu>, Shaobo Li, and Adam Maidman

#### References

Zou H, Li R (2008). "One-step sparse estimates in nonconcave penalized likelihood models." *Ann. Statist.*, **36**(4), 1509-1533.

Yi C, Huang J (2017). "Semismooth Newton Coordinate Descent Algorithm for Elastic-Net Penalized Huber Loss Regression and Quantile Regression." *J. Comput. Graph. Statist.*, **26**(3), 547-557.

Belloni A, Chernozhukov V (2011). "L1-Penalized quantile regression in high-dimensional sparse models." *Ann. Statist.*, **39**(1), 82-130.

Peng B, Wang L (2015). "An iterative coordinate descent algorithm for high-dimensional nonconvex penalized quantile regression." *J. Comput. Graph. Statist.*, **24**(3), 676-694.

rq.pen.cv

### **Examples**

```
n <- 200 p <- 8   
x <- matrix(runif(n*p),ncol=p)  
y <- 1 + x[,1] + x[,8] + (1+.5*x[,3])*rnorm(n)  
r1 <- rq.pen(x,y) #Lasso fit for median  
# Lasso for multiple quantiles  
r2 <- rq.pen(x,y,tau=c(.25,.5,.75))  
# Elastic net fit for multiple quantiles, which must use Huber algorithm  
r3 <- rq.pen(x,y,penalty="ENet",a=c(0,.5,1),alg="huber")  
# First variable is not penalized  
r4 <- rq.pen(x,y,penalty.factor=c(0,rep(1,7)))  
tvals <- c(.1,.2,.3,.4,.5)  
#Similar to penalty proposed by Belloni and Chernouzhukov.  
#To be exact you would divide the tau.penalty.factor by n.  
r5 <- rq.pen(x,y,tau=tvals, tau.penalty.factor=sqrt(tvals*(1-tvals)))
```

rq.pen.cv

Does k-folds cross validation for rq.pen. If multiple values of a are specified then does a grid based search for best value of  $\lambda$  and a.

#### **Description**

Does k-folds cross validation for rq.pen. If multiple values of a are specified then does a grid based search for best value of  $\lambda$  and a.

### Usage

```
rq.pen.cv(
 х,
 у,
  tau = 0.5,
  lambda = NULL,
 penalty = c("LASSO", "Ridge", "ENet", "aLASSO", "SCAD", "MCP"),
  a = NULL,
  cvFunc = NULL,
  nfolds = 10,
  foldid = NULL,
  nlambda = 100,
  groupError = TRUE,
  cvSummary = mean,
  tauWeights = rep(1, length(tau)),
  printProgress = FALSE,
 weights = NULL,
)
```

rq.pen.cv 33

#### **Arguments**

X	Matrix of predictors.
у	Vector of responses.
tau	Quantiles to be modeled.
lambda	Values of $\lambda$ . Default will automatically select the $\lambda$ values.
penalty	Choice of penalty between LASSO, Ridge, Elastic Net (ENet), Adaptive Lasso (aLASSO), SCAD and MCP.
a	Tuning parameter of a. LASSO and Ridge has no second tuning parameter, but for notation is set to 1 or 0 respectively, the values for elastic net. Defaults are Ridge () $\frac{1}{2}$
cvFunc	Loss function for cross-validation. Defaults to quantile loss, but user can specify their own function.
nfolds	Number of folds.
foldid	Ids for folds. If set will override nfolds.
nlambda	Number of lambda, ignored if lambda is set.
groupError	If set to false then reported error is the sum of all errors, not the sum of error for each fold.
cvSummary	Function to summarize the errors across the folds, default is mean. User can specify another function, such as median.
tauWeights	Weights for the different tau models. Only used in group tau results (gtr).
printProgress	If set to TRUE prints which partition is being worked on.

### **Details**

weights

. . .

Two cross validation results are returned. One that considers the best combination of a and lambda for each quantile. The second considers the best combination of the tuning parameters for all quantiles. Let  $y_{b,i}$ ,  $x_{b,i}$ , and  $m_{b,i}$  index the response, predictors, and weights of observations in fold b. Let  $\hat{\beta}_{\tau,a,\lambda}^{-b}$  be the estimator for a given quantile and tuning parameters that did not use the bth fold. Let  $n_b$  be the number of observations in fold b. Then the cross validation error for fold b is

Weights for the quantile loss objective function.

Additional arguments passed to rq.pen()

$$\mathrm{CV}(b,\tau) = \frac{1}{n_b} \sum_{i=1}^{n_b} m_{b,i} \rho_\tau(y_{b,i} - x_{b,i}^\top \hat{\beta}_{\tau,a,\lambda}^{-b}).$$

Note that  $\rho_{\tau}()$  can be replaced by a different function by setting the cvFunc parameter. The function returns two different cross-validation summaries. The first is btr, by tau results. It provides the values of lambda and a that minimize the average, or whatever function is used for cvSummary, of CV(b). In addition it provides the sparsest solution that is within one standard error of the minimum results.

The other approach is the group tau results, gtr. Consider the case of estimating Q quantiles of  $\tau_1, \ldots, \tau_Q$  with quantile (tauWeights) of  $v_q$ . The gtr returns the values of lambda and a that minimizes the average, or again whatever function is used for cvSummary, of

$$\sum_{q=1}^{Q} v_q \text{CV}(b, \tau_q).$$

34 rqPen

If only one quantile is modeled then the gtr results can be ignored as they provide the same minimum solution as btr.

#### Value

An rq.pen.seq.cv object.

**cverr:** Matrix of cvSummary function, default is average, cross-validation error for each model, tau and a combination, and lambda.

cvse: Matrix of the standard error of cverr foreach model, tau and a combination, and lambda.

**fit:** The rq.pen.seq object fit to the full data.

**btr:** A data.table of the values of a and lambda that are best as determined by the minimum cross validation error and the one standard error rule, which fixes a. In btr the values of lambda and a are selected seperately for each quantile.

**gtr:** A data.table for the combination of a and lambda that minimize the cross validation error across all tau.

geve: Group, across all quantiles, cross-validation error results for each value of a and lambda.

call: Original call to the function.

### Author(s)

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# **Examples**

```
## Not run:
x <- matrix(runif(800),ncol=8)
y <- 1 + x[,1] + x[,8] + (1+.5*x[,3])*rnorm(100)
r1 <- rq.pen.cv(x,y) #lasso fit for median
# Elastic net fit for multiple values of a and tau
r2 <- rq.pen.cv(x,y,penalty="ENet",a=c(0,.5,1),tau=c(.25,.5,.75))
#same as above but more weight given to median when calculating group cross validation error.
r3 <- rq.pen.cv(x,y,penalty="ENet",a=c(0,.5,1),tau=c(.25,.5,.75),tauWeights=c(.25,.5,.25))
# uses median cross-validation error instead of mean.
r4 <- rq.pen.cv(x,y,cvSummary=median)
#Cross-validation with no penalty on the first variable.
r5 <- rq.pen.cv(x,y,penalty.factor=c(0,rep(1,7)))
## End(Not run)</pre>
```

rqPen

rqPen: A package for estimating quantile regression models using penalized objective functions.

#### **Description**

The package estimates a quantile regression model using LASSO, Adaptive LASSO, SCAD, MCP, elastic net, and their group counterparts, with the exception of elastic net for which there is no group penalty implementation.

rqPen 35

# rqPen functions

The most important functions are rq.pen(), rq.group.pen(), rq.pen.cv() and rq.group.pen.cv(). These functions fit quantile regression models with individual or group penalties. The cv functions automate the cross-validation process for selection of tuning parameters.

# See Also

Useful links:

• https://github.com/bssherwood/rqpen/blob/master/ignore/rqPenArticle.pdf

# **Index**

```
bytau.plot, 2
bytau.plot.rq.pen.seq, 3
bytau.plot.rq.pen.seq.cv, 4
coef.rq.pen.seq, 5
coef.rq.pen.seq.cv, 6
plot.rq.pen.seq, 7
plot.rq.pen.seq.cv, 8
predict.gic.select,9
predict.rq.pen.seq, 10
predict.rq.pen.seq.cv, 11
print.qic.select, 12
print.rq.pen.seq, 13
print.rq.pen.seq.cv, 14
qic.select, 14
qic.select.rq.pen.seq, 15
qic.select.rq.pen.seq.cv, 17
rq.gq.pen, 18
rq.gq.pen.cv, 21
rq.group.pen, 23
rq.group.pen.cv, 26
rq.pen, 28
rq.pen.cv, 32
rqPen, 34
rqPen-package (rqPen), 34
```